

# IMO 2007 in Vietnam

## UK Leader's Report by Geoff Smith

The International Mathematical Olympiad is the annual world championship of secondary school mathematics. This year the UK team comprised 5 boys and one girl. They were selected on the basis of their performances in many competitions and practice examinations.

The team representing the UK this year was as follows:

Tim Hennock	Christ's Hospital
Tom Lovering	Bristol Grammar School
Takaki Oshima	Westminster School, London
Jack Shotton	Portsmouth Grammar School
Dominic Yeo	St. Paul's School, London
Alison Zhu	Simon Langton Girls' Grammar School, Canterbury

The reserves were:

Jonathan Lee	Loughborough Grammar School
Preeyan Parmar	Eton College
Julia Robson	Perse School for Girls

An army of volunteers run the mentoring schemes and camps which are the basis of the British effort. We are truly fortunate to have such a generous and talented community, supported by the professional administrative staff in the Leeds office of the United Kingdom Mathematics Trust. These days nearly three-quarters of a million students participate in our competitions, and I thank them all for supporting the organization.

The IMO has finally adjusted to the existence of the internet. This has consequences for this annual report, for there is little point carefully printing information which is freely available at a well designed web site. We must thank the organizers of IMO 2006 in Slovenia who took on the task of producing an official IMO site. They have done a tremendous job.

Here are the problems of IMO 2007:

**Problem 1.** Real numbers  $a_1, a_2, \dots, a_n$  are given. For each  $i$  ( $1 \leq i \leq n$ ) define

$$d_i = \max\{a_j : 1 \leq j \leq i\} - \min\{a_j : i \leq j \leq n\}$$

and let

$$d = \max\{d_i : 1 \leq i \leq n\}.$$

(a) Prove that, for any real numbers  $x_1 \leq x_2 \leq \dots \leq x_n$ ,

$$\max\{|x_i - a_i| : 1 \leq i \leq n\} \geq \frac{d}{2}. \quad (*)$$

(b) Show that there are real numbers  $x_1 \leq x_2 \leq \dots \leq x_n$  such that equality holds in (\*).

**Problem 2.** Consider five points  $A, B, C, D$  and  $E$  such that  $ABCD$  is a parallelogram and  $BCED$  is a cyclic quadrilateral. Let  $\ell$  be a line passing through  $A$ . Suppose that  $\ell$  intersects the interior of the segment  $DC$  at  $F$  and intersects line  $BC$  at  $G$ . Suppose also that  $EF = EG = EC$ . Prove that  $\ell$  is the bisector of angle  $DAB$ .

**Problem 3.** In a mathematical competition some competitors are friends. Friendship is always mutual. Call a group of competitors a *clique* if each two of them are friends. (In particular, any group of fewer than two competitors is a clique.) The number of members of a clique is called its *size*.

Given that, in this competition, the largest size of a clique is even, prove that the competitors can be arranged in two rooms such that the largest size of a clique contained in one room is the same as the largest size of a clique contained in the other room.

**Problem 4.** In triangle  $ABC$  the bisector of angle  $BCA$  intersects the circumcircle again at  $R$ , the perpendicular bisector of  $BC$  at  $P$ , and the perpendicular bisector of  $AC$  at  $Q$ . The midpoint of  $BC$  is  $K$  and the midpoint of  $AC$  is  $L$ . Prove that the triangles  $RPK$  and  $RQL$  have the same area.

**Problem 5.** Let  $a$  and  $b$  be positive integers. Show that if  $4ab - 1$  divides  $(4a^2 - 1)^2$ , then  $a = b$ .

**Problem 6.** Let  $n$  be a positive integer. Consider

$$S = \{(x, y, z) : x, y, z \in \{0, 1, \dots, n\}, x + y + z > 0\}$$

as a set of  $(n + 1)^3 - 1$  points in three-dimensional space. Determine the smallest possible number of planes, the union of which contains  $S$  but does not include  $(0, 0, 0)$ .

The UK performances were:

UNK1	Tim Hennock	7	0	0	7	0	0	14	Bronze
UNK2	Tom Lovering	7	2	0	7	0	0	16	Bronze
UNK3	Takaki Oshima	0	0	0	7	0	0	7	Hon Mention
UNK4	Jack Shotton	7	7	2	7	7	1	31	Gold
UNK5	Dominic Yeo	4	1	0	7	0	1	13	Hon Mention
UNK6	Alison Zhu	7	0	0	7	0	0	14	Bronze

The UK team was supported at the IMO by Deputy Leader Ceri Fiddes, Observer A Joseph Myers and Observer C Pam Hunt.

Two UK questions made the IMO shortlist this year. The excellent unselected G5 was by Christopher Bradley. Problem N6 was on the paper as Problem 5. The history of the problem is complicated, and perhaps it is best to attribute it as a joint effort by Kevin Buzzard and Edward Crane.

The running five year IMO rank order at the back of the American Booklet now has the UK at 14th, which is not so bad. In 2007 we had a relatively low ranking of equal 28th. At IMO 2007 there was something of a shock when Russia pipped China for first place. Congratulations to Nazar Agakhanov, his organization and students. Note the continuing strong performance of Italy. At the UKMT retreat the Italian Leader Roberto Dvornicich made out that their excellent showing in 2006 was a lucky chance. This was, of course, nonsense.

For all other details, I refer you to

[www.imo-official.org](http://www.imo-official.org)

It would be particularly helpful if more senior readers could look at information concerning past IMOs, and supply corrections and missing data to the organizers of this website. There was a special plea at IMO 2007 for this to happen.

## Leader's Diary

The extra cost of flying on to Australia is not that great, so we decide to repeat our 2003 scheme of adjusting to far eastern time by spending a few days in Queensland, Australia. James Cook University has a campus in Cairns where we stayed before, and had a happy time. Of course the fact that the team of 2003 performed very well may have something to do with my enthusiasm for this plan.

**July 11th** We arrive at Sydney International at 5:30am. We have a mid-afternoon internal flight to Queensland, so we take our bags over to the domestic terminal and leave them in left luggage. We catch a double decker train to the city in order to take a ferry across Sydney harbour.

In the afternoon we take an internal flight to Cairns. We sit towards the front of the plane, mixed up with a jovial bunch of hearty Australian blokes who quickly drink the plane dry. The fun starts when the exhausted and jetlagged Alison Zhu falls fast asleep, and for a few moments uses one of the drinking party as a snuggle rug. He is a very happy man, and looks rather pleased with himself as his jealous colleagues celebrate his good fortune. The banter disturbs Alison, and she straightens up in her sleep. It turns out that this happy band of brothers are on their way to Queensland to hunt feral pigs with spear guns. They invite us to join them in the bush for some murderous fun. I am all for it, but Ceri mutters pathetically about health and safety issues and the absence of any reference to wild boar in the risk assessment document, and so I am pressed to decline. What can you expect from a vegetarian?

We go on a snorkelling trip on the Great Barrier Reef. When the time comes to enter the water, we are told to simply jump in from the slowly moving boat. I go first, and enter the water with a certain force for some reason. I was expecting this, and my body (with, thanks to Descartes, mind in tow) plunged deep beneath the surface. As team member Dominic happily pointed out, this was presumably a life-changing experience for the single celled organisms making up the reef. As photosynthesis was suddenly switched off, it must have seemed like aquatic Armageddon was imminent. The four seahorses of the apocalypse swam by. I had taken a deep breath on the way down, and without too much discomfort I eventually kicked to the surface to be greeted by a concerned professional diver. Heaven knows what must have happened to the seismological arrays which now monitor the Pacific. This was not the ideal start to a snorkelling tour, but after grabbing a float

to boost my confidence, I eventually forced my heart rate down and even started to enjoy the experience. Praise be to Archimedes.

While all this going on, Joseph Myers has gone Ocean walking. Being a man of doubtful buoyancy, he has taken the *Futurama* option. His head is inside a giant transparent bubble, and teams of Australian lackeys pump life support down plastic tubes while Joseph strolls beneath the waves. Presumably oxygen, ice cream and combinatorics problems are pumped down separate tubes by retainers.

We also went on a short trip to a local zoo, and Ceri busies herself playing housemother to vast numbers of amiable kangaroos. The most remarkable inmate is the crocodile keeper, who seems to think that he deserves his corn for poking a croc with a stick while delivering patter which only exceptionally strays from the vacuous to the contradictory. Any celebrity bestowed by Warhol should in this case be confined to 15 nanoseconds.

By way of contrast, as we are royally entertained by an erudite bus driver who takes us to Kuranda in the rainforest. His casual familiarity with the social and economic history of Cairns, and working knowledge of local natural history leaves me impressed.

From Kuranda we make a prodigious journey by cable car over the forest back to settled Australia. We are in competition for seats with an army of teenaged Americans who are on a Pacific tour as some kind of goodwill ambassadors. We never quite work out how they are selected, but apparently it is all to do with being good. This tourism leavens the diet of daily exams which form the staple of our days.

Well before IMO 2007, the shortlist of questions for IMO 2006 leaked onto the internet. This disrupted our preparations, because one of our students looked at the offending document. It seems perfectly clear who facilitated this breach of IMO protocol and etiquette. If this happens again (future organizers please note), then surely the credentials of the person or people involved should be withdrawn, and they should not be permitted to attend IMOs. Growl.

**July 17th** We all fly to Sydney to join the Australian team. In case you are not from Australia, think of Madrid. However, Sydney is in the grip of a completely untypical cold snap. We find that our accommodation at the University of New South Wales is not really adapted to antarctic living, and it is not at all funny. We arrive late in the evening and very hungry. To my astonishment, pizza delivery seems difficult. We are driven to walk the streets, and can only find that burger chain open.

**July 18th** Joseph and I are heading north towards the IMO today. The Australian team and leaders are very hospitable, and we leave amid cheery schemes to deal with the cold. Breaking up the furniture, and tossing in a petrol soaked Tom Lovering or Tim Hennock with a magnesium flare is the obvious way forward. Ceri and the Australian deputy Norm seem to have more sensible plans. Time to go. We spend the night in Singapore.

**July 19th** We head for Hanoi. At Hanoi airport team leaders and Observers A are directed to a dedicated queue. Immigration and customs formalities are dealt with at speed, and very quickly Joseph and I find ourselves in the arrivals hall. We are engulfed by charming IMO volunteers who are determined to make us feel welcome. There is much jovial banter from the Vietnamese, and for some reason the words *Father Christmas* keep being mentioned. Maybe we should go as Lapland next year. Local organizers help us to change money and purchase water. Although I have a little wad of one dollar bills (universal currency units) tucked away, it seems sensible to purchase some local currency. This is called the *đồng*, and you get about  $2^{15}$  *đồng*s to the pound. Thus a pound is 32 *kilođồng*s and in turn this means that a million *đồng*s is easy to work out and a *megađồng* is 32 pounds. There is a TV camera and much shaking of hands. The arrival hall has a mild form of air-conditioning which is good enough to stop foreigners from wandering off.

Leaders are pouring in from all over the world, and my old pal Jim Cruickshank of Ireland suddenly bursts on the scene. Now the crack can begin. He and particularly his Observer A Mark will become the engines of many an adventure. I can see that Mark is a man of destiny, and will soon thrust Jim aside in the harsh world of Irish mathematics enrichment politics.

After a while we are shown to our bus. This involves negotiating a pedestrian crossing. First of all an IMO official jumps into the road in an attempt to stop the traffic. He merely diverts the flow as vehicles swerve around him. Then a policeman steps out in an impressive uniform and raises his hand. This has an entirely similar effect. Eventually we spot a gap and make a dash for the coach. This is air-conditioned, comfortable and under-populated. By now we have discovered that the jury site is at Halong Bay. Different people give different opinions as to how long it will take, but clearly it will be several hours. Nomatter, with air-conditioning and the promise of a comfort stop en route, this will be fine.

Apart from taxis, there are not many cars on the roads. There are big buses and trucks and small motorcycles, but not much in between. It is quite

remarkable what you can carry on a motorcycle (a refrigerator for example). On a later occasion we see a young blade moving four female companions round Hanoi on his motorbike—the packing strategy was one in front and three behind, in case you are wondering.

We drive away from Hanoi and through an agricultural area. People labour beneath broad conical hats to tend rice paddies. We see many brick factories actually built in a great river. From time to time we pass through villages, each one of which has a motorcycle franchise.

The jury site is in a very luxurious hotel in Halong Bay. This coastal resort is north of Hanoi, towards the Chinese border. From the point of view of natural science, it is a celebration of exotic geology. The bay is littered with so many hemispherical islands that there is no visible route to the open sea. I first read about Halong Bay on the long drive from the airport. Someone has some astonishingly frank tourist literature which makes disparaging remarks about the particular tourist clientele which Halong Bay attracts. Actually I may be letting my prejudices show again, and the remarks may not have been intended as disparaging at all. The phrases in question concerned karaoke bars and sleazy nightclubs. I know that several members of the jury will flourish in such an environment.

It is always a concern to protect the IMO shortlist, and especially the IMO paper, from unauthorized access. There are contrasting attitudes around the world. Last year the postmodern Slovenian organizers took the view that given the state of modern communications, the only possible defence of the IMO paper is the collective honesty of the jury and co-ordinators. This year the Vietnamese organizers take a different view. The hotel is surrounded by the army and no-one is allowed out of the hotel. In order to give a relaxed and friendly impression, the soldiers are only carrying handguns. We have access to an emasculated version of the internet which allows us to trawl for information but not to send e-mail.

Personally I find being outside in Vietnam a difficult experience, the air being hot and thick, and need little encouragement to stay in air-con heaven. After several days of incarceration, following a plea from the Leader of Trinidad and Tobago, a concession was made and we were allowed out in groups of five, subject to making no attempt to communicate with the outside world. Against my better judgement, I agree to join one of these tropical expeditions. Donning a Panama hat, I ambled gently to the city centre, and returned via a bar. I was a little surprised to find other westerners wandering round the town. Presumably they were taking karaoke breaks.

The hotel lifts are the weak point of the establishment. There are three of them, but two sets of calling buttons. Two of the lifts respond to one set of buttons, and the third lift is controlled by the other set. Therefore if you want a lift, all but the Quakers (of which we had an empty collection) selfishly press both buttons, thereby ensuring that half the calls to which the lifts respond are hoaxes. The net effect was that the lifts flocked. There was the option of using the outside stairs, but you would only do this if you were planning a shower in the immediate future.

To be serious, notwithstanding minor shortcomings, the hotel is of fabulous standard, and the leaders and observers are stunned by the hospitality of our Vietnamese hosts who are doing their very best to make this a great IMO.

The quest to set the IMO paper soon began. We first receive the shortlist without solutions, and next day the extended booklet with solutions is issued. I am afraid that it is a fantasy to think of the jury a wise collective mind, selecting a delicious and exquisite paper aimed at discriminating between students at all medal boundaries. There is definitely no controlling mind of this unwieldy body, and it proceeds using a mixture of good judgement, herd instinct and obsession with geometry. Democracy is not an effective method to set a mathematics exam, and it remains a mystery how the jury manages to do quite a good job most years.

We seem to have a shortage of good relatively easy questions. My heart sinks when I see G1, ostensibly the easiest Geometry question. The jury has a weakness for G1, irrespective of what year it happens to be. I have noticed that when ordering Chinese food, number 76 is often quite delicious, and am happy to order it without enquiring as to what it is. The jury's attitude to G1 is similar. Now, since it is a racing certainty that the jury will select G1, it is the duty of the Problem Selection Committee to ensure that it is a good question. This year G1 (proposed by the Czech Republic) is not the worst problem on the shortlist, but it is not absolutely ideal. This is because although a little ingenuity is needed to find a solution by synthetic geometry, one can instead simply lower ones head, scrape ones feet against the ground, and charge forward deploying either trigonometry or areal coordinates. Either way, a determined professional approach will quickly yield a solution. Actually a solution by calculation turns out to be safer, because one then need not allow for the singular geometric configuration that arises in the isosceles case.

The jury selects a second question deemed to be easy from the algebra



list. This question is not hard, but it does involve the casual manipulation of subscripts, and thereby plays to the more experienced and well trained students. This A1 was the brain child of Michael Albert, the New Zealand leader.

Next the jury moved to select the hard questions, the candidates for Problems 3 and 6. Following a slight diversion where an originally selected problem proves to be already known, they select one from Russia, an excellent but unusual combinatorics problem because all known solutions involve devising an algorithm. The other choice is one which masquerades as combinatorics, but the known solutions are actually algebraic. The devastating effectiveness of algebra in this context wins many friends on the jury. Later there will be chatter on the internet that the problem is unsuitable because it is trivialized by the *combinatorial Nullstellensatz*. This is a recurrent issue, when a theorem of relatively advanced mathematics renders a problem straightforward, should the jury throw out the question? In this case there was no problem, because no-one alerted the jury to the possibility of invoking this technique. As far as I know, only a handful of students attempted to invoke this advanced theorem in a solution. The Russian Problem 3 was manifestly combinatorial. Thus when Problem 6, its paired hard question also looked combinatorial, I reasoned that astute students would ‘smell a rat’, and realise that the solution to Problem 6 would not be combinatorial at all. Problem 6 is a Dutch proposal.

Finally the jury moves to selecting the medium questions. It plumps for G4, the creation of the Luxembourg Leader Charles Leytem (he of the dynamic  $z$ -co-ordinate) and the *lemma of N6*. This reminds me of the way that, many years ago, there was a pop singer called *the artist formerly known as Prince*. Now we have *the problem formerly known as N6*. The jury likes the hard problem N6 so much that it selects the key lemma from its solution, turns that into the problem, and reclassifies it as medium so that it can be selected. In fact N6 is a UK proposal, but being either saintly, or at least well aware of my own fallibility, I refuse to look at the UK submissions so that I can speak freely in the jury. Indeed, I spoke in favour of N5.

Sometimes there are dark mutterings when students from country  $X$  do exceptionally well in response to a problem posed by country  $X$ . Eyebrows are bent, lips are pursed, and leaders exchange meaningful glances. No doubt the UK will fall prey to similar suspicions. Our six students will go on to rake in a total of 7 marks between them on this problem.

The jury sends one of the proposed marking schemes back for revision,

but the jury chair seems surprised that we will not vote to accept it anyway. There is a minor cultural clash here, but happily it is easily resolved. Some of the jurors are getting restive, and resentment at being locked up in the hotel is starting to build up. Unsurprisingly, some of the leaders banging on about their human rights come from countries which habitually lock up people in industrial quantities. Eventually Indra, Leader of Trinidad and Tobago, makes a health and safety based case that we be allowed out for exercise. Our hosts readily agree to *excats* for large group as mentioned earlier.

**July 24th** It is time for the opening ceremony. It is hard to be sure who is in charge here, since Vietnam has parallel government and communist party structures. We have ministers at all ceremonies, the PM at the opening jamboree and the president will turn up at the medalfest. We are escorted to the opening ceremony by cops with *blues and twos* switched on. This cuts down the journey time, and unimaginative leaders such as myself seek front seats to enjoy the chaos as oncoming traffic hurls itself into ditches as we drive on random sides of the road and use traffic lights only for illumination. We have an excellent ceremony, and our team's hats make quite a splash.

**July 26th** The second exam finishes today, and we start to sweat over the scripts. Ceri joins us at the leaders' hotel, leaving Pam in charge of looking after the team. Results are mixed. Jack Shotton has done very well. He has solved 1, 2, 4 and 5 completely, and has part marks on both 3 and 6. It is quite clear that he will get a gold medal. The other students have mostly underperformed slightly, and looking back I can see that we should have gone pig hunting. The other five students are about one category down from what one might have expected. They get three bronzes and two honourable mentions, where we hoped for at least three silvers and a couple of bronzes. Most of them have thrown away easy marks, and missed solutions within their grasp. It is hard to know why this has happened. I blame Pam, Ceri, Joseph and global warming.

Actually, it could have been a lot worse. Ceri did a fantastic job of working out that a lot of incoherent gibberish associated with two of our solutions to Problem 1 is actually correct mathematics written down by orang-utans on acid. She writes out what these two candidates were trying to say, and manages to get the Vietnamese co-ordinators to agree that these solutions are indeed correct. Joseph and I are stunned. She has done nothing dishonest, and her case is sound, but without her insight into what passes for the student mind, many marks would have been lost.

Jack's script for the hard combinatorics Problem 3 claims a solution but has a fatal flaw in the last line and might score zero. The marking scheme for this problem offends Observer Joseph. The only known solution involved making very special initial moves, for which part marks are available. Joseph reckons that this is unfair to Jack's script because he speculates that there are solutions which involve completely different initial moves. He disappears for several hours and returns with a new solution which is compatible with various initial moves. This is dynamite, and he circulates it the evening before co-ordination to stir up the marking scheme. Jack gets 2/7.

I lead during the co-ordination of Problem 4. Here our students have all solved the problem, but a stern co-ordinator might want to take a mark off one script where the student has been sloppy in two different ways. This is a particular concern because the author is Takaki Oshima, and this is his only chance of marks. If he gets 7/7 he will get an honourable mention. Otherwise he will get nothing. The co-ordinators are indeed programmed to subtract marks for listed imperfections, but it turns out that, somewhat characteristically, Takaki's flakiness is so bizarre as not to be in the marking scheme. Happily he scrapes a 7/7.

Our Vietnamese hosts have done a wonderful job. There have been some difficulties on the way, and some leaders are annoyed by particular incidents. No IMO is perfect (though admittedly some are less perfect than others). There was a suggestion that there was some small inconsistency in the marking of one problem. This seems to have been true, but I can remember an IMO when the co-ordination was a real disaster: idleness, stupidity and petulance seemed to be the co-ordinators motto. This was certainly not the case in Vietnam, where diligent and intelligent co-ordinators were all trying their best, and were mostly doing an excellent job.

We moved hotels to Hanoi after co-ordination. The new site is next to a large lake in the centre of town. Happily BBC World Service TV is available at this new hotel, so there is a relatively cerebral alternative to the pap put out by CNN. At the closing ceremony I am interviewed by *Voice of Vietnam*, a national radio station. The IMO has been front page news in the papers throughout the competition, and has had much TV coverage. I meet the team's excellent local guide, a young woman called Quy. Her English is excellent, and according to Pam and the team she has been a first class guide.

**July 30th** The closing ceremony is surprisingly relaxed, and the president does not seem to be concerned with formalities. We dash for a table at the

banquet, and have a marvellous time. Happily I have failed to hold on to the *microphone d'or*, the prize for the most garullous juror. The Romanian leader Radu Gologan has made one more speech than I have, so he holds the shiny plastic this year. All the students mix up in a deplorable festival of goodwill. Later we go out to celebrate Jack Shotton's gold medal and 18th birthday with ice cream, a party sponsored by Mr and Mrs Shotton.

There were some unusual features to the entertainment at the ceremonies, including a song by a Vietnamese game show host. However, I leave you with the words of a song which featured at the opening ceremony. It was sung by a girl band, all jump suits and lip gloss, who wriggled and writhed through the following number (thanks to the Leader of Norway, seven times contestant Dávid Kunszenti-Kovács, for the text).

A new horizon is waiting for us  
The broad horizon is inviting us over  
The horizon for science is right in front of us  
Awakening our passions, challenging our thirsts. (x2)

Together, let's realize our dreams about creativity  
Together, let's indulge in the joy of science  
With our hearts and brains, victory is within our reach  
Shoulder to shoulder, we build our nation.

Together, let's realize our dreams about creativity  
Together, let's indulge in the joy of science  
With our hearts and brains, victory is within our reach  
Fly to the future, young men and women.