The Romanian Master of Mathematics 2013
Student Report

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It has become something of a tradition for a student to write a report on a competition, to give people a better idea of what the experience is like for us. This competition took place from the 27^{th} of February to the 4^{th} of March (although papers were only sat on the 1^{st} and 2^{nd}), and it was hosted by Tudor Vianu School. Our team of six was Andrew Carlotti, Gabriel Gendler, Daniel Hu, Sahl Khan, Warren Li and Matei Mandache (me). Our leader was Jonathan Lee, who was also a contestant at this competition five years ago, and our deputy was Beverley Detoeuf. Dan Schwarz was observer with leader. The Romanian Master of Mathematics competition is a relatively new competition, as it was started in 2008. It is generally a more relaxed competition than the IMO or even the Balkan Mathematical Olympiad, but it is also a very tough competition, and is considered to be harder than the IMO. Our results are displayed below. They are good considering the toughness of the competition.

Contestant	Q1	Q2	Q3	Q4	Q5	Q6	Total	Award
Andrew	0	7	0	7	7	7	28	Silver
Gabriel	0	0	0	7	0	7	14	Honourable Mention
Daniel	2	7	7	7	7	4	34	Gold
Sahl	7	0	0	1	7	0	15	Honourable Mention
Warren	5	0	0	6	7	0	18	Bronze
Matei	7	7	0	7	7	7	35	Gold
Total	21	21	7	35	35	25	144	
Min	0	0	0	1	0	0	1	
Max	7	7	7	7	7	7	42	Gold

I know what you're thinking: Who is this Max? How is he so good? And how did Min get on the team? Unfortunately neither Min nor Max are real contestants, they are purely fictional people, but this doesn't stop them from sitting the paper. For each question, Min gets the lowest mark out of the team, and Max gets the highest. It is unusual that their scores differ so greatly, as there is usually a problem none of us manages to complete (normally this is the geometry problem), but this year we managed to solve all the questions between us.

I have recorded the events of the competition in chronological order, in a

diary format.

27^{th} February

Today begins early for us, as we have to be at the airport for 7:45 a.m. For me, travel to Luton consists of taking a train to Luton Airport Parkway, followed by a bus to the airport itself. On the bus, I am pleasantly surprised to bump into fellow team member Daniel Hu, who is using a similar strategy to get to the airport. At the airport we find Sahl, who has been awake even longer than us, and are soon joined by Jonathan, Beverley, Warren and Andrew. It seems Gabriel is running 24 hours late: he will be flying over on the 28^{th}. We later find out this is due to a debating competition.

We are soon through check-in and security, and the flight is uneventful. It is not long before we arrive in Bucharest. All my grandparents live in Bucharest, and three of them have come to the airport to greet us. They have also brought bottles of water, covrigi (a typical Romanian snack, effectively a large pretzel made of bread) and cake. These are very welcome, as we are thirsty, and, as it turns out, there is no other lunch organised for us.

We share a minibus with the USA team to get to the student hall where we will be staying. They too are missing a team member, and he will be coming over on a different flight. Various things are discussed on the minibus, among them a maths problem which the Americans have been struggling with. One of them wagers fifteen dollars that nobody will be able to solve it, claiming it to be impossible. Naturally, we do not take his word for it, and spend a good deal of time over the next few days trying to solve it. Feel free to have a go at it yourself, although I cannot guarantee that you will receive fifteen dollars if you do solve it: find all positive integers m such that $5^m + 3^m$ is divisible by $m^2 - 1$.

We say goodbye to Jonathan, who will be staying at a different hotel, and are taken to our student hall, where we meet the first of our guides, Andrei. Unlike other competitions, where there is only one guide per team, most teams at the RMM have several guides. We have four: Andrei, Silvia, Ioana and Dan. They are all surprised to see I am also Romanian. As in previous years, there are four people in each room, so two of us will be sharing with two Brazilians. We agree these will be Sahl and Andrew, as they have been to this competition before. Sahl has been last year and Andrew has been the last three years, so counting this year, he will have attended two thirds of all RMMs.

After going to our rooms and leaving our bags, we have a late lunch of covrigi, and get used to our rooms. We discover that the toilet in our room (not Andrew and Sahl's) is broken, and water is flowing continuously. Luckily, we can stop this by closing the valve while the toilet isn't in use. After a couple of rounds of card games, we leave for dinner. We have all our meals at a canteen which is one minute's walk away from the hall, which is quite convenient. I am taken aback when a dish with meat is placed in front of me, as I am vegetarian. However, the situation is quickly explained, and the dish is soon replaced with a mushroom risotto. This is a process that happens at every meal, and we get

used to it.

After dinner, Andrew shows us a couple of games on his laptop. The first can be described as a one button real time strategy game. We are intrigued to find out how this works: as Andrew explains, you deploy different types of spaceship depending of the period of time the button is kept pressed. It is surprisingly fun. The second game, Liquid War, uses four times as many buttons. You control a cursor which some liquid of your colour follows, and when your liquid collides with an opponent's, some of the opponent's liquid switches sides, changing to your colour. The aim is to take over all your opponents' liquid by having your liquid collide with it and surround it. Andrew's experience means he always wins, so we decide to stop playing fair and all gang up on his liquid.

28^{th} February

After breakfast, we leave for the Institute of Mathematics, where we are given a talk on hyperbolic geometry, which I found very interesting. It covers strange objects such as "Ideal triangles", all of whose angles are 0, and whose sides are infinitely long. We then go to our student hall complex for lunch.

There are a few things happening this afternoon: as well as a visit to the Antipa natural history museum and the opening ceremony, Gabriel will be arriving. Beverley will take Gabriel from the airport, so we join the rest of the contestants at the museum. The museum is full of all kinds of animals: mammals, birds, reptiles, amphibians, stuffed animals, plastic model animals and even pickled animals.

We then leave for the opening ceremony, which is held at Tudor Vianu School. Tudor Vianu has played a very large part in organising the competition, and both the papers will be sat in its classrooms. At the opening ceremony, our team is given a few things: backpacks, paper, lanyards and Gabriel. We are glad to receive them. The opening ceremony comprises of several speeches, and certificates are presented to Zuming Feng, József Pelikán, and Geoff Smith, for helping the Romanian Masters of Mathematics competition over the years, and always being there to help out. Ironically, Geoff isn't there to collect his award. One speaker informs us that Tudor Vianu is the best school in Romania. As he is a teacher at Tudor Vianu school, we suspect he may be a little bit biased. This is followed by a talent contest between some Tudor Vianu students, which involves singing, piano playing and an electric violin solo. We are told the contest will conclude at the closing ceremony (I guess we will then be able to vote for our favourite act by phone).

Upon returning to our student hall, we discover that the toilet situation is worse than before, and the toilet won't flush. We inform our guide Andrei about this. Until it is fixed, we will use the toilet in Sahl and Andrew's room instead.

1st March

Today is the first day of the contest. After breakfast, we walk to Tudor Vianu again, where we are allocated to four classrooms to sit the paper. This is what we puzzle over for the next four and a half hours:

1. For a positive integer a, define a sequence of positive integers $x_1, x_2, \ldots$ by letting $x_1 = a$ and $x_{n+1} = 2x_n + 1$ for $n \geq 1$. Let $y_n = 2^{x_n} - 1$. Determine the largest possible k such that, for some positive integer a, the numbers $y_1, \ldots, y_k$ are all prime.

2. Does there exist a pair (g, h) of functions $g, h : \mathbb{R} \to \mathbb{R}$ such that the only function $f : \mathbb{R} \to \mathbb{R}$ satisfying $f(g(x)) = g(f(x))$ and $f(h(x)) = h(f(x))$ is the identity function $f(x) \equiv x$?

3. Let $ABCD$ be a quadrilateral inscribed in a circle ω. The lines AB and CD meet at P, the lines AD and BC meet at Q, and the diagonals AC and BD meet at R. Let M be the midpoint of the segment PQ, and let K be the common point of the segment MR and the circle ω. Prove that the circumcircle of the triangle KPQ and ω are tangent to one another.

The problems look tough, and the general opinion is that the paper is harder than a standard IMO paper. As problem 3 is geometry, I spend almost all my time on the first two problems: a number theory problem about Mersenne primes and an abstract-looking question about functions. For the first two hours I don't get much progress, but then have a few ideas, and manage to solve questions 1 and 2 with little time to spare. The others also found it a tough paper: Gabriel hasn't made much progress, while Warren and Sahl claim solutions to question 1. Andrew has solved question 2, as has Daniel. Daniel's solution to question 2 sounds radically different from mine or Andrew's, as it involved simple functions, unlike the strange ones constructed by me and Andrew. Daniel also claims a solution to question 3, by a trig bash.

For lunch, we agree to go to town instead of eating at the canteen. This is a much more lengthy process than we anticipate. First we must exchange money into lei, the Romanian currency. Interestingly, "lei" literally translates as "lions". I guess that a long time ago Romanians used to pay for things by giving each other pet lions. Gabriel and Silvia go into the bank first, but are unsuccessful, as someone who is both 18 and has a passport is required. Andrew fits the bill, so he joins them in the bank, and eventually he emerges, holding a wad of lions in his hand. We had originally planned to go to Pizza Hut, but as the restaurant is full, we go to Subway instead.

Good news awaits us on our return to the student hall: the toilet has been fixed, or at least partially fixed. There is still a continuous flow of water, but we all agree this is much better than not being able to flush the toilet. In the evening, some of us continue to work on the American problem, but the solution (and the fifteen dollars) remains elusive. Andrew shows us a game of life program on his computer, and we explore the various patterns, and see what happens when we modify them: it turns out that one missing glider can

cause a huge convoy of spaceships to disintegrate. Later, we are confronted with a mystery when the ladder to Daniel's bunk disappears. It is nowhere to be found. However, when Daniel has a shower he finds the ladder dangling from the shower rail. Apparently the ladder felt it was too dirty, so it went to have a shower. In any case, Gabriel had nothing to do with the ladder's movements.

2nd March

Today is the second day of the contest, and strangely enough the paper today is to be sat at 8:30, a full hour earlier than yesterday's. After an early start, and breakfast, we head towards Tudor Vianu. This is what we find waiting for us:

4. Let P and P' be two convex quadrilateral regions in the plane (regions contain their boundary). Let them intersect, with O a point in the intersection. Suppose that for every line ℓ through O the segment $\ell \cap P$ is strictly longer than the segment $\ell \cap P'$. Is it possible that the ratio of the area of P' to the area of P is greater than 1.9?

5. Given an integer $k \geq 2$, set $a_1 = 1$ and, for every integer $n \geq 2$, let a_n be the smallest $x > a_{n-1}$ such that:

$$x = 1 + \sum_{i=1}^{n-1} \left\lfloor \sqrt[k]{\frac{x}{a_i}} \right\rfloor .$$

Prove that every prime occurs in the sequence a_1, a_2,... .

6. $2n$ distinct tokens are placed at the vertices of a regular $2n$-gon, with one token placed at each vertex. A *move* consists of choosing an edge of the $2n$-gon and interchanging the two tokens at the endpoints of that edge. Suppose that after a finite number of moves, every pair of tokens have been interchanged exactly once. Prove that some edge has never been chosen.

My first reaction when I saw the paper was surprise, as the results asked for in question 5 and 6 seemed at first rather surprising. However, once I had played around with the problems for a bit it became clear that although the results look tricky, especially question 5's result about primes, they wouldn't be so difficult to prove. Today's paper seems somewhat easier, in fact, than yesterday's. I manage to complete solutions to all three questions in little over four hours.

We discuss the problems just outside the classrooms once we are finished. It seems others have found today's paper easier as well. Gabriel has solutions for 4 and 6, while Warren, Sahl and Daniel claim solutions for 4 and 5. Unfortunately, we find a flaw in Sahl's solution to problem 4. It seems he overlooked a single line in his configuration, but the solution cannot easily be fixed. Andrew seems to have disappeared: he was not among the contestants as we all left the classrooms. We find him later with Jonathan in the school's central hall. It turns out he solved all three questions and walked out after using only half

his time. Daniel also mentions he made some progress towards an inductive solution for problem 6, but he is not too optimistic about it. We report our performances to Jonathan, while he informs us of our scores from yesterday's paper. Most of the scores are as expected. Warren lost a couple of marks on question 1, due to a simple mistake, while Daniel gained a couple on the same question for a useful observation, so the balance is maintained.

We head back to our hall for lunch, and find out that after lunch, a trip to a military museum is organised for us. Not all of us are intrigued to find out about the development of the Romanian military, and how their uniform changed over the years, but the trip is not optional. As well as finding out about the military, our guides tell us some of Romania's history. It was initially three separate countries: Wallachia, Moldova and Transylvania. In 1859, Wallachia and Moldova chose the same person, Alexandru Ioan Cuza, as their king, thus uniting the two. Transylvania, however, was still part of the Austro-Hungarian Empire. Union with Transylvania was only achieved in 1918, after the war, when the Austro-Hungarian Empire was broken up. As well as finding out about these things, we talk to some of the other teams. Daniel finds out that the Chinese team is, like in previous years, a regional team. This year it is from Wuhan, and they called themselves a 5^{th} or 6^{th} best team. If China had sent their first team, their performance would have been very impressive. I guess they decide to give someone else a chance to win the competition.

After the museum visit is finished, our guides take us to a bowling alley. Nearby there are a large number of pool tables, and a few table tennis tables. While Sahl, Daniel, Gabriel and Andrei choose to play table tennis, the rest of us opt for pool. It is very enjoyable, although the 8-ball has a tendency to avoid the pockets when we are trying to pot it (this may be because I'm not hitting it in the right direction, but I blame the ball). On the other hand, it happily rolls in when we don't want it to.

Afterwards, we head back to our canteen for dinner. The hall in which we eat has a large number of TVs, and by now we have noticed a pattern in the channels they are on: they are always on music channels, with the volume turned down very low. Someone should really go and find the remote.

Later, we play more Liquid War. By now we know more about the tactics and strategy, and the battles are more interesting and less one-sided. In fact, with all six of us playing, Andrew only wins half the time. As well as playing, we work on some problems: we listen to "I am the Walrus", and try to work out what the seemingly nonsensical lyrics mean. However, we just can't work them out. We also muse over the American problem, which now looks like it is an extremely hard problem, but still a bit easier than making sense of the lyrics.

3^{rd} March

The organisers have left this morning as free time, so I take this opportunity to visit my grandparents. I don't know for sure what the others did this morning, but I hear I didn't miss much. They had a rushed lunch at a restaurant,

just before the closing ceremony. The closing ceremony is in the afternoon, at two o'clock, although the timetable originally had it down at 5 o'clock. My grandparents also want to attend, so they take me directly to Tudor Vianu for the ceremony. We arrive early, and the rest of the team isn't here yet. I find Jonathan, and get news of our marks, and medals. Daniel and I have both got Gold medals, which is a very good result, as the UK has only had two previous gold medallists at the RMM: Jonathan Lee and Adam Goucher. Andrew and Warren have also won medals: Silver and Bronze, but Gabriel and Sahl narrowly miss out. The medal boundaries are rather high this year, perhaps due to the second paper being slightly easier than normal.

The others arrive, and we inspect the results in more detail. Daniel got four marks on question 6, for his attempt at an inductive solution. Jonathan explains: he got these marks for Daniel by finding three lines that would complete Daniel's solution, and presenting them to the coordinators, who then agreed on giving four marks, i.e. a deduction of one mark per missing line. There were two contestants with perfect scores, Omer Cerrahoglu from Romania and Dmitri Krachun from Russia. According to the statistics, problem 2 was hardest, and problem 5 easiest, which suggests that the problems weren't selected in the right order, as usually 1 and 4 are the easiest, and 3 and 6 are the hardest. The team ranking is done by summing the scores of the top three scorers in each team. We come third as a team, behind the USA and Russia. In past years we have come first, last, penultimate, second, and somewhere in the middle – a range of performances.

While waiting for the ceremony to begin, we notice some lists on the walls of the hall. They are lists of Tudor Vianu students who had participated in international Olympiads and competitions. The lists are surprisingly long: it seems Tudor Vianu has an outstanding record of good students.

The opening ceremony starts and we are presented with medals and certificates. In most competitions, ironically, honourable mentions are not mentioned in the closing ceremonies, but this is not the case here. After all the individual awards are given out, we go up on stage again as a team, for coming third. This time, there is no talent contest, only a few speeches. Perhaps not everyone was a fan of "Vianu's got talent", and someone decided to reschedule the ceremony without telling the talent show contestants.

After the ceremony, we go out into a courtyard next to the school, to be photographed. We all get thoroughly photographed, because as well as us wanting photographs, so do the guides and my grandparents. There are probably more photographs of me taken in these five minutes than in any other five minutes of my life.

Afterwards, our guides suggest we visit the old centre of Bucharest, which is actually nicer than the rest of the city. We sightsee a bit, and then go into a bar for some drinks. When we get back to our student hall, it is time to say goodbye to our guides. As they plan to be guides again, there is a good chance Gabriel or Warren will see them again, but for the rest of us the goodbye is final. They tell us that they had a lot of fun being our guides, and are sad the experience has come to an end.

This evening we have to pack, as tomorrow's flight is early. My grandparents have a selection of parcels for me to take back to the UK, and they come to our student hall to give them to me to put in my suitcase. They have also very generously prepared sandwiches and snacks for all of us to eat tomorrow morning. The extra parcels mean that I have considerably more luggage than I came with, and I am now confronted with a tricky-looking packing problem. At first I am unsure of how to solve the packing problem, but then I realise it can easily be solved with a case bash.

4th March

Today we wake up at 5 a.m. Romanian time (3 a.m. British time). The journey home starts with a minibus ride to the airport, where we check in. I am worried about the weight of my bag, but luckily the weight limit is very large. After check in, we have a breakfast of the sandwiches and apples my grandparents have given us. The flight is uneventful, and most of us use it to get some extra

sleep. We are soon standing in Luton airport with our luggage, and we say our goodbyes. The Romanian Master of Mathematics competition was a great experience for all of us, and we thoroughly enjoyed it, but such things don't happen by accident. I would therefore like to thank the following:

- The organisers of the competition, for doing a good job.
- Tudor Vianu School, for hosting the competition.
- The jury and the problem selectors, for selecting a wonderful batch of problems.
- Our guides Andrei, Silvia, Ioana and Dan, for their help throughout the competition.
- Moxa student hall, for providing us with rooms and food.
- My grandparents, also for providing us with food.
- Beverley, for being there to sort things out.
- All the volunteers at maths camps, who taught us a lot of the stuff we used during the competition.
- Jonathan, for leading the team, and Dan Schwarz, for helping him out.
- And finally Gabriel, Warren, Sahl, Daniel and Andrew for being great company.