

Correction to the solution of Problem 1, UK IMO report 1970

GCS

22-vi-2011

Background

In 2011 the British Mathematical Olympiad decided to award Prof Walter Hayman FRS and the late Margaret Hayman a medal to commemorate their role in launching the BMO in the 1960s.

As current chair of the BMO subtrust of UKMT, I read the report on the 1970 IMO which Margaret attended as deputy. One of the solutions given in the report is garbled, and this note is a correction.

Discussion of the error

Using standard triangle notation,

$$r = 4R \sin A/2 \sin B/2 \sin C/2$$

and

$$s = 4R \cos A/2 \cos B/2 \cos C/2$$

so that

$$[ABC] = rs = 2R^2 \sin A \sin B \sin C.$$

Also

$$r_c = 4R \cos A/2 \cos B/2 \sin C/2$$

so

$$\frac{r}{r_c} = \tan A/2 \tan B/2.$$

The 1970 report uses ρ as notation for r_c . Moreover the formula given for ρ_c is wrong, and is in fact a formula for $s - c$. Now

$$[ABC] = rs = r_c(s - c),$$

so the author of the solution ends up working with the reciprocal of the intended ratio, and that is why the algebra appears to work correctly.

If the 'solution' in the report were handed in at an IMO, there would be a strong case for putting it in the 7- marking regime.