28th INTERNATIONAL MATHEMATICAL OLYMPIAD

CUBA 1987

Report by Mr. Robert Lyness, Leader of the British Team

The Olympiad was held in Havana, Cuba from Sunday 5th July to Thursday 16th July 1987. Forty two countries took part; generally each delegation contained a Leader, Deputy Leader and a maximum of six competitors. In the Appendix, a list of countries is given (with their total marks). This list also shows the countries which had less than six competitors. The total number of competitors was 236 (a record high). About a dozen were girls.

Also attending were observers from Eire (to take part next year), West Germany (to be host in 1989) and representatives of the Site Committee from Great Britain, the USSR and Australia (host in 1988).

Travelling difficulties led to some delegations arriving and/or departing early or late. Apart from the British Leader, who flew out earlier, the British delegation travelled out on July 4th and returned on July 15th. It contained Mr. Terry Heard (City of London School, Deputy Leader), the six competitors and Mr. John Hersee (Secretary of the IMO Site Committee).

The Jury met first on July 6th in a seaside hotel some 30 kilometres from the boarding school which housed all competitors. Until July 11th, the Jury consisted only of Leaders. Deputies were accommodated in a hotel in Havana and were in contact with competitors. They joined the Jury after the second paper had been taken.

The six problems were chosen by the Jury from those proposed by participating countries, a preliminary selection having been made by a subcommittee of the Cuban organizers. Once the questions were settled, translated and typed out, the competitors had four-and-a-half hours on each of two successive mornings to solve them. Then Leaders and Deputies marked their team's papers and submitted their markings to Cuban co-ordinators. Terry Heard presented our scripts excellently.

Prizes were awarded as follows: '1st prizes' went to each of the 22 competitors who attained a maximum total of marks, 42. There were forty two '2nd prizes' and fifty four '3rd prizes'. This meant that the British Team gained one 1st, two 2nd and two 3rd prizes. Our best performer, Kevin Buzzard, scored full marks (42) and Andrew Smith dropped only one mark, with 41.

The prize-giving took place in a fine assembly hall. On the platform were the Chairman of the Jury and members of the Site Committee. Prize-winners went up in groups to be garlanded simultaneously by the seated members - a much faster procedure than usual. Following the prize-giving there was a concert of popular music.

Our team was selected by means of the National Mathematics Contest and the British Mathematical Olympiad, followed by some postal tuition and a residential selection/training session which included a further test. This session was held at the Ship Hotel, Reading from Friday 8th May to Sunday 10th May 1987. It was staffed by Judita Cofman, David Cundy, Terry Heard, John Hersee, Paul Woodruff, and myself. The training programme consisted of short lectures and tutorial periods during which the participants had opportunities to expound their own solutions to problems. It proved extremely helpful. All these activities are the responsibility of the Mathematical Association's 'National Committee for Mathematical Contests'.

A programme of non-mathematical events was provided including, for the Leaders, a visit to the Botanical Gardens, an evening cocktail party given by the President of the Assembly of the People's Power of Havana City in the courtyard of a beautiful 'Spanish colonial' house, and a visit to Ernest Hemingway's Cuban Estate. With our teams, we went for the day to Playa Giron (by the Bay of Pigs) and to a most enjoyable evening reception offered by the Minister of Education of the Cuban Republic.

After a couple of days acclimatisation, the team found the accommodation and food at the 'V.I. Lenin School for Exact Sciences' perfectly adequate. There were good on-site recreational facilities, especially for swimming, and several evening entertainments, but some team members missed the freedom to explore the neighbourhood and Havana. The layout of the dormitories and the early arrival of many teams encouraged the mixing of nationalities which produced an exceptionally relaxed and friendly atmosphere at the school. The members of the British team got on well together: ideas were swapped, eccentricities tolerated, and much mutual support and encouragement given.

Our thanks are due to the Cuban organisers who succeeded in giving us an interesting and enjoyable time. The purpose of improving international understanding and friendship has been achieved.

Robert Lyness

APPENDIX

The British Team

Question	1	2	3	4	5	6	Total	Prize
James Angus (Wells Cathedral School)	1	3	0	5	0	7	16	---
Kevin Buzzard (RGS, High Wycombe)	7	7	7	7	7	7	42	1st
Gareth McCaughan (Lincoln Christ's Hospital S)	7	3	7	6	5	3	31	3rd
George Russell (King Edward's S, Birmingham)	7	0	7	7	7	4	32	2nd
Andrew Smith (City of London School)	7	6	7	7	7	7	41	2nd
Gerard Thompson (St. George's Coll, Weybridge)	0	7	0	5	7	1	20	3rd
Total marks	29	26	28	37	33	29	182 out of a maximum total of 252	

Team Totals

Romania	250	Australia	143	Colombia	68
W. Germany	248	Canada	139	Mongolia	67
USSR	235	Sweden	134	Poland (3)	55
E. Germany	231	Yugoslavia	132	Iceland (4)	45
USA	220	Brazil	116	Cyprus	42
Hungary	218	Greece	111	Peru	41
Bulgaria	210	Turkey	94	Italy (4)	35
China	200	Spain	91	Algeria	29
Czechoslovakia	192	Morocco	88	Kuwait	28
Great Britain	**182**	Cuba	83	Luxembourg (1)	27
Vietnam	172	Belgium	74	Uruguay (4)	27
France	154	Iran	70	Mexico (5)	17
Austria	150	Norway	69	Nicaragua	13
Holland	146	Finland	69	Panama	7

Numbers ( ) indicate teams of less than 6.

28 OLYMPIADA INTERNACIONAL DE MATHEMATICA - LA HABANA, CUBA

First Day - July 10, 1987

1. Let $p_n(k)$ be the number of permutations of the set $\{1, ..., n\}$, $n \geq 1$, which have exactly k fixed points. Prove that
$$\sum_{k=0}^{n} k \cdot p_n(k) = n!.$$
(Remark: A permutation f of a set S is a one-to-one mapping of S onto itself. An element i in S is called a fixed point of the permutation f if $f(i) = i$.)

2. In an acute-angled triangle ABC the interior bisector of the angle A intersects BC at L and intersects the circumcircle of ABC again at N. From point L perpendiculars are drawn to AB and AC, the feet of these perpendiculars being K and M respectively. Prove that the quadrilateral $AKNM$ and the triangle ABC have equal area.

3. Let $x_1, x_2, ..., x_n$ be real numbers satisfiying $x_1^2 + x_2^2 + ... + x_n^2 = 1$. Prove that for every integer $k \geq 2$ there are integers $a_1, a_2, ..., a_n$, not all 0, such that $|a_i| \leq k - 1$ for all i and
$$|a_1x_1 + a_2x_2 + ... + a_nx_n| \leq \frac{(k-1)\sqrt{n}}{k^n - 1}.$$

Second Day - July 11, 1987

4. Prove that there is no function f from the set of non-negative integers into itself such that $f(f(n)) = n + 1987$ for every n.

5. Let n be an integer greater than or equal to 3. Prove that there is a set of n points in the plane such that the distance between any two points is irrational and each set of three points determines a non-degenerate triangle with rational area.

6. Let n be an integer greater than or equal to 2. Prove that if $k^2 + k + n$ is prime for all integers k such that $0 \leq k \leq \sqrt{n/3}$, then $k^2 + k + n$ is prime for all integers k such that $0 \leq k \leq n - 2$.

Time: 4.5 hours

Each problem is worth 7 points.