INTERNATIONAL MATHEMATICAL OLYMPIAD 1988

Report by Dr David Monk, Leader of the UK team

The Olympiad took place in Australia from 9th to 21st July 1988 and was designated an Australian Bicentennial Event, with the Prime Minister of Australia as Patron. The main venue was Canberra but teams and leaders spent two or three days in Sydney on arival.

The team reached Sydney on 10th July accompanied by Paul Woodruff (Dulwich College), the Deputy Leader. I had arrived the previous day, in accordance with the programme for leaders, and I accompanied the team on the homeward journey, reaching London early on 25th July. Three days outside the official Olympiad dates were spent in Australia in order to take advantage of an excursion fare. Travel was by Qantas and all arrangements went smoothly.

The organisation of the Olympiad was extremely efficient, with a comprehensive programme of leisure activities to complement the academic work. Innovations included visits by teams to schools, students' homes and to Embassies and High Commissions. There was a special programme for "accompanying persons". The teams were accommodated, and the contest took place, at the Canberra College of Advanced Education. The first part of the Jury's time was spent at University House and the Australian Academy of Sciences building on the campus of the Australian National University. The Jury moved to CCAE after the second examination. The team's accommodation in Sydney on arrival, and for the extra days, was at the University of New South Wales.

The Jury met under the chairmanship of Professor Renfrey Potts of the University of Adelaide, who guided its proceedings with friendly efficiency. The selection, translation and typing of the problems were completed well within the the time allocated. Of the five problems submitted by the United Kingdom, four were on the short list preselected by the Jury, and one of these was among that committee's "starred" choices. This problem was selected a number 3 for the Competition. Judged by the number of contestants gaining the full mark of 7 for it, this proved to be the second hardest problem. The decisions on prizes were also made without undue difficulty. As Leader of a team submitting a contest problem, I was a member of the subcommittee to adjudicate on special prizes. Only one such prize was awarded, to a Bulgarian contestant for his solution to the extremely difficult, but very interesting, problem 6.

49 countries competed, several others being represented by observers. Our team, with a total mark of 121, gained three second prizes (Malcolm Law, Gareth McCaughan and Oliver Riordan) and two third prizes (Colin Bell and Christopher Nash). We were 11th in "national order", after USSR (217 out of 252), Romania (201), China (201), West Germany (174), Vietnam (166), USA (153), East Germany (145), Bulgaria (144), France (128) and Canada (124). This is much the same position as in recent years. In an attempt to improve it, we hope to improve our publicity to ensure that we attract the best candidates available into our selection procedure and to make some modifications to our tests.

Everybody greatly enjoyed the trip. Warmest thanks are due to the Olympiad organisers under Mr Peter J O'Halloran for their splendid arrangements and hospitality. I add especially my own thanks to Paul Woodruff for his mathematical work and careful attention to the well-being of the team. We are all most grateful to the sponsors who made our participation possible.

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APPENDIX

The UK Team

Question	1	2	3	4	5	6	Total	Prize
Colin Bell (Trinity School)	1	7	7	0	5	0	20	Bronze
Malcolm Law (King Edward's S, Birmingham)	5	7	2	1	7	1	23	Silver
Gareth McCaughan (Lincoln Christ's Hospital S)	1	3	7	6	7	1	25	Silver
Christopher Nash (King Edward's S, Birmingham)	7	7	1	1	1	2	19	Bronze
Oliver Riordan (St. Paul's School)	7	7	1	7	4	1	27	Silver
Joshua Ross (City of London School)	1	3	1	1	0	1	7	
Total marks	22	34	19	16	24	6	121 out of a maximum total of 252	

Team Totals

USSR	217	Singapore	96	Italy (4)	44
Romania	201	Yugoslavia	96	Algeria (5)	42
China	201	Iran	86	Mexico	40
West Germany	174	Netherlands	85	Brazil	39
Vietnam	166	Republic of Korea	79	Iceland (4)	37
United States	153	Belgium	76	Cuba	35
East Germany (5)	145	Hong Kong	68	Spain	34
Bulgaria	144	Tunisia (4)	67	Norway	33
France	128	Colombia	66	Ireland	30
Canada	124	Turkey	65	Philippines (5)	29
United Kingdom	**121**	Greece	65	Kuwait	23
Czechoslovakia	120	Finland	65	Argentina (3)	23
Sweden	115	Luxembourg (3)	64	Cyprus	21
Israel	115	Morocco	62	Indonesia (3)	6
Austria	110	Peru	55	Ecuador (1)	1
Hungary	109	Poland (3)	54		
Australia	100	New Zealand	47		

Numbers ( ) indicate teams of less than 6.

XXIX INTERNATIONAL MATHEMATICAL OLYMPIAD

Canberra, Australia - July 1988

1. Consider two coplanar circles of radii R and r $(R > r)$ with the same centre. Let P be a fixed point on the smaller circle and B a variable point on the larger circle. The line BP meets the larger circle again at C. The perpendicular ℓ to BP at P meets the smaller circle again at A (if ℓ is tangent to the circle at P then $A = P$).

   (i) Find the set of values of $BC^2 + CA^2 + AB^2$.

   (ii) Find the locus of the midpoint of AB.

   (Luxembourg)

2. Let n be a positive integer and let $A_1, A_2, \ldots, A_{2n+1}$ be subsets of a set B. Suppose that

   (a) each A_i has exactly $2n$ elements,

   (b) each $A_i \cap A_j$ $(1 \leq i < j \leq 2n+1)$ contains exactly one element,

   and (c) every element of B belongs to at least two of the A_i.

   For which values of n can one assign to every element of B one of the numbers 0 and 1 in such a way that each A_i has 0 assigned to exactly n of its elements?

   (Czechoslovakia)

3. A function f is defined on the positive integers by

$$f(1) = 1, \quad f(3) = 3$$
$$f(2n) = f(n)$$
$$f(4n+1) = 2f(2n+1) - f(n)$$
$$f(4n+3) = 3f(2n+1) - 2f(n)$$

   for all positive integers n. Determine the number of positive integers n, less than or equal to 1988, for which $f(n) = n$.

   (United Kingdom)

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4. Show that the set of real numbers x which satisfy the inequality

$$\sum_{k=1}^{70} \frac{k}{x-k} \geq \frac{5}{4}$$

is a union of disjoint intervals, the sum of whose lengths is 1988.

(Ireland)

5. ABC is a triangle right-angled at A, and D is the foot of the altitude from A. The straight line joining the incentres of the triangles ABD, ACD intersects the sides AB, AC at the points K, L respectively. S and T denote the areas of the triangles ABC and AKL respectively. Show that $S \geq 2T$.

(Greece)

6. Let a and b be positive integers such that $ab + 1$ divides $a^2 + b^2$. Show that $\frac{a^2 + b^2}{ab + 1}$ is the square of an integer.

(West Germany)

Problems 1 - 3 first day; 4 - 6 second day. 4½ hours each day.

Each problem is worth 7 points.