# International Mathematical Olympiad 1989
# Braunschweig, West Germany
# Report by Dr. D. Monk, Leader of the UK team

**The UK team**

| | |
|---|---|
| Katherine Christie | (Portsmouth Grammar School) |
| Vin de Silva | (Dulwich College) |
| Clive Jones | (Dulwich College) |
| Christopher Nash | (King Edward's School, Birmingham) |
| Oliver Riordan | (St. Paul's School) |
| John Simcox | (The Chase High School, Malvern) |
| Leader: | Dr. David Monk, Edinburgh University |
| Deputy Leader: | Mr. Paul Woodruff, Dulwich College |

The 30th International Mathematical Olympiad took place in Braunschweig, Lower Saxony, from 13th to 24th July 1989. Fifty countries competed. Among these, India and Portugal were taking part in an IMO for the first time. There were also observers from Denmark and Japan.

The team leaders arrived on Thursday 13th July and were accommodated very comfortably in the Hotel Mercure Atrium. All meetings of the Jury, apart from the brief sessions to deal with contestants' queries at the start of each examination, were held there and the facilities were extremely good. The teams arrived on Sunday 16th July with the Deputy Leaders, who joined the Jury at their hotel immediately upon arrival. The British team, with some twenty other, largely English-speaking teams, were housed in a Youth Hostel some distance away. The two competition papers were taken on Tuesday and Wednesday 18th and 19th July at the Technische Universität which was also the venue for the opening ceremony on the Monday. The closing and prize-giving session, as well as the splendid final dinner, took place in the Stadthalle on Sunday 23rd July, and the delegations dispersed on the following day.

The Jury met under the kindly and efficient chairmanship of Professor Arthur Engel of Frankfurt University, whom many knew from his work as leader of West German teams in previous years. Business was conducted almost entirely in English, with a minimum of translation into or from French and occasionally German or Russian. From over one hundred problems submitted, a shortlist of 32 had been produced for the Jury to consider. The task of choosing the six contest problems and translating them into the candidates' languages went very smoothly. After the competition, in accordance with established procedure, Paul Woodruff of Dulwich College, the Deputy Leader, and I assessed the scripts of our team and then submitted them to the German coordinators to determine the final marks.

We found the coordination generally fair and satisfactory, if occasionally severe. At the final meeting, the Jury decided on the award of prizes without too much difficulty. First (gold) prizes went to those with scores of 38 and upwards, with second (silver) prizes down to 30 and third (bronze) prizes down to 18. The maximum possible score was 42. It was further agreed that any non-prizewinning contestant with a perfect score of 7 on at least one problem should be given an 'honourable mention'. As a result, 20 first, 55 second and 72 third prizes were awarded. The total number of competitors was 291.

The UK team gained two second prizes (Christopher Nash and Oliver Riordan) and one third (Vin de Silva). John Simcox and Katherine Christie qualified for honourable mention. Our total of 122 was 20th in the (unofficial) ranking of the teams. We did much better on the second day than on the first. The coordination was very strict in that no credit was given for 'good ideas' unless they could be shown to be part of a feasible solution. This meant that our policy of encouraging the team to set out their thoughts clearly even if they had not made much progress, in an attempt to avoid zero scores, was less successful than previously. Countries appear to be taking the competition increasingly seriously and providing more training. It is noteworthy that ten of the first twelve places went to 'Eastern bloc' countries which have always been very efficient in that respect.

Our team had an excellent guide, Maria Rech, who watched over them most conscientiously. She dealt extremely competently with a couple of minor accidents and was generally pleasant and helpful. We were most grateful to her. Besides the academic schedule there was the customary programme of sightseeing trips and receptions, including a lengthy excursion to Hannover. The organisation was admirable throughout and made this one of the most enjoyable Olympiads I have attended.

At the closing ceremony, the Chinese delegation extended the invitation to the 1990 Olympiad in the People's Republic of China. Other forthcoming venues are 1991: Sweden, 1992: USSR and 1993: Turkey.

Our thanks are due to Hans-Heinrich Langmann, the Organiser, and to Arthur Engel for their work in making the event such a success. We are grateful to all our sponsors who enabled the team to travel to West Germany, but particular mention may perhaps be made of Trinity College, Cambridge, which provided generous prizes and excellent facilities for the Training Session held there from 14th to 16th April 1989.

Finally, it is a great pleasure to acknowledge the cooperation of Paul Woodruff as Deputy Leader, not only in connection with the team's travel arrangements and welfare, but also academically and particularly during coordination. His earlier work in organising the training session was also much appreciated.

D. Monk,
Department of Mathematics,
Edinburgh University.

23 October 1989

# APPENDIX

## The UK Team

| Question | 1 | 2 | 3 | 4 | 5 | 6 | Total | Prize |
|---|---|---|---|---|---|---|---|---|
| Katherine Christie (Portsmouth Grammar School) | 0 | 1 | 0 | 7 | 0 | 0 | 8 | H |
| Vin de Silva (Dulwich College) | 0 | 5 | 1 | 7 | 7 | 7 | 27 | III |
| Clive Jones (Dulwich College) | 5 | 0 | 1 | 0 | 5 | 0 | 11 | – |
| Christopher Nash (King Edward's S, Birmingham) | 3 | 3 | 3 | 7 | 7 | 7 | 30 | II |
| Oliver Riordan (St. Paul's School) | 2 | 7 | 1 | 7 | 7 | 7 | 31 | II |
| John Simcox (The Chase High School, Malvern) | 3 | 5 | 0 | 0 | 7 | 0 | 15 | H |
| Total marks | 13 | 21 | 6 | 28 | 33 | 21 | 122 out of a maximum total of 252 | |

H = Honourable mention

## Team Totals

| | | | | | |
|---|---|---|---|---|---|
| China | 237 | Italy | 124 | Luxemburg | 65 |
| Romania | 223 | Canada | 123 | Brazil | 64 |
| USSR | 217 | **United Kingdom** | **122** | Norway | 64 |
| East Germany | 216 | Greece | 122 | Morocco | 63 |
| USA | 207 | Australia | 119 | Spain | 61 |
| Czechoslovakia | 202 | Colombia | 119 | Finland | 58 |
| Bulgaria | 195 | Austria | 111 | Thailand | 54 |
| West Germany | 187 | India | 107 | Peru | 51 |
| Vietnam | 183 | Israel | 105 | Philippines | 45 |
| Hungary | 175 | Belgium | 104 | Portugal | 39 |
| Yugoslavia | 170 | Republic of Korea | 97 | Ireland | 37 |
| Poland | 157 | Netherlands | 92 | Iceland | 33 |
| France | 156 | Tunisia | 81 | Kuwait | 31 |
| Iran | 147 | Mexico | 79 | Cyprus | 24 |
| Singapore | 143 | Sweden | 73 | Indonesia | 21 |
| Turkey | 133 | Cuba | 69 | Venezuela | 6 |
| Hong Kong | 127 | New Zealand | 69 | | |

# XXX. INTERNATIONALE MATHEMATIK-OLYMPIADE

13.–24. Juli 1989

Bundesrepublik Deutschland
Braunschweig · Niedersachsen

English version

## FIRST DAY

Braunschweig, July 18th 1989

1. Prove that the set $\{1, 2, \ldots, 1989\}$ can be expressed as the disjoint union of subsets $A_i$ $(i = 1, 2, \ldots, 117)$ such that
   (i) each $A_i$ contains 17 elements;
   (ii) the sum of all the elements in each $A_i$ is the same.

2. In an acute-angled triangle ABC the internal bisector of angle A meets the circumcircle of the triangle again at $A_1$. Points $B_1$ and $C_1$ are defined similarly. Let $A_0$ be the point of intersection of the line $AA_1$ with the external bisectors of angles B and C. Points $B_0$ and $C_0$ are defined similarly.
   Prove that
   (i) the area of the triangle $A_0B_0C_0$ is twice the area of the hexagon $AC_1BA_1CB_1$;
   (ii) the area of the triangle $A_0B_0C_0$ is at least four times the area of the triangle ABC.

3. Let n and k be positive integers and let S be a set of n points in the plane such that
   (i) no three points of S are collinear, and
   (ii) for every point P of S there are at least k points of S equidistant from P.
   Prove that

$$k < 1/2 + \sqrt{2n}.$$

Time: 4.5 hours
Each problem is worth 7 points.

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SECOND DAY

Braunschweig, July 19th 1989

4. Let ABCD be a convex quadrilateral such that the sides AB, AD, BC satisfy AB = AD + BC.
There exists a point P inside the quadrilateral at a distance h from the line CD such that AP = h + AD and BP = h + BC.
Show that

$$\frac{1}{\sqrt{h}} \geq \frac{1}{\sqrt{AD}} + \frac{1}{\sqrt{BC}} .$$

5. Prove that for each positive integer n there exist n consecutive positive integers none of which is an integral power of a prime number.

6. A permutation $(x_1, x_2, \ldots, x_{2n})$ of the set $\{1, 2, \ldots, 2n\}$, where n is a positive integer, is said to have property P if $|x_i - x_{i+1}| = n$ for at least one i in $\{1, 2, \ldots, 2n-1\}$.
Show that, for each n, there are more permutations with property P than without.

Time: 4.5 hours
Each problem is worth 7 points.