32ND INTERNATIONAL MATHEMATICAL OLYMPIAD

SIGTUNA SWEDEN

12TH - 23RD JULY 1991

Each year the Leader of the UK IMO team writes a report for the UK organising committee. This year's report is slightly different in that

- *it is deliberately informal in style;*
- *it tries to summarise the whole sequence of events leading from the NMC up to the IMO so that the reader can see how the different stages interact;*
- *copies will be sent to all schools which took part in the BMO in the hope that they will make their own copies for interested colleagues, for students who took part in last year's BMO, and for those who might be interested in taking part in the coming year;*
- *copies will also be sent to all who have contributed in some way to the success of the 1990/1 UK Olympiad program.*

It may be worth stressing that the UK organisation has changed dramatically during 1991.
(1) The sequence of events from the BMO onwards is now run by a committee - the British Mathematical Olympiad Committee - **which is entirely separate from the Mathematical Association.**
(2) The old MA committee which used to run everything - the National Committee for Mathematical Contests - is now responsible **only for the NMC.** *They hope that, with the introduction of certificates, improved publicity and more attractive questions, the NMC will expand considerably in the years ahead.*
The BMOC had its first meeting in June this year and is in the process of rethinking its strategy. (If you have any thoughts on how you think the BMOC should develop its activities, please write to me before 14th September 1991.)

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General Background

For many countries the International Mathematical Olympiad marks the climax to their own domestic program of Mathematical Olympiads. At the IMO, each country can enter up to six students. This year 318 students from 56 countries took part.

It is easy to dismiss an event which involves only six students from each country as being *irrelevant* to most able mathematicians. However, the students who represent each country are chosen from a much larger group - namely all those who take part in the various stages of that country's national Olympiad sequence. Most of these students do not expect to reach the final six; they simply enjoy the experience of trying to solve unusual and challenging problems up to the highest level they can reach.

Thus one should not view the IMO in isolation. The sequence of events which leads up to the selection of the UK IMO team is intended to stimulate and challenge a much larger group. In the process, hundreds of students achieve remarkable results. All are to be congratulated.

Mathematicians come in all shapes and sizes, and develop in different ways and at different rates. Olympiad competitions - especially the experience of being completely stumped by harmless looking problems - should encourage all of us to aim a little higher. Some have shown by their success what they *could* achieve in the future if they apply themselves; others have glimpsed a world of hard but worthwhile problems which are at present beyond their powers, but which - given time and effort - could be mastered.

One member of this year's IMO team illustrates this point to perfection. Michael Fryers (Altrincham Grammar School) got into last year's team for the first time by the skin of his teeth. In the 1990 IMO in Beijing he acquitted himself well, but certainly not outstandingly, by scoring 18 out of 42. Fortunately the team also contained two students (Oliver Riordan from St Pauls and Vin de Silva from Dulwich College) who showed just what could be achieved, scoring 40 and 39 respectively out of 42. The point was not wasted on young Michael. Read on!

The UK Selection Process

On 30th November 1990 nearly 17000 students took the ***National Mathematics Contest*** - a 1½ hour multiple choice paper for students in their last two or three years at school. Almost 1000 of these received Gold certificates, 2000 received Silver and 3000 received Bronze.

On 16th January 1991 400 of the best students tackled seven problems on the 3½ hour *British Mathematical Olympiad*. Olympiad problems are not just hard A level problems: they force students to *think* and cannot usually be solved by merely applying the right standard method. For students who have been trained on a diet of A level papers, and who have come to expect all mathematical problems to be as predictable as A level questions, the experience of facing genuinely hard problems for the first time can be rather daunting.

When faced with a hard problem, it is always tempting to give up too easily. But the important thing in any Olympiad (as in real mathematics!) is *not* to give up, but to struggle on and to try to solve one or two problems *completely*. Those who managed this on the BMO had already achieved something substantial, even if it was only the top 26 students who received book prizes.

In the light of each student's performance in both the NMC and the BMO (with more emphasis on the BMO score!) 60 students were invited to take another 3½ hour paper (FIST) on February 28th 1991. This paper contained just four harder questions (though still somewhat easier than real IMO problems). Those chosen were not simply the top 60 students. Some allowance was made for age, since we felt that some younger students who had shown notable promise needed the experience of trying to solve mathematical problems at this level under timed conditions.

Selection is always difficult. Limitations on numbers mean that we often have to choose between individuals who cannot be easily 'ranked'. We hope those we leave out appreciate this, and find encouragement in what they *have* achieved rather than disappoinment at just missing out.

The next stage was to choose 20 students for a 3 day residential "training" session held in Trinity College, Cambridge from April 4th-7th 1991.

IMO problems are substantially harder than BMO or FIST problems, so some kind of further preparation is essential. However, there is a limit to how much one can achieve in just three days. As long as the summer term continues to be dominated by public examinations, it is hard to see how the UK team can do much by way of serious, intensive, residential training. Thus, unlike many other countries, the UK has little option but to look for ways of encouraging students to do their own preparation in their own time.

The Cambridge session is very valuable in providing basic instruction in Algebra, Combinatorics, Geometry, Inequalities, and Number Theory, and general sessions on *How to attack problems which look horribly hard*, or *How to write out solutions*. But if the session is to bear fruit, participants must then go home and do a lot more work on their own.

On the last morning of the training session the students take the final selection test (SIST) after which the IMO team of six is selected.

Many of those who are invited to the session have one or more years left at school. We depend on these younger participants being sufficiantly motivated to go home and do the necessary extra work for themselves. We are delighted that this year the younger students who did not make this year's IMO team have continued to send in solutions to problems thoughout the summer. We hope that next year their sights will be set that much higher. But, above all, we hope that they will have benefitted mathematically from what they have done this year.

The 32nd International Mathematical Olympiad

This year's UK IMO team consisted of

Michael Fryers	(Altrincham Grammar School)
Oliver Johnson	(KES, Birmingham)
Robin Michaels	(Haberdashers Askes Boys School)
Luke Pebody	(Rugby School)
Adam Shepherd	(KES, Birmingham)
Stephen Wilcox	(Portsmouth Grammar School)

The Team Leader was Tony Gardiner (University of Birmingham) and the Deputy Leader Paul Woodruff (Dulwich College).

Having succeeded in our own Olympiad sequence these six students were now faced with a much tougher challenge. The IMO problems they were to face in Sweden in July would be harder than anything they had seen up to that point. Moreover, the IMO requires not only creativity, but also reliability: it is not enough that one *could in principle* solve this or that problem - you have to be able to come up with the right ideas *there and then*.

The six were encouraged to do all the preparation they could. But with exams and other pressures, the only formal requirement was that they should send in solutions to three IMO-type problems every two weeks between late April and late June. The team also met in Birmingham for 2½ days (3rd-5th July) to 'warm up' and to generate a sense of communal purpose shortly before leaving for Sweden.

The IMO competition consists of two 4½ hour papers taken on successive mornings (17th and 18th July). Each paper contains three problems, and each problem is worth 7 marks. The problems are tough, and some are tougher than others. My guess is that most experienced professional mathematicians would be lucky to solve two problems completely and correctly on each day. Thus a score of more than 30 is a remarkable achievement.

Many official contestants have the 'advantage' of having had weeks of intensive preparation and training. If one is going to enter an international competition of this type at the very highest level, then it is sensible to do enough serious preparation to allow one's students to get the most out of taking part. This is not quite as simple as it sounds. Olympiads are meant to *encourage* the mathematical development of exceptional students. There is obviously a danger that some countries may become so obsessed with 'success' at the IMO that they will adopt intensive training methods over long periods which may *distort* the mathematical development of their best students. Most countries understand this danger, and I was, on the whole, most impressed with the sensitive way many countries select and prepare their teams: we have a lot to learn from them.

Next we come to this year's two IMO papers.

Paper 1

1. Given a triangle ABC, let I be the centre of its inscribed circle. The internal bisectors of the angles A, B, C meet the opposite sides in A′, B′, C′ respectively. Prove that

$$\frac{1}{4} < \frac{AI.BI.CI}{AA'.BB'.CC'} \le \frac{8}{27}.$$

2. Let $n>6$ be an integer and $a_1, a_2, \ldots, a_k$ be all the natural numbers less than n and relatively prime to n. If

$$a_2-a_1 = a_3-a_2 = \ldots = a_k-a_{k-1} > 0$$

prove that n must be either a prime number or a power of 2.

3. Let S = {1,2,3,...,280}. Find the smallest integer n such that each n-element subset of S contains five numbers which are pairwise relatively prime.

Paper 2

4. Suppose G is a connected graph with k edges. Prove that it is possible to label the edges 1,2,3,...,k in such a way that at each vertex which belongs to two or more edges the greatest common divisor of all the integers labelling those edges is equal to 1.

5. Let ABC be a triangle and P an interior point in ABC. Show that at least one of the angles ∠PAB, ∠PBC, ∠PCA is less than or equal to 30°.

6. Given any real number $a>1$, construct a bounded infinite sequence $x_0, x_1, x_2, \ldots$ such that

$$|x_i - x_j| \cdot |i-j|^a \ge 1$$

for every pair of distinct non-negative integers i, j.

Of the six problems, Question 2 is probably the kindest (though it may not look that way under IMO conditions).

Two of the problems (1 and 5) involve *geometry* and *inequalities* (though in a very different way, and neither problem involves much *pure* geometry). Any reasonably prepared team from a country the size of the UK should have scored almost full marks on these two problems, whereas we scored only 42 out of a possible total of 84! There is plenty of scope here for improvement. Our weakness in Question 5 (where we scored only 15 out of a possible 42) contrasts starkly with our performance on the very tough Question 3.

On Question 3 we scored a remarkable 26 out of a possible 42 - a score which was bettered only by USSR (34), FRG (33), China (31), Hungary (30), Japan (28), and Roumania (27). This shows what our students were capable of mathematically, and suggests that marks were simply thrown away on Question 5.

Question 4 was an attempt to create a precedent, so that in future questions explicitly involving graphs might be accepted as conforming to the (*unwritten!*) IMO syllabus.

At the IMO roughly 1/12 of participants receive *First Prizes* (=Gold medals), 1/6 receive *Second Prizes* (=Silver), and 1/4 receive *Third Prizes* (=Bronze). This year 20 students (those with scores ≥ 39/42) received Gold, 51 students (those scoring ≥31) received Silver, and 84 (those scoring ≥19) received Bronze. There is also a category called *Honourable Mention* for any participant who does not receive a medal but who scores full marks for solving at least one problem completely and correctly.

From time to time any country as big as the UK is bound to throw up the odd truly exceptional student. If we are lucky we may even find that our IMO team regularly contains at least one such student. However, such students should not distract us from what I believe to be the main task, which is to *raise the performance of the team's lowest scoring member* in a predictable and controlled way. Too often good students go to the IMO and turn in what one can only describe as a *shell-shocked* performance, scraping up scores of 1, or 2 (and maybe one 3) on each question.
I believe we should concentrate our efforts in the immediate future on devising ways of selecting and preparing students which will allow us to reliably produce teams whose minimum score is in the low 20s.

I set this year's UK IMO team a simple goal. I suggested that all six students should aim at achieveing either a medal or an *Honourable Mention*. (Those who knew they should be aiming much higher than this did not need to be told!) They worked hard throughout the summer term, and I am delighted to be able to report that they did exactly what I asked, at the same time raising the minimum score to 17. Here are the results:

Michael Fryers	42	(Gold)
Robin Michaels	29	(Bronze)
Stephen Wilcox	20	(Bronze)
Oliver Johnson	17	(H.M.)
Luke Pebody	17	(H.M.)
Adam Shepherd	17	(H.M.)

It would have been nice to come home with six medals (Oliver, Luke, and Adam missed Bronze by just 2 points, and Robin missed Silver by the same margin). But their achievement should not be underestimated. They deserve our hearty congratulations.

Clearly the most remarkable achievement is that of Michael Fryers in obtaining a perfect score. Eight other students also scored full marks: 4 from the Soviet Union, and 1 each from China, France, Hungary, and Roumania. These are all remarkable students (for example, Vincent Lafforgue from France also had a perfect score last year, and one young girl from the Soviet Union - Evgeniya Malinnikova - has had perfect scores for three years in a row). But all, except possibly the student from France, have had the advantage of intensive preparation provided by experienced adults. In contrast, Michael's success (like that of our two outstanding students last year) is largely due to his own efforts. The enclosed article celebrating his achievement comes from the Daily Telegraph of 25th July 1991.

The IMO is full of lovely surprises. One other outstanding achievement was that of the young Swiss girl Bea Wollenmann. She found out about the IMO only to discover that the Swiss had never taken part. However she was so determined to go, that she chose her own Team Leader, raised the money to get them both to Sweden, and went home with a Bronze medal having achieved the second highest score by a female participant!

How the 32nd IMO was run

That the IMO takes place at all is an annual miracle. It is not run by any organisation, nor guided by any international quango. It started in 1959 and for the first nine years was restricted to a small group of East European countries (Bulgaria, Czechoslovakia, East Germany, Hungary, Poland, Roumania, Soviet Union, and Yugoslavia). Some of these had a long tradition of mathematical Olympiads: for example, the Hungarian Eötvös Competition started way back in 1892. Moreover, these countries all make a habit of encouraging academics to work with and to write for bright High School students. Just as we began to ignore our own 1944 Education Act with its bold vision of providing for children "according to their needs", these rather different countries were busy taking the challenge seriously.

In 1967 13 countries took part in the IMO, including *for the first time* four western countries - among them the UK and this year's hosts Sweden. The 1979 IMO was held in London with around 25 countries. The number of participating countries expanded massively at the end of the 80s, and at this year's IMO 56 countries took part, with 3 more having official *Observers* with a view to joining in next year.

Each year one country agrees to act as host and is then responsible for all expenses from the time teams arrive at the airport on Day 1 to their departure after the IMO. This represents a substantial commitment in both money and manpower, and I suspect that it takes the host country three or four years to prepare for each IMO. This year's official hosts were the *Swedish Mathematical Society*. They did a very fine job.

All practical arrangements clearly have to be made by the local organisers long in advance. But the body with formal responsibility
for choosing the problems
for marking the scripts and for approving the awards
for making any other relevant decisions
is the *IMO Jury*. This year's Jury came into existence on Day 2 (13th July) and ceased to exist on Day 11 (22nd July) at the closing ceremony. Membership of the Jury is restricted to the Leaders of Participating Countries for that year. Noone else is allowed to vote (or to contribute in any way unless invited to do so).

How are the problems chosen? Each participating country is asked to submit up to six problems of an appropriate standard. The resulting collection is whittled down to a shortlist (this year of 30 problems) by a committee of experienced mathematicians from the host country. The Jury could theoretically overturn their judgement, but is usually extremely grateful for this preparatory work: without it the delicate task of choosing problems agreeable to *all* participating countries would be almost impossible. This year was no exception. One valuable innovation this year was that on arrival, members of the Jury were given the shortlisted problems *but not the solutions*. This gave us 12 hours to work on the problems and to decide for ourselves how hard, or how suitable, the problems were without being distracted by 'official' solutions. (In this time I managed to solve 8 or 9 of the problems and got a reasonable impression of what the others involved.)

The Jury has to work quite hard. First unsuitable problems are eliminated:
- problems which are much too hard or much too easy, or
- problems which will lead to solutions which will be impossible to mark (in 36 different languages!), or
- problems which have already appeared somewhere in the literature, or
- problems which have been used in previous competitions unknown to the shortlisting committee, or
- problems which are very close to problems which one or more countries have used during training.

Next problems which are popular with many Jury members are identified. These are then classified as being either *easyish*, *middling*, or *hard*. Roughly speaking, the Jury tries to choose the first problem on each paper (Questions 1 and 4) to be relatively straightforward, but feels free to make the third problem each day (Questions 3 and 6) genuinely tough.

The whole system depends on trust and honesty. Team Leaders have all worked long and hard with their teams and clearly want their team to do as well as possible. With modern communications technology it would be easy to cheat if one was determined to do so. Despite this, the atmosphere in general struck me as being very open with remarkably little petty rivalry. However, to help protect us from temptation, the Jury arrives three days early and is held incommunicado at an unknown location away from the teams while it does its work. This year the Jury was based in Uppsala from July 12th-18th and had completed most of its work before the teams arrived. The teams arrived on July 15th and were housed in Sigtuna (50kms away!).

Once the 6 problems have been chosen and approved the Jury has to agree on the precise wording of each question, and has to check and approve versions in the five official languages - English, French, German, Russian and Spanish. It is very important to get these right, as countries whose students usually work in other languages must then translate one of these official versions into their own language. These translations must then be checked and approved. This year there were 36 different language versions in seven or eight different alphabets.

All this and other Jury business takes three very full days. On July 16th, while the teams were settling in and the papers were being duplicated for the following day, the Jury was taken off to visit a famous 17th century ironworks and a 20th century nuclear power station. After three days with our heads down, this provided opportunities for Leaders and Observers to get to know each other more informally and to share their experiences from running their own national Olympiads and from training their own IMO teams. We returned in the evening and dressed for the Opening Ceremony, which was held in the main hall of Uppsala University. The University was founded in 1477 and is the oldest in northern Europe. The hall was built to commemorate the 400th anniversary in 1877. For the Opening Ceremony the Jury and the teams are kept strictly apart. I found this gave rise to strange emotions. We were only too aware of what our teams would face the very next morning. We had been an integral part of their preparations, and would share in their success or failure. Yet we were unable to join them!

The 56 teams had not yet had much opportunity to get to know each other. Little could one imagine how different things would be just six days later at the Closing Ceremony, Banquet, and Entertainment.

Two days later the teams had finished their hard work -though ours had begun in earnest. The fact that each participant had gone through fire along with 317 other students provided a common bond which made it easier for groups of students from different cultural backgrounds to discover that they were all made of the same flesh and blood. The second exam was followed by a football knockout competition in which flesh collided with flesh and blood flowed freely. We were beaten 5-0 by Spain!

For some reason this particular encounter did not seem to blossom into more cordial Anglo-Hispanic relations. But solid links were soon forged with the Icelandic, the Irish, the New Zealand, the Australian teams, and others. One UK team member invested a considerable amount of effort pursuing a cuddly Koala, while another definitely preferred Kiwi, and a third was rumoured to have played bridge continuously for 18 hours. The Canadians and Americans collected signatures on their T-shirts, while a Slovenian wore a Slovenian independence T shirt to collect his medal, explaining that - no matter what the official program might say - he saw himself as representing Slovenia rather than Yugoslavia.

The emerging patchwork of social interactions was only glimpsed by the hard-driven Leaders and Deputy-Leaders who were now embroiled in the process of marking and grading students' scripts. The logistics of coordinating and judging the marking of 6 problems from each of 6 students from 56 countries in 36 languages - all in less than 48 hours - is truly mind-boggling.

The Swedish judging process struck me as impressively firm and admirably fair. I found the whole procedure - in which Leaders have the responsibility to present their students' solutions clearly and in the best possible light to the judges, whose job it is to ask all sorts of awkward questions to identify the weaknesses and limitations of each solution before finally deciding on a mark - a new and rewarding experience.

As each new set of marks (say, for the six UK students on Question 1) were approved, they were pinned up in the foyer of the main buiding. The intense interest shown by students in the successes of other countries as well as their own was quite moving.

On Friday afternoon our youngest student admitted that he had had a curious pain in his lower stomach ever since Tuesday morning! He was whisked off to hospital and diagnosed as having a twisted lower intestine. Prompt action eventually avoided the need for surgical intervention.

On Sunday all 450 students and staff were let loose on Stockholm. We visited the impressive Vasa museum, went on a long harbour cruise, and ambled through the narrow streets of the old town. Months of hard work and preparation were at an end, and most of us drifted through the day. We all had our successes (individuals or groups who had excelled themselves, surpassing our expectations). And most of us also had our disappointments. But there was nothing we could do now except pick ourselves up slowly and begin to look ahead to what we might do in the coming year.

On Monday most people went to Uppsala for the day. But some preferred to enjoy their first leisurely breakfast, followed by a stroll round Sigtuna itself - an old small town which was for hundreds of years the capital of Sweden, until that role passed to Uppsala and Stockholm. After lunch we packed and dressed for the Closing Awards Ceremony, which was followed by the banquet and entertainment in the City Theatre.

We returned to Sigtuna, happy but tired, around 11.30. Our bus to the airport was to leave at 6.30 a.m., so a good night's sleep was already out of the question. I sat round talking to other Leaders and Deputies, drinking tea (or was it Tequila? I forget.) until about 3.00 before going to bed. The students decided it was safer not to got to bed at all!

I have enjoyed working with them and with all those who took part in earlier rounds of our Olympiad program. I wish you all well in the coming year. I hope those of you who will still be at school next year will be back to do battle with some good problems in next year's Olympiad sequence. If you can find time to reread your BMO Link Booklet and work on some BMO past papers *before* the Christmas holidays, so much the better!

But first make sure your school has entered the NMC on November 22nd 1991. Entry forms may be obtained by sending an SAE to:

NMC, c/o Dulwich College, London SE21 7LD

Any enquiries about the BMO should be addressed to:

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