

# 41st INTERNATIONAL MATHEMATICAL OLYMPIAD

## TAEJON, SOUTH KOREA

13-25 July 2000

### Report by Imre Leader (UK Team Leader)

This is a report on the 41st International Mathematical Olympiad, which was held in South Korea in July 2000. The IMO is the pinnacle of excellence in mathematics for school pupils the world over. Every year, around 80 countries each send a team of 6 contestants to the IMO. There they sit two  $4\frac{1}{2}$ -hour exams, each containing just three questions. Medals are awarded for good performances. This year the host country was South Korea, which has not hosted an IMO before.

Let us start with some of the events leading up to the IMO. The selection of the team started with the Senior Mathematics Challenge, a multiple-choice paper sat by more than 30000 students, taken in November 1999. The SMC lasts 90 minutes, and consists of 25 questions, of which the first 15 are meant to be widely accessible and the last 10 rather more testing. Based on their performance in the SMC, around 800 contestants proceed to the next round, the BMO1. This is a far, far harder paper, containing just 5 questions to be done in  $3\frac{1}{2}$  hours. Anyone who solves a BMO1 question has reason to feel pleased with himself/herself! The BMO1 is held in mid-January, and is followed by an amusing weekend in which 20 or so academics, teachers and ex-olympians gather together to mark the scripts.

After BMO1, about 100 pupils qualify for BMO2, which is a still harder exam, consisting of 4 questions to be attempted in  $3\frac{1}{2}$  hours. Based on BMO2, 20 pupils are selected for the Trinity Training Session at Easter – these 20 include those who we feel are realistic contenders for the team, and also some younger pupils who we believe are good prospects for the future.

The Trinity Training Session is an extremely intense and exciting experience for everyone. It lasts four days. For the first three days, the students have a variety of sessions,

some taught to the whole group of 20 and some taught in groups of 6 or 7. The emphasis is on the students trying problems: the actual amount of ‘lecturing’ is kept to a bare minimum. The final day is by contrast rather different. The main event is the last of the selection exams, the Final Selection Test. The FST is designed to resemble a real IMO paper: there are just 3 questions, and the time allowed is  $4\frac{1}{2}$  hours.

In the next day or so, a squad of 8 is selected. The choice is based on performance in FST, BMO2 and BMO1, and also on how the students have performed during the Training Session. The 8 are notified within a few days of leaving Trinity, and they then embark upon the final and most gruelling part of the selection. This is the dreaded Correspondence Course. Each week to 10 days, the students are sent a sheet of about 8 hard problems. They send in their solutions, which are marked by the Leader (myself) and the Deputy Leader (Richard Atkins, Head of Maths at Oundle). After about five rounds of this, the team of six is chosen, with the other two acting as reserves. Of course, the two reserves contribute immeasurably to the success of the team, as their presence during the training course has forced people to work hard for their places in the team!

In the week before the IMO itself, the team gather at Birmingham, where the Summer School for younger pupils is held. As well as participating in some of the events of the Summer School, the team receive some final training and preparation.

This year, the squad of 8 was as follows.

**Team:**

Thomas Barnet-Lamb (Westminster School)

Stephen Brooks (Abingdon School)

David Collier (King Edward VI School, Southampton)

David Knipe (Sullivan Upper School)

Michael Spencer (Lawnswood High School)

Oliver Thomas (Winchester College)

**Reserves:**

Hannah Burton (City of London Girls School)

Kerwin Hui (Berkhamsted Collegiate School)

Of these 8, Thomas and Ollie were ‘returners’ from last year’s IMO in Romania (where

each won a Silver medal). Stephen and Michael were last year's reserves.

Next came the IMO itself. The IMO, for the students, was to start in Taejon on July 16th, but the Team Leaders flew in three days early, to select the questions that would be used. Each country has the right to submit some questions (months in advance); the host country then narrows these down to a short-list of about 25 questions, and it is from these that the six must be chosen. For those three days, the Leaders were kept in a secret location, far from Taejon – in fact, they were allowed no contact at all with the teams or the Deputy Leaders until the last exam had finished, for obvious reasons! This year, the Leaders met in Chonan, in a large complex just outside town that is normally used for training communications industry workers.

Some countries send an 'Observer' with the Leader or Deputy: this is usually someone who will do the job in a later year, and is coming along to see how things work. This year, however, things were very different for the UK, as we will be hosting the IMO ourselves in 2002. For this reason, we sent plenty of Observers. As Observers with Leader, there were Adam McBride, Duncan Harvey and Robert Smart, while as Observer with Deputy we had Alex Barnard.

The Jury chose the questions, and supervised the various translations. This year, 82 countries participated, which necessitated more than 50 languages. The Jury consists of all the 82 Leaders: as one can imagine, a committee of this size functions in a rather chaotic way, but it does seem to reach sensible decisions.

Meanwhile, the team, led by Richard Atkins, had arrived in Taejon. After acclimatisation, and a pleasantly short Opening Ceremony, the actual exam dates were July 19th and 20th. There then followed a period of 48 hours of intense activity for the Leaders and Deputies. The Leaders move to a hotel in Taejon, and the Deputies join them there to help with the marking of the exams (a considerable increase in luxury from their previous accommodation with the students). Each country marks its own students' scripts, and then goes to 'coordination' for each question: this involves meeting with two Korean mathematicians and agreeing on marks. Finally, totals are worked out, and the cutoffs for medals established. The rough principle is that the ratio of Gold to Silver to Bronze to no medal should be very close to 1 to 2 to 3 to 6.

After a day of socialising, there was a Closing Ceremony, at which the medals were

awarded. Everyone flew home the next day.

Now on to the papers. Each day has three problems, to be done in  $4\frac{1}{2}$  hours, with each question worth 7 points.

### FIRST DAY

Problem 1. Circles  $S_1$  and  $S_2$  intersect at  $M$  and  $N$ . Let  $L$  be the common tangent to  $S_1$  and  $S_2$  that is closer to  $M$  than to  $N$ , and let  $L$  touch  $S_1$  at  $A$  and  $S_2$  at  $B$ . Let the line through  $M$  parallel to  $L$  meet  $S_1$  again at  $C$  and  $S_2$  again at  $D$ . Let the lines  $CA$  and  $DB$  meet at  $E$ , let the lines  $AN$  and  $CD$  meet at  $P$ , and let the lines  $BN$  and  $CD$  meet at  $Q$ . Prove that  $EP = EQ$ .

Problem 2. Let  $a, b, c$  be positive real numbers such that  $abc = 1$ . Prove that

$$\left(a - 1 + \frac{1}{b}\right) \left(b - 1 + \frac{1}{c}\right) \left(c - 1 + \frac{1}{a}\right) \leq 1.$$

Problem 3. Let  $n \geq 2$  be a positive integer and let  $\lambda$  be a positive real. Initially, there are  $n$  fleas on a horizontal line, not all at the same point. A *move* is defined as follows: choose any two fleas, at points  $A$  and  $B$ , with  $A$  to the left of  $B$ , and let the flea at  $A$  jump to the point  $C$  to the right of  $B$  with  $BC/AB = \lambda$ . For each value of  $n$ , determine all values of  $\lambda$  such that, for any point  $M$  on the line and any initial positions of the  $n$  fleas, there is a finite sequence of moves that will take all the fleas to positions to the right of  $M$ .

### SECOND DAY

Problem 4. A magician has 100 cards, numbered 1 to 100. He puts them into three boxes, a red one, a white one and a blue one, in such a way that each box contains at least one card. A member of the audience selects two of the three boxes, chooses one card from each, and announces the sum of the numbers on these cards. Given this sum, the magician identifies the box from which no card has been chosen. How many ways are there to put the cards into the boxes in such a way that this trick always works?

Problem 5. Does there exist a positive integer  $n$  having exactly 2000 prime divisors such that  $2^n + 1$  is divisible by  $n$ ?

Problem 6. Let  $A_1A_2A_3$  be an acute-angled triangle, with altitudes  $A_1H_1, A_2H_2, A_3H_3$ . The incircle of  $A_1A_2A_3$  touches the sides  $A_2A_3, A_3A_1, A_1A_2$  at  $T_1, T_2, T_3$  respectively. Let the lines  $L_1, L_2, L_3$  be the reflections of the lines  $H_2H_3, H_3H_1, H_1H_2$  in the lines  $T_2T_3, T_3T_1, T_1T_2$  respectively. Prove that the lines  $L_1, L_2, L_3$  determine a triangle whose vertices lie on the incircle of  $A_1A_2A_3$ .

*You are invited to send in solutions, enclosing an SAE please, to:*

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The countries of origin of these questions were Russia, USA, Belarus, Hungary, Russia and Russia, respectively. Although none of the United Kingdom's problems were included, a remarkable three of them made the short-list: two by our veteran problem-setter David Monk (Edinburgh University), and one jointly by myself and Peter Shiu (Loughborough University). For one person to have two questions on the short-list is extremely rare!

The team did very well, with everyone securing a medal. The total haul was two Silver and four Bronze. As a whole, the UK team's total score of 96 points (out of a possible 252) put us 22nd of the 82 countries. When looking at such results, it is important to bear in mind that our training programme is tiny compared with those of other countries (most of which have training camps lasting one or several months, and in some cases even a whole year!). Indeed, perhaps a more revealing statistic is that, among the Western European countries (where long training camps are the exception rather than the rule), we came 2nd, beaten only by Germany. Here are the top 10 teams, with their scores:

China 218, Russia 215, USA 184, South Korea 172, Bulgaria and Vietnam 169, Belarus 165, Taiwan 164, Hungary 156, Iran 155.

Here are our individual scores, with marks per question:

	Q1	Q2	Q3	Q4	Q5	Q6	Total	
Thomas Barnet-Lamb	6	0	0	7	7	1	21	Silver
Stephen Brooks	5	0	4	7	1	0	17	Bronze
David Collier	7	1	0	7	7	2	24	Silver
David Knipe	0	7	1	4	0	0	12	Bronze
Michael Spencer	2	2	0	6	1	0	11	Bronze
Oliver Thomas	7	1	0	1	0	2	11	Bronze

Four contestants scored full marks: two from Russia, one from China and one from Belarus. The cutoffs for medals were 30, 21 and 11 for Gold, Silver and Bronze respectively.

The IMO was a magical experience for the Leaders as well as for the Teams. For the Leaders, there was the wonderful initial 3 days of meeting as the Jury, all thinking and arguing about the problems and discussing different ways to solve the questions, then the unbelievably hard two days of marking and coordination, and then the last few days socialising with the team and other teams. For the contestants, there was the rather awe-inspiring arrival, being plunged among the best few hundred school pupils in mathematics from around the world, then the tension of the actual exams, then the sometimes equally great tension of waiting for the results, and in amongst all this the meeting with other teams from different parts of the world.

Perhaps the best way to convey some of the atmosphere of the IMO is to record my daily diary of some of the things that happened.

**July 12:** I meet up with Duncan and Robert at Stansted. Duncan is coming with a view to getting more involved with the role of Deputy – he would normally be Observer with Deputy, but that place is filled by Alex Barnard (who is coming along to see what the IMO is like, as possible preparation for IMO2002). Robert is one of the main people involved with the organisation of IMO2002, so he is coming to see how the IMO works in practice. The third Observer with Leader, Adam McBride, has left two days earlier, as he is on the IMO Advisory Board, a committee of about ten respected people who make sure that the IMO progresses successfully from year to year.

We change planes in Frankfurt, and it seems that our flight on to Seoul is the ‘IMO

flight', as I spot the Leaders from Belgium, Germany, Italy, Luxembourg and Spain on the same plane. We settle down for the long flight.

**July 13:** After some terrible in-flight movies, we arrive at Seoul around noon local time (Korea is eight hours ahead of the UK). Just before landing, I rush off to the toilet to put on gallons of mosquito repellent. Everyone laughs at me: although my doctor had told me to wear it 24 hours a day, it seems everyone else has been told that there is no problem as long as we are not out in the bush. (No-one has any problems with mosquitos, so it seems the majority were right.)

As we walk out of Customs we are astonished to see an array of TV cameras and bright lights. The IMO seems to be big news in South Korea. We head for the large IMO placard, and meet up with old friends from last year. Until that moment, we have not known where we will be going (for security reasons: the one thing we know is that it will *not* be Taejon!). We are told that we are going to Chonan, a medium-sized city about halfway to Taejon.

The air is incredibly hot and humid as we step across the tarmac to the coach. En route to Chonan, we stop to pick up the IMOAB, who have been meeting in Seoul but are now coming with to Chonan, so we meet up with Adam.

The venue at Chonan is very impressive. It looks like something out of a James Bond film. A huge institute, in acres of grounds, with no other habitation anywhere near, has been given over to us exclusively. This is where the future Korean leaders in the field of communication are trained. There are about 1000 rooms, but it is being used just by the 150 or so of us (Leaders, Observers, and local organisers). No-one can believe the sheer scale of the place.

As we check in, along with our room keys we are given the shortlisted problems, minus solutions. We have about 24 hours to look at these problems and form our opinions of them before the solutions are given out. This is one of the best parts of the IMO. There are so many beautiful questions that one does not know where to start! It is utter joy as we go off to our rooms to have a think about some of them.

The time passes quickly, and soon it is time for dinner. This is a huge buffet, containing both Western dishes and, more interestingly, Korean ones. Most of the food seems very

spicy (like Chinese food, but hotter), and rather delicious, particularly a beef stir-fry called bulgogi. This is something of a national dish, it seems, as we have it several times in our stay. Another national dish is kimchi, which is pickled cabbage with a very hot red sauce on it. This is served as an accompaniment to *every* meal, even breakfast.

After dinner, off to play table tennis with the Leader from Ecuador, who is well known for his Chinese-style serve. Then back to thinking about the problems. It is rapidly becoming clear that they are a *very* nice set.

**July 14:** Breakfast includes, as well as kimchi, most other possible kinds of vegetable (cauliflower, broccoli, carrots, tomatoes, cucumber). Plus plenty of eggs, bacon, sausages and so on, luckily. Then back to the problems. Adam is obsessed with a functional equation, and Duncan is looking at a three-dimensional shape-packing problem. I discover that the New Zealand Leader (new this year) and I get on well together, and we work together on several problems.

At 4.30 comes the first Jury meeting. After various unexciting speeches of welcome, we get on with our preliminary work. This consists in our throwing out questions that have been seen before (or that are very close to such questions). However, the Problems Committee (who had selected the short-list) have done a very thorough job, trawling through every possible book or magazine of problems, so only one problem on the short-list is rejected at this stage.

Dinner has lots of fantastic sushi. Including some hard-to-identify things that turn out to be sea cucumber (delicious, but very chewy) and marinated squid (even chewier, so that one really has to ‘chew to exhaustion’).

The solutions have been given out before dinner, so everyone is keenly flicking through the booklet. Then more table tennis and also some drinking. There is a large computer room, and various Leaders who cannot live a day without email go off there.

**July 15:** Over breakfast, it turns out that several Leaders have been ill in the night. After some questioning, it transpires that they are exactly the Leaders who have drunk water direct from the tap. I am thankful for my bottled water.

Now comes a whole day of Jury meetings. We are starting off with a ‘difficulty poll’:

each Leader is given a form on which to grade each problem on the short-list as easy, medium, hard or very hard. Then it is decided that we should have another column, marked ‘good for IMO?’. We are all set to go when someone points out that we do not know what ‘good for IMO’ means. Does it mean ‘suitable for an IMO question’ or ‘I like this question’ or ‘I would like this to be one of the six questions chosen’? There is much discussion, mainly by people saying it is ‘obvious’ what it means. We eventually go for ‘suitable’, and then it is suggested that we also mark six questions as being our favourites – note that this may well be different from the six questions we would like to see on the paper, for example because of balance.

Suddenly, a Leader requests that the Organising Committee should be making notes of what everyone is saying, as ‘it is hard to remember afterwards’. I think ‘Oh no, that is absurd, and will slow things down incredibly, but the Koreans are so polite that they will think they have to oblige’. Luckily, the Australian Leader immediately gets up and say words to the effect of ‘What utter nonsense’. So the day is saved. In the coffee break, he is surrounded by people saying ‘Well done’.

The results of the vote are announced. There are five problems that stand out head and shoulders above the rest in popularity, and rather amazingly they are a beautiful spread of subjects and a good mixture of levels of difficulty. It seems an obvious choice to go for those five straightaway. But immediately several Leaders speak about ‘caution...must not rush these important decisions...there are many other problems’, so nothing is decided. (As it turns out, all five will later be chosen).

There is a long afternoon break, in which Adam goes ten-pin bowling with the New Zealand and Australian Leaders. The Australian Leader manages to fall. Not just a fall onto the wooden floor, but a fall onto the edge of the gutter that runs alongside the lane, so a nasty sharp edge. He is hobbling, and teased a lot, for the next few days.

That evening there is a banquet, hosted by the Regional Governor. Dish after dish is brought, and we lose count of the number of courses. Then a Korean orchestra plays, the highlight being some very loud drums, played violently yet carefully.

**July 16:** Over breakfast, several of us hatch a plan to get the five popular questions chosen as a package. It will save huge amounts of time, and it seems criminal to waste the

opportunity presented by having popular questions that have good balance in subject and simultaneously in difficulty. So the plan is that the Finnish Leader will, right at the start of the Jury meeting, propose a motion that we adopt these five questions. Then the rest of us will chime in in support.

But, to our horror, the Chairman sees another hand raised first, and so calls on a different Leader, one who had not been privy to our breakfast-time plotting. He proposes that first we select two easy questions. (In the IMO it is traditional to have two easy, two medium and two hard questions, although of course this is very hard to judge, especially if one has not tried the question for oneself but merely read the solution. In any case all of these are relative terms – as none of the shortlisted questions are trivial!)

This seems to meet with assent around the room, so the moment to go for all five together has passed, and now we must get on with choosing in drips and drabs. Two ‘easy’ problems are duly selected, one by one. The breakfast group who had wanted a quick resolution meet in the coffee break, and we are pleased that at least some progress has been made.

On we go. Someone makes a formal motion to adopt problem N3 (one of the five popular ones), but so many Leaders say ‘wait, this may tie our hands in terms of the subject-matter of other questions’ that he withdraws his motion. Then the Turkish Leader makes a plea about problem G8 (one of the least popular, with only one Leader putting it on his list of six favourites), saying that it would make a wonderful hard problem. He speaks very persuasively, and when he makes a motion and we vote the majority are in favour.

Well, except that it seems unclear what the voting rules are. For a motion to succeed, does it need a majority of the votes cast (for and against), or a majority of all votes (including abstentions)? There are different views, but it is decided that in the past the phrase ‘absolute majority’ has meant ‘majority including abstentions’ (although some Leaders have different recollections). So we are told that the motion did not pass after all. (In fact, this problem *will* end up being chosen.)

Someone now proposes a package of N3 and A5 (A5 is another of the favourites). Amazingly, given the earlier (utterly unfounded) worries about N3 by itself, no-one objects

to this, and the pair are voted through easily.

The one remaining favourite that has not been voted through is A1, an inequality question. It is proposed, and there is now a discussion about whether it is really ‘medium’ or not. The trouble is that it has been placed at the top of the ‘algebra and analysis’ page (at position 1), denoting that the Problems Committee think it is the easiest of those questions. Now, it is certainly *not* easy, and in fact is pretty hard as inequalities go. But many Leaders just think ‘Oh, it is number 1, so it must be easy’, with the result that on the difficulty poll it has been overwhelmingly rated as ‘easy’. So now there are some speeches about how A1 is actually not that easy. These seem to carry the day, and A1 is voted onto the paper.

Thus, after lots and lots of discussion, we have the five favourites after all. Now we just need one ‘difficult’ question and the paper will be finished. There are three or four leading candidates, of which the front-runner is probably the geometry question that we had almost voted in before. Another is a strange and unusual question about polynomials. At first I have no strong feelings about this question. But then the Hungarian Leader makes a speech pointing out that, as the question is so unusual, it is a very fair question: it does not give an advantage to over-trained teams. I am completely persuaded, and as he sits down I turn to the American Leader, who sits next to me (in between UNK and USA there should be URU, but the Uruguay Leader is sitting near the other Spanish-speakers), to say how much I agree. But he just says “If you saw this problem, would you want to work on it?” And I realise that it is not a good IMO question after all! His point is that one should have appealing, attractive questions: questions that, if you heard them, you would want to tell your friends.

In general, the American Leader is quite amazing to sit next to. He came to America from Romania, and so is incredibly well versed in all things to do with mathematics competitions. He is full of wise comments, and I get the benefit of many whispered asides. We seem to agree on many things, and often support each other.

The geometry question runs out an easy winner. So now we have our paper. At this point the Problems Committee announce to us the countries of origin of the short-listed problems. We have not known them until now, to discourage ‘political’ voting. We are

astonished at the Russian contribution: not only are *three* of the chosen problems Russian (a record), but all six of Russia's problems had made the short-list. And in fact each of the other three Russian problems had been strongly in the running to be chosen.

Next comes the preparation of the official English-language version. The ten or so Leaders from English-speaking countries retire to a quiet room to haggle over wording. The biggest problem turns out to be 'number of prime factors': does this mean with or without multiplicity? We go to incredible contortions to try to make it clear which one we mean. Then there are several arguments about things like 'which' versus 'that'. It is a typical gathering of mathematicians: pedantry abounds. Although at the same time everyone is doing their best to make things as clear to the contestants as possible.

Later that afternoon, we learn that our team have arrived in Korea. The Cuban team have not been so lucky: their flight has been cancelled, so they must wait a few days for another one.

**July 17:** Now it is time for the English-speakers to relax, while the other official versions are produced (in French, Spanish, German and Russian). It turns out, however, that there is unhappiness about our English version. We (the English speakers) think 'How dare non-native speakers try to tell us how to write English?', but they do have a point. And the Georgian Leader comes up with a choice of wording that we had not thought of but is clearly superior to our own version.

After lunch, more relaxation, while all the translations are made. The computer room is filled with Leaders whose computer screens show a bewildering array of alphabets. There are about 50 languages needed!

**July 18:** The translations are pinned up for us to look at, and we wander around gazing at all of them. Some of them are very beautiful. Then, after lunch, off we go to the Opening Ceremony. This is in Taejon, so we go by coach. (It seems that coach journeys are an integral part of the IMO.) When we arrive at the auditorium, we have to wait without getting off the coach, because some contestants are still milling around outside. Only when they are safely seated are we allowed out. Some of those milling around are our team, so we manage to wave to them.

The ceremony itself is mercifully short. Impressively, the Prime Minister is present, and gives the opening speech. Then some dancing, and a choir of young girls called the ‘Little Angels’ come on. They sing (very nicely) such traditional Korean songs as ‘Banjo on my Knee’ and ‘Old MacDonald’.

Then back to Chonan. After dinner, a lot more table tennis, with the Leaders from Mongolia, Kuwait and Ecuador. The Ecuador Leader has found his special table tennis shoes, and this seems to make him very hard to beat.

Later that evening, we wander into the bar, to find some animated discussion in progress. It turns out that the final problem we had chosen bears some similarity to an IMO problem from 18 years previously (some Leaders have an encyclopaedic knowledge!). There is much argument about how similar it really is, and it is decided that we will discuss it in the Jury the next day.

**July 19:** This is the first day of the competition. For the first half-hour, the contestants have the right to ask questions. They must submit their questions in writing, and these are then faxed to us (as they are in Taejon while we are still in Chonan). When a question arrives in the Jury room, the relevant Leader reads out the question and proposes his response. The Jury discuss this and may alter the wording or even completely change the reply.

Most of the questions are pretty hopeless: questions whose answer is ‘Read the question again’. Some cause great amusement, such as the student who asks ‘What is a flea?’. His Leader proposes ‘A small insect’, but we decide that this does not give the right information, so we amend this to ‘A small insect that jumps’. One important thing is to be consistent: if we answer one question with say ‘No comment’ then we must answer all identical questions with ‘No comment’ as well. This causes some heated moments, if a Leader thinks his student has been unfairly treated.

After the question-and-answer session, we turn our attention to the similarity issue about the final question. But the overwhelming view is that the similarity is very slight, so it is decided that the question will not be changed.

Now is the time for the mark-schemes to be introduced and discussed. The Chief Coordinator comes to tell us the proposed schemes. (Each question has 6 Coordinators,

who work in pairs, plus a Senior Coordinator who moves between the pairs during coordination. And overseeing all of them is the Chief Coordinator.) Some of the mark-schemes look rather vague, and there is much heated discussion. In particular, it seems as though the magician question has a strange mark-scheme: in their efforts to give a few points to people who have just done a bit, the setters have ended up penalising people who have got the question right but with only minor mistakes. In particular, it seems that one can get up to 5 points without having the key ideas, whereas one slip in an otherwise fully correct proof can lead to a score of only 3.

In the afternoon, we are taken to a folk museum: a recreated old village (this may be an IMO tradition, as in the 1999 IMO we did the same thing on the same day). Lots of dancers, plus a spectacular tightrope walker, who at one point shuffles along on his knees. A camera crew ask me to say something ‘short, very short’ about Korea. I say something like ‘Korea is wonderful, people very friendly, sushi delicious’, and they say ‘Yes, thank you, but shorter, please’.

Back at Chonan, we are waiting for the first day’s scripts to arrive. They are due around 10pm, the lateness being because every script must be photocopied before we see them (so that the Coordinators can have a look in advance). But there is a hitch. It seems that two students have inadvertently taken their scripts out of the exam room in their backpacks. The organisers do not want to start photocopying until all the scripts are in one place, which involves sorting this matter out. It turns out that there is no suspicion of foul play, so the two scripts are added to the general pile and photocopying can begin. The new estimated arrival time is midnight.

Our rooms have an intercom system connected to the main reception desk, so we are promised that as soon as the scripts arrive we will be summoned from our rooms. Sure enough, at a quarter to midnight we learn that the scripts have arrived. Everyone rushes to pick up their scripts – the reception area is a sea of people and chatter.

I find a corner and sit down to have a look. I start with the geometry problem, as it is the one I expect the team to have done the worst on. But we have solutions from Thomas, David C and Ollie. Plus a nearly-solution from Stephen. He has clearly been under time pressure, as his work ends with a proof that three triangles are similar and a statement

that ‘the result ought to follow from this’. It does, easily, and with one more minute he would have done it. But he will only lose a point or two for this. Four solutions: I am absolutely delighted.

Then I look at the inequality, which we ought to be good at. And my mood changes. David K has solved it, with a highly original and beautiful proof. But no-one else has. I cannot believe it. Michael has made some progress, but not much.

Finally, I look at the fleas question, and this is even worse. There is only one solution offered, by Stephen. And, on closer inspection, this proof has a fatal flaw. The team have also disobeyed a key instruction: to always hand in all one’s rough work. Either that, or they have spent almost no time thinking about this question, as almost no work has been handed in. The actual answer is easy to guess (although proving it correct is the whole meat of the question), but apart from Stephen only David K has even done that. I am devastated.

Robert has stayed up as well, and we try to find out how other teams have fared. We are next to the American Leader, and soon find out that his team have done well – he is quite cheerful.

**July 20:** The second day of the contest. Another question-and-answer session. Some amazing questions: one student asks, about the magician question, ‘Is the magician told which boxes the two cards have come from?’! We decide that the only possible answer is ‘Read the question again’. Actually, this is almost certainly *more* helpful than just saying ‘No’, because to even ask such a question the student must have made a severe error in his reading, so we do need to signal this to him. One student asks about the number theory question so many times that it is suggested that we should send the reply ‘Read the question for the fourth time’.

Then we are transferred to Taejon. We settle into our hotel, where the Deputies will be joining us to help with the marking. Then we take a taxi to the competition site, to greet our team when they come out from the exam. We see Richard and Alex, and they tell us that the team had real problems on the first day from lack of sleep: the rooms are not air conditioned, and the humidity is very high.

Out come the team, relieved to have finished. We go to lunch with them, and dis-

cover that their food is surprisingly good. In addition to the nice main courses, there are unlimited soft drinks and unlimited ice cream.

Richard and Alex come back with us to the hotel, and we look at yesterday's solutions a bit more. Then the day's scripts arrive, and we rush off to get them. We tear open the folders. Thomas, Stephen, David C and Michael have all done the magician question. Each has done it in his own inimitable style. Thomas has found a wonderful induction way of doing the question, David C has the 'standard' solution, but in very short form, Stephen has a variant on this and Michael has an extremely clever very unusual proof. Our spirits rise.

Then on to the number theory question. A long and ingenious solution from Thomas (so strange that, to this day, I have no idea how he thought of it!), and a short elegant solution from David C. Michael has some clever ideas, but there is an error in his proof, and almost nothing can be salvaged.

The geometry question, Problem 6, is the disaster we had feared: no-one has really got anywhere on it. So, all in all, we are not very happy. But now we must hurry up and prepare for the coordination sessions. These will start tomorrow morning. We go and look to see what our times will be. For each Problem, there are three columns on the noticeboard, giving the times for each team (each column corresponding to one pair of Coordinators). It takes us ages to work out the system the organisers have used. They have tried to put all the top teams together with the *same* pair of Coordinators for each question – clearly sensible for consistency. But this would overload the timing: there is half an hour per team per question, and of course the good teams tend to have more answers, more written, more to discuss. So in among the good teams are a few of the not-so-good teams, just to speed things along! This is all extremely sensible. We are in the top group, and it seems that for alphabetical reasons we will be followed into almost every coordination by the Americans.

**July 21:** We are starting with a bang: 9am is coordination time for us for Problem 6. Actually, we are quite lucky in this, as we don't have much to argue about, so this will probably be the easiest coordination session of all. Of course, however, every point is vital.

Richard, Duncan and I gather outside the room where Problem 6 is being coordinated.

The rules are that only two of us may speak during coordination, so Duncan will be the non-speaking person (we always have to ask permission to bring in an Observer, but this is always granted). In fact, I will also be a more-or-less non-speaker, as it is Richard who has gone over the scripts in detail for this problem. I am very nervous – it is that butterflies-in-the-stomach feeling that everyone who has waited to be called in to an important meeting must know.

We are called in, shake hands with the two Coordinators, and off we go. One of the Coordinators is Marcin Kuczma, who had been the Polish Leader the previous year (the Koreans imported two foreigners to help with problem selection, and also act as Coordinators). He has become a good friend of mine, so it is strange to be facing him across the coordinating table! Ollie and David C have made a bit of progress, and get 2 points each. Thomas has done less, and gets 1 point, which is fair. The rest get 0. So this has gone as expected.

Back to our room we go, to prepare for Problem 2. There may be some trouble with David K's solution. He has two 'without loss of generality's. The first is fine, but the second should come earlier than he has placed it. This is not a problem, as later he has inserted something saying 'oh, please move that WLOG to the relevant place'. But the Coordinators may not have seen that, so we may have to show it to them.

The other script that we are spending time on is Michael's. He has made some preliminary reductions, and has given some vague 'growth speed' reasons as to why what he is left with to prove is true. This will score a maximum of 2 marks. But it occurs to us that 'growth speed' suggests calculus, so that if his proof can be finished off by calculus we can at least say 'look, here is how to make it into a proof'.

I try to do it by calculus. One gets a horrible expression that one has to show is always negative, but I can see no reason why it should be. So I ask Alex to have a look, and in a few minutes he has come up with a clever argument that establishes it. It is for things like this that it is so nice to have plenty of people helping with the marking. Alex, Adam and Duncan all give lots of help to Richard and myself – it is quite a luxury to be able to say to someone else 'look at this script, please'!

Straight after lunch we are on for Problem 2 coordination. For David K's script, the Coordinators say that they have a problem with the 'without loss of generality'. We try

to point out where the error is corrected, but they seem not to understand. It eventually turns out that we have been talking at cross purposes: they are happy about the second WLOG, but it is the first one they want explained. We explain it, and David gets his 7.

When we get to Michael's question, we ask whether a continuation by calculus would be grounds for an extra point for Michael's 'growth speed' nonsense. No, we are told. And this is no surprise, to be honest. So Alex's nice argument never sees the light of day.

Now we have a couple of hours before our final coordination of the day, which will be on the dreaded Problem 4 – the one with the silly mark-scheme. Thomas, Stephen and David C will get 7. There is a gap in Michael's proof, but it is easy to fill (not needing any new ideas), so he will score 5 or 6. The one that will be difficult is David K's script. His neat work is nonsense, but in his rough work there are plenty of ideas. In fact, his rough work contains all the ideas one needs for the proof, but in a strange jumbled-up order. Plus there is one case that he has failed to consider. In a sensible mark-scheme, this would be worth 4 points, but this mark-scheme will only give him 2 or 3, depending on how it is interpreted.

As we wait to go in, I am finding it very hard to keep calm, because I hate the mark-scheme so much: it is so unfair. Anyway, we start off easily enough: 7 for Thomas, 7 for Stephen, 7 for David C. Then we get to David K's solution. The Coordinators offer 2 points, and we explain why we want 3. They stick to 2: they are not being at all flexible. I try to explain that the mark-scheme could be interpreted as giving 2 or 3 (depending on what one counts as 'significant progress'), but that, since the whole scheme is biased towards people who have enumerated a few cases and against people who have tried to solve the problem, we must interpret the scheme in favour of the latter class of people. The Coordinators listen and say 'Yes, we understand what you say, but we are offering 2 marks'.

After some more heated discussion, I sense that we are getting nowhere. It is terribly unfair that David's attempt is not getting the credit it deserves, but I suspect that if we go on arguing then the Coordinators will just say 'well, this is how we are treating everyone', and of course that cannot be argued with. So, to speed things up, I say 'OK, I am prepared to sign if you tell me that you have had similar scripts before that you also gave 2 points to'. (Signing means signing the mark-list, which is the official endorsement that I and the

Coordinators agree to the given marks.)

The Coordinators produce a script and invite me to read it. It is a Russian script, a beautiful solution, with one case overlooked (and remember that this mark-scheme savagely punishes overlooked cases). I say that I imagine this is worth 6 points, and they say ‘Well, we gave this 3 points’. I say that that is terrible, but that of course that solution deserves far more points than David’s, so I am sadly ready to take the 2 points.

Then Svetoslav Savchev (who is from Bulgaria, and is the other foreign Coordinator) says ‘The Russian Leader refused to sign, by the way’! So I also refuse to sign, and we carry on arguing. By now our half-hour is nearly up, and we are getting nowhere. Again I sense that eventually the ‘this is how we have decided to do it’ argument will carry the day, so I say ‘OK, I will sign, but with heavy heart’. I am just reaching out to put the script away when Svetoslav says ‘Remember, once you sign, that is the point of no return’. He is essentially telling me to refuse to sign! This is great evidence that the Coordinators’ role is not just adversarial. We arrange to come back at 9pm that evening.

Over dinner, I chat to the Russian Leader, who is outraged about his student’s script, and also to the American Leader, who has a similar story to tell. Dinner calms us down a bit, as it is so nice: lots of sushi, including some incredibly soft and delicate tuna, and also a large dish of snails (cooked, not part of the sushi).

When we come back at 9pm, we discover that there is a queue of Leaders who are all coming back after refusing to sign for Problem 4. The 8.30 return slot is overrunning, so we are asked to come back the following afternoon instead.

We go back to our room to prepare for the next day. Problem 5 is ready for coordination, the only question being whether Michael will score 1 or 2. It certainly deserves at least 2, but I am afraid the mark scheme will be interpreted to give only 1. And Problem 3 is easy to prepare as well, as the only script that needs attention is Stephen’s. Meanwhile, the foursome of Alex, Richard, Adam and Duncan are working hard on the Problem 1 scripts. They have discovered a slip in Thomas’s solution. It has been caused just by a typographical problem (maybe misreading his own writing), but it is definitely a gap. Of course, if one had pointed this out to him then he would have instantly fixed it, but the question is what what he has actually written is worth. After a huge amount of agonising

between 4, 5 and 6 points, the foursome decide that 5 is the right mark.

At midnight, I decide to have one more look at David K's Problem 4, just to make sure I am still on top of it when we go back to argue the next day. I have been perplexed by the fact that there is so much good stuff in his rough work, while his neat solution is such rubbish. Perhaps he thinks, after his rough work, that he *has* solved the problem, and then he thinks (mistakenly) that he has found some amazing 'short cut', to make the neat solution so short. So I decide to go back over his rough work and see if I cannot actually find a proof in there. And, after lots of reading, something amazing emerges. The rough work is full of paragraphs that seem to end in strange ways. If one *completely* ignores all those paragraphs, and then follows various signposts David has written like 'Now go to foot of previous page', then one does obtain, well, not quite a proof, but a proof with a gap. Elation! We decide to ask for 4 points on the basis of this.

**July 22:** The day starts with coordination for Problem 3. The Coordinators offer 2 points for Stephen's failed proof. But we explain that actually what his attempt does prove is rather more than the work for which the mark-scheme gives 2 points (although it is far from a solution to the problem), and they quickly agree to give 4 points.

After lunch it is time for Problem 5. As expected, Michael's script gets only 1 point, which I still feel is unjust (although it is consistent with the mark-scheme). The Coordinators are as impressed as we are with David C's super-short solution and Thomas's how-did-he-think-of-it solution.

Now comes the moment we have been waiting for. We are back for Problem 4. The Coordinators start by saying that they are sorry, but they are still only prepared to give 2 points. I say that I apologise enormously for what I am about to say, but I must up my request from 3 to 4. They are intrigued, and enjoy it when I explain the contorted way one has to read the script to see the proof-with-gap. After conferring, they say 'Yes, 4 points'. We are delighted.

Then we deal with the rest of the Problem 4 scripts. We come to Michael's script, and they say 'There is a gap. What mark do you propose?'. This shows again how carefully they have done their homework, reading hundreds of scripts in advance. We say 6, and they say that that is what they had thought as well.

Now there is only Problem 1 to go. We take a break to go and look at the huge scoreboards where the results of each contestant are being displayed. Several people are making charts and tables to try to predict the medal cutoffs. Of course, this is very inexact, because of the partial nature of the information. In addition, many teams have taken postponements on coordinations. This is often for the better teams (as there is more for them to discuss), and trying to compensate for this in the estimates is very difficult. Anyway, the Irish Deputy and the Dutch Deputy are keeping count, and they seem to think it will be 13 for Bronze, 22 or 23 for Silver, and 30 to 32 for Gold. This is indeed the impression most Leaders have.

We go back to our room to take stock. David C has 17 points so far, with 7 to come from Problem 1, so he is a safe Silver. Thomas has 15, with 4 or 5 to come, so a high Bronze. Stephen has 12, with about 5 to come, so this is a safe Bronze. Michael and Ollie seem to be out of medal contention: Michael has 9 so far, with 2 to come, while Ollie has 4 with 7 to come. So these two will narrowly miss out. And David K has 12 so far, so just 1 point from Problem 1 will give him a Bronze. Unfortunately, his script looks depressingly like a zero. He has stated that a certain quadrilateral is cyclic (which is relevant), but has not written down a proof of this! This looks like it will cost him a point. We are devastated. All that hard work over his points on Problem 4 in vain!

So in we go for Problem 1 with heavy hearts. The Coordinators ask us how many points we want for Thomas's solution, and we say 5, but they say 'We thought 6, actually'. So that is nice. After 5 for Stephen and 7 for David C, it is time for David K. Richard argues persuasively for 1 point, but the Coordinators say that they cannot give a point without an argument on the page for why this quadrilateral is cyclic. Richard argues some more. We are almost begging for this point, as it seems it will make the difference between medal and no medal. But we are bound to lose the argument, and indeed we do. We go out of the coordination room rather sad. When we see the Coordinators at tea, we apologise to them for wasting their time arguing about that lost cause, and we explain why it had been important to us. They say 'Oh yes, we guessed that'!

After dinner, the excitement is building, as everyone is clustered round the scoreboards. The predictions are now 13 for Bronze, 22 for Silver and 31 for Gold. This means

that the team will get one Silver and two Bronzes, with the other three narrowly missing a Bronze and Thomas just missing a Silver. We feel terrible for the team – four near-misses.

But, as the late results trickle in, it seems that the postponed sessions have not led to such high marks as the experts had thought. By 9pm, it is certain that 21 will be enough for a Silver, so Thomas has done it. And by 9.30 the cutoff for Bronze has come down to 12. This is great – our work on David K's Problem 4 was not in vain after all. We are still sad for the other two, who will miss out on a medal by one point.

Then, with only about 20 contestants' scores incomplete, a rumour starts going round that maybe 11 points will be enough for Bronze. For this to happen, we would need half of all contestants to score 10 or less. Suddenly we are galvanised into action. The first question is how many contestants there are in total: some people say 461 and some say 463. Alex and I decide to count it ourselves. We rush up and down the scoreboards, getting in everyone's way, and come up with 461. So we need to find 231 people scoring 10 or less.

We count 226 confirmed scores of 10 or less - meaning either completed scores of 10 or less or scores of 3 or less with one mark to come. The big grey area is the people with scores of from 4 to 10, with a mark to come. One of these is updated, and to our joy (although not the joy of that country's Leader, of course) the score is under 10. So up to 227.

We see the Czech Leader, and ask him about him one blank: a student of his on 5 with one mark to come. He says that yes, they are still fighting over his mark for one problem (Problem 4, of course), but that the fight is over whether it should receive 3 or 4 marks. So now we are on 228.

Then comes a period of a few minutes in which some scores are updated, but none in our favour: people being pushed over 10 points by their last mark. Eventually, by 10.30pm, there are just five contestants left who could affect things, and we are still on 228. Three of these are from Macedonia. So we frantically look for the Macedonian Leader, and ask him about those postponements. We know he has power of life and death over us. He asks why we want to know about his students, and we breathlessly explain why. And he tells us that none of the three has any chance of getting to 11 points. I kiss his hand (an event unfortunately captured on film by the Dutch Deputy). And off we go to the bar to celebrate. Everyone has got a medal (for the first time since 1996).

**July 23:** The day starts with our final Jury meeting. First there are two contestants whose marks on one question have still not been agreed (this is extremely rare), so we vote on these. Then we ratify the cutoffs. The Gold cutoff is officially 31, as to allow 30 would mean 39 Golds, which is very slightly above one-twelfth of 461. But it is so close ( $461/12$  being about 38.5) that we decide to allow the extra half-person, bringing the cutoff down to 30. At last everyone can relax.

In the afternoon I go shopping with the New Zealand Leader and Deputy and the Canadian Deputy. And this is my introduction to bargaining. In one shop I buy a fan for 3000 won (about 2 pounds). When I come out, my companions ask me if I bargained. I say no, and they laugh and explain that one should always bargain.

So in the next shop, when I need to buy a disposable umbrella (against the sudden torrential downpour), I communicate via counting-by-showing-fingers to reduce an umbrella in price from 900 won to 700 won. I try to pay, and the assistant shows that I have not paid enough. Of course, I am out by a zero, as I have actually gone from 9000 to 7000.

I watch my companions bargaining, and feel I have absorbed enough to be an expert. So, in the next shop, when a price of 15000 won is given (for an *extremely* nice fan), I start to bargain. But the assistant is not willing to give an inch, and says ‘No bargain’. I decide that this must be a hard case, and persevere. Then she says, in perfect English, ‘We are not a bargaining shop’. It seems that certain shops are designated as non-bargaining. I feel very embarrassed, and am laughed at even more by my colleagues.

On the way out, we meet the Russian Leader, looking for shoes. His Deputy has brought his black suit from Russia, instead of his white one, and he only has his white shoes with him. He has spent half the day looking for black shoes, but without success, as no shoe will fit him – Korean shoe sizes are rather small.

After dinner there is a ‘Korean Cultural Evening’, to which the teams have been invited as well. We meet up with ours, and congratulate them. Then comes a fast and furious performance, with the highlight being some more loud and skilful drumming.

**July 24:** In the morning the team come to visit us at our hotel. We go onto the roof (illegally, I suspect) to admire the view. Then off they go to change into smart clothes for the Closing Ceremony.

The Ceremony is great fun. The South American teams give huge cheers when one of them goes up on stage to receive a medal. There is already a party atmosphere. Then back to the students' accommodation, which is where the Closing Dinner will be. We want to take photographs of the team in their smart clothes and also in their team T-shirts. We tell the team to pick up their T-shirts from their rooms but to keep their smart clothes on for the moment. It takes 10 minutes for them to understand this, and many of them change at the wrong time or into the wrong clothes, but eventually it is all sorted out.

Then the outdoor banquet, with a beautiful ice sculpture the focal point. After dinner some very loud music starts. The DJ invites contestants on to the stage if they would like to sing. After some harmless attempts, one contestant starts to sing a song with highly unpleasant lyrics. The DJ cuts off his microphone and says to the audience 'That is censored, so please forget what you just heard'. I set off to find YUG3 and ROM6. These are two contestants who are famous among the Leaders, as they are the only ones who have managed to solve Problem 6 but not Problem 1 (which seems impossible, as they are on similar topics but Problem 6 is so much harder). Eventually, with the help of their Leaders, I find them, and shake their hands. I also find USA6, who has managed to do Problem 6 by coordinates!

We head back to our hotel, and the bar. The team are staying up late, as the Americans are leaving at 4am and our team want to stay up with them.

**July 25:** A bus to the airport, and a long flight to Frankfurt. We say goodbye to Adam, who is flying to Edinburgh, and we fly off to Heathrow. We arrive at 10pm, exhausted but happy.

IMO2000 was extremely interesting, both for the mathematics and for the opportunity to see what Korea is like. The whole event ran very smoothly, which is a real credit to all the organisers. This is particularly impressive in view of the fact that Korea has never hosted an IMO before. We are very grateful to all of the Koreans for this fantastic experience.

Closer to home, I would like to thank

- all the pupils who took part in any stage of the UK competition

- all the teachers who encouraged them
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- the DfEE for a grant covering travel to and from the IMO
- Ben Meisner for producing the BMO 2000 booklet, copies of which are now owned by practically every Leader and Deputy
- Richard Atkins for assistance with the Correspondence Course
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Perhaps most of all, I would like to thank Tony Gardiner and Adam McBride for continuing to give me advice on the role of Leader. Whatever I ask them, I always receive thoughtful and useful replies.

This just leaves the squad of 8. I have found them an amazing squad to work with. All 8 are extremely bright and enthusiastic. Each of them has come up with novel and exciting answers in the Correspondence Course. I have particularly enjoyed the pleasure when one of them starts off a proof in a way that I ‘know’ will not work, yet manages to make it work! It has been wonderful to deal with them.

Hannah Burton and Michael Spencer will still be around next year. Of the others, two are off to Oxford: Stephen Brooks to read Maths at Trinity College and Ollie Thomas to read Classics at New College. The other four, Thomas Barnet-Lamb, David Collier, Kerwin Hui and David Knipe, will read Maths at Trinity College, Cambridge. We wish them all the very best for the future.