42nd INTERNATIONAL MATHEMATICAL OLYMPIAD

WASHINGTON D.C., U.S.A.

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Report by Imre Leader (UK Team Leader)

This is a report on the 42nd International Mathematical Olympiad, which was held in Washington D.C. in July 2001. The IMO is the pinnacle of excellence in mathematics for school pupils the world over. Every year, around 80 countries each send a team of 6 contestants to the IMO. There they sit two $4\frac{1}{2}$-hour exams, each containing just three questions. Medals are awarded for good performances. This year the host country was America, which had last hosted an IMO 20 years ago.

Let us start with some of the events leading up to the IMO. The selection of the team started with the Senior Mathematics Challenge, a multiple-choice paper with more than 50000 entries, taken in November 2000. The SMC lasts 90 minutes, and consists of 25 questions, of which the first 15 are meant to be widely accessible and the last 10 rather more testing. Based on their performance in the SMC, around 800 contestants proceed to the next round, the BMO1. This is a far, far harder paper, containing just 5 questions to be done in $3\frac{1}{2}$ hours. Anyone who solves a BMO1 question has reason to feel pleased with himself/herself! The BMO1 is held in mid-January, and is followed by an amusing weekend in which 20 or so academics, teachers and ex-olympians gather together to mark the scripts.

After BMO1, about 100 pupils qualify for BMO2, which is a still harder exam, consisting of 4 questions to be attempted in $3\frac{1}{2}$ hours. Based on BMO2, 20 pupils are selected for the Trinity Training Session at Easter – these 20 include those who we feel are realistic contenders for the team, and also some younger pupils who we believe are good prospects for the future.

The Trinity Training Session is an extremely intense and exciting experience for everyone. It lasts four days. For the first three days, the students have a variety of sessions,

some taught to the whole group of 20 and some taught in groups of 6 or 7. The emphasis is on the students trying problems: the actual amount of 'lecturing' is kept to a bare minimum. The final day is by contrast rather different. The main event is the last of the selection exams, the Final Selection Test. The FST is designed to resemble a real IMO paper: there are just 3 questions, and the time allowed is $4\frac{1}{2}$ hours.

In the next day or so, a squad of 9 is selected. The choice is based on performance in FST, BMO2 and BMO1, and also on how the students have performed during the Training Session. The 9 are notified within a few days of leaving Trinity, and they then embark upon the penultimate, and in some ways most gruelling, part of the selection. This is the dreaded Correspondence Course. Each week to 10 days, the students are sent a sheet of about 8 hard problems. They send in their solutions, which are marked by the Leader (myself) and the Deputy Leader (Richard Atkins, Head of Maths at Oundle).

In usual years, after about five rounds of this the team of six are chosen. This year, however, there was an extra training session. The whole squad came back to Trinity, scene of their triumph in April, for an extra four days of training in late May. The last two mornings of this consisted of exams, each of the same format as the FST (the main problem being what to call two exams that come after the 'Final' Selection Test – we eventually settled on 'Team Selection Tests'!).

Shortly after this had finished, the team of six was chosen, with the other three acting as reserves. Of course, the three reserves contribute immeasurably to the success of the team, as their presence during the training course and exams has forced people to work hard for their places in the team!

This year, the squad of 9 was as follows.

Team:

Nathan Bowler (Knutsford High School)

Hannah Burton (City of London School for Girls)

Edward Catmur (Hinchley Wood Secondary School)

Stuart Haring (Haberdashers' Aske's)

Paul Jefferys (Berkhamsted Collegiate School)

David Loeffler (Cotham School)

Reserves:

Robert Backhouse (King Edward VI Camp Hill School)

James Cranch (Elizabeth College)

Andrew Fisher (Eton College)

None of these 9 had been to an IMO before, although Hannah had been on the squad last year, as one of the reserves.

Next came the IMO itself. The IMO, for the students, was to start in Washington on July 3rd, but the Team Leaders flew in two days early, to select the questions that would be used. Each country has the right to submit some questions (months in advance); the host country then narrows these down to a short-list of about 25 questions, and it is from these that the six must be chosen. For those few days, the Leaders were kept in a hotel at the other end of Washington, far from the where the students were staying – in fact, they were allowed no contact at all with the teams or the Deputy Leaders until the last exam had finished, for obvious reasons! This year, the Leaders met in a nice hotel near the White House.

Some countries send an 'Observer' with the Leader or Deputy: this is usually someone who will do the job in a later year, and is coming along to see how things work. This year, however, things were very different for the UK, as we will be hosting the IMO ourselves in 2002. For this reason, we sent plenty of Observers.

The Jury chose the questions, and supervised the various translations. This year, 83 countries participated, which necessitated more than 50 languages. The Jury consists of all the 83 Leaders: as one can imagine, a committee of this size is a rather curious animal, but it does seem to reach sensible decisions most of the time.

Meanwhile, the team, led by Richard Atkins, had arrived in Washington, and moved to George Mason University, where they would be staying. After the Opening Ceremony on July 4th, there were a few days for the students to get acclimatised, with the actual exam dates being July 8th and 9th. There then followed a period of 48 hours of intense activity for the Leaders and Deputies. The Deputies join the Leaders at their hotel, to help with the marking of the exams (a considerable increase in luxury from their previous accommodation with the students). Each country marks its own students' scripts, and then goes to 'coordination' for each question: this involves meeting with two American

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mathematicians and agreeing on marks. Finally, totals are worked out, and the cutoffs for medals established. The rough principle is that the ratio of Gold to Silver to Bronze to no medal should be very close to 1 to 2 to 3 to 6.

After a day of socialising, there was a Closing Ceremony, at which the medals were awarded. Everyone flew home the next day.

Now on to the papers. Each day has three problems, to be done in $4\frac{1}{2}$ hours, with each question worth 7 points.

FIRST DAY

Problem 1. Let ABC be an acute-angled triangle with circumcentre O. Let P on BC be the foot of the altitude from A. If $\angle BCA \geq \angle ABC + 30°$, show that $\angle CAB + \angle COP < 90°$.

Problem 2. Prove that

$$\frac{a}{\sqrt{a^2 + 8bc}} + \frac{b}{\sqrt{b^2 + 8ca}} + \frac{c}{\sqrt{c^2 + 8ab}} \geq 1$$

for all positive real numbers a, b, c.

Problem 3. Twenty-one boys and twenty-one girls took part in a mathematical contest. Each contestant solved at most six problems, and, for each girl and each boy, at least one problem was solved by both of them. Prove that there was a problem that was solved by at least three girls and at least three boys.

SECOND DAY

Problem 4. Let n be an odd integer greater than 1, and let $k_1, k_2, \ldots, k_n$ be given integers. For each of the $n!$ permutations $a = a_1, a_2, \ldots, a_n$ of $1, 2, \ldots, n$, let $S(a) = a_1 k_1 + a_2 k_2 + \ldots + a_n k_n$. Prove that there are distinct permutations b and c such that $n!$ is a divisor of $S(b) - S(c)$.

Problem 5. In a triangle ABC, let AP bisect $\angle BAC$, with P on BC, and let BQ bisect $\angle ABC$, with Q on CA. It is known that $\angle BAC = 60°$, and that $AP + BP = AQ + QB$. What are the possible angles of triangle ABC?

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Problem 6. Let a, b, c, d be integers, with $a > b > c > d > 0$. Show that if $ac + bd = (b + d + a - c)(b + d - a + c)$ then $ab + cd$ is not prime.

You are invited to send in solutions, enclosing an SAE please, to:

Imre Leader, Department of Pure Mathematics and Mathematical Statistics, Centre for Mathematical Sciences, Wilberforce Rd., Cambridge CB3 0WB.

The countries of origin of these questions were South Korea, South Korea, Germany, Canada, Israel and Bulgaria, respectively. Although none of the United Kingdom's problems were included, two of them made the short-list: one by our veteran problem-setter David Monk (Edinburgh University), and one by Kevin Buzzard (Imperial College), based on an idea by Alan West (University of Leeds).

The team came away with a haul of four medals: one Silver and three Bronze. As a whole, the UK team's total score of 79 points (out of a possible 252) put us 31st of the 83 countries. When looking at such results, it is important to bear in mind that our training programme is tiny compared with those of other countries (most of which have training camps lasting one or several months, and in some cases even a whole year!). Here are the top 10 teams, with their scores:

China 225, Russia and USA 196, Bulgaria and South Korea 185, Kazakhstan 168, India 148, Ukraine 143, Taiwan 141, Vietnam 139.

Here are our individual scores, with marks per question:

	Q1	Q2	Q3	Q4	Q5	Q6	Total	
Nathan Bowler	0	1	0	7	0	0	8	Hon. Mention
Hannah Burton	5	0	0	6	2	1	14	Bronze
Edward Catmur	7	0	1	1	7	0	16	Bronze
Stuart Haring	0	0	0	2	2	0	4	
Paul Jefferys	1	0	1	7	2	0	11	Bronze
David Loeffler	7	7	1	7	4	0	26	Silver

Four contestants scored full marks: two from China and two from the United States. The cutoffs for medals were 30, 20 and 11 for Gold, Silver and Bronze respectively. An Honourable Mention is awarded to any student who does not get a medal but has managed to get one question completely right.

The IMO was a magical experience for the Leaders as well as for the Teams. For the Leaders, there was the wonderful initial 5 days of meeting as the Jury, all thinking and arguing about the problems and discussing different ways to solve the questions, then the unbelievably hard two days of marking and coordination, and then the last few days socialising with the team and other teams. For the contestants, there was the rather awe-inspiring arrival, being plunged among the best few hundred school pupils in mathematics from around the world, then the tension of the actual exams, then the sometimes equally great tension of waiting for the results, and in amongst all this the meeting with other teams from different parts of the world.

Perhaps the best way to convey some of the atmosphere of the IMO is to record my daily diary of some of the things that happened.

July 1: My check-in time at Heathrow is 9.30am, but a combination of London Underground strikes and Railtrack engineering work means that I have to leave Cambridge at 6am. I get to Heathrow at 9am, to find that the check-in queue is the largest I have ever seen at an airport (well, outside Russia, anyway). After one hour of queuing, I get to the front. I will be travelling with Geoff Smith (from the University of Bath), who is coming as an Observer because he will be taking over from me as Leader next year. As it is now 10am, and I have not seen him, I assume he must have got here super-early, but am told he has not arrived. So I go off to the departure lounge to wait for him.

Then the flight is called, and the plane starts to board, but still no Geoff. I pester the attendants, who can tell me nothing. Finally, when the gate is getting pretty empty, along comes Geoff. He has sauntered into the airport way past the check-in time – so late, in fact, that he has been whisked to the front of the queue and avoided any delay at all. I am green with envy.

The flight passes quickly, and we arrive at Dulles airport. We head for the large IMO placard, and are met by a bigwig – Dan Ullman, who is in charge of the whole students'

site. He explains that *everyone* has to do meeting-flights duty. I meet three other Leaders, and then we are shown to our bus. We have a whole air-conditioned bus for the 5 of us. Geoff prods the driver into pointing out some of the Washington sites on our trip to the hotel.

The hotel, the Embassy Suites, is quite extraordinary. The entrance foyer is 9 storeys high (all of the corridors in the hotel look down over this foyer), and enormous – full of trees and a fountain and other interesting bits. Most impressive of all, this incredible space is air-conditioned.

I find my room, which is very large. In the bedroom is a wardrobe, but when I open it there is no space for clothes, as all it contains is a big television. This is in addition to the other television in my main room. Then I go back downstairs, to find Geoff still there. He has been told that his room is not ready, because there is no key waiting for him. We sit around, and then after an hour we suddenly remember that he is sharing a room with Adam McBride (our representative on the IMO Advisory Board, which is a committee that oversees things between IMOs – he has arrived a day early, as the Advisory Board always meet a day before the IMO starts). So we ask 'Has Professor McBride arrived yet?', and instantly everything is sorted out! Adam is highly amused when we turn up in his room an hour later than planned.

Upon arrival, we are given the booklet of shortlisted problems. So everyone is walking around in the foyer meeting old friends and at the same time trying to tackle some of the problems. There are lots of wonderful enticing questions, so we feel like children let loose in a sweetshop. At the back of the book are the solutions, and it is very hard to avoid the temptation to look there – this would spoil the fun, but more importantly it is *very* difficult to judge the difficulty level of a problem if one has just read a solution, rather than working it out for oneself.

The hotel has a 'happy hour', lasting for 2 hours, before dinner. It is really a 'very happy hour', as all snacks and drinks are free. Everyone thinks this is fantastic. Then dinner, with a large spread of things, buffet-style. The highlight is some grilled vegetables – courgettes and aubergines and squashes. We meet up with three more of our Observers, Angela Gould and Peter Saunders, both of whom will be heavily involved with the organisation of next year's IMO, and Bill Richardson, who will be dealing with the academic

side of things.

Then on to more thinking about the problems. I have gone straight to one of the combinatorics problems which looks interesting. It looks as though it should be easy, so I think 'I'll just do this and then get on with the others'. Meanwhile, Geoff is looking at another combinatorics question, for which he has already found several proofs, and Adam is wrestling with a tortuous functional equation.

Five hours later, it is 2am and I have still not solved the problem. The only progress I have made is to upgrade it from 'easy' to 'difficult'! So I go to bed full of respect for the problem, but very frustrated as well.

July 2: Breakfast is an enjoyable typical luxury-hotel-in-America affair. There are eggs of every description, bacon, sausages, pancakes with maple syrup, about a dozen different kinds of bagel, and of course plenty of fruit. Then comes our first Jury meeting. After some formalities and welcomes, we are left for the rest of the morning to think about the problems. I meet up with the New Zealand Leader, with whom I had got on very well the previous year. I say 'I've been trying problem C6 for ages', and he says 'So have I'. So we sit down together, and in an hour have solved it – two heads are better than one. It is a beautiful question.

When we reconvene in the afternoon, the first thing for us to do is to establish a voting rule. The question is: when a motion is voted on, how should it pass? Is it if there are more votes in favour than against, or alternatively if an absolute majority of those voting are in favour (ie. the fors beat the againsts and the abstentions combined). Last year, we had had some problems when it was discovered halfway through that we had not fixed a voting rule, so here we are doing this first. The Australian Leader makes a speech in favour of 'absolute majority' (the latter alternative), and this is adopted (after a comical few minutes while we decide what the rules are for voting on this particular issue). Unfortunately, it becomes apparent within a day or so that this method of voting is really not very good: it causes terrible delays and blockages, and the Australian Leader remarks 'What is this monster I have created?'.

The only other thing on the agenda for the afternoon is a 'difficulty poll'. The idea is that each Leader should give a difficulty score to each question, so that a profile can

be established of how difficult each problem is perceived to be. I think that this is far too soon, and luckily the Australian Leader, who is very efficient at stopping things being done at the wrong time, stands up and says so. So we have the rest of the day to think about the problems. I retire with the New Zealand Leader for more work.

July 3: After another excellent breakfast, down to business. Several of the problems are discussed, mainly with the aim of removing from the list those problems that have been seen before, or are too close to such problems. Some of the Leaders have an amazing knowledge of the problems literature. stretching back many years.

We hand in our difficulty poll sheets, and we then move on to the beauty poll: we each select our favourite questions, with the result that everyone can see which the popular questions are. It is becoming clear that there are some *very* nice questions on the list. During all this I am sitting next to the American Leader, who is full of excellent sense and also cutting remarks that leave me in hysterics: I am well entertained. The UK Observers at the back of the room are not allowed to vote in all this, of course, but that does not stop them coming forward and giving me advice as well.

After the Jury meeting, we go over to the boards showing which teams have arrived. Eventually, after an anxious wait (the team's travel has not always gone smoothly at previous IMOs), the UK box is ticked.

July 4: Today is the Opening Ceremony. Which means that, after discussions from 9 to 10.30, we break for the earliest lunch I have ever had. Then we board the buses to the ceremony, which is taking place at the contestants' site (this is very sensible: one wants to minimise the amount of disruption for the contestants in the days before the exam). We arrive just after noon, and are shown in to the upper tiers of the auditorium – with many guards about to make sure that we do not go downstairs, where the students are, by mistake.

Several Leaders lean over the balcony to catch sight of their teams, and we have soon established eye contact with Richard and the team. I shout down to Richard to throw me my T-shirt (Richard has got some special 'team T-shirts' made, but they had not been ready until the day before the team travelled). He throws it up, and (with Adam's

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assistance) I catch it first time. Then someone tells Richard off for communicating with the Leaders upstairs. We laugh at his being caught, but then we too are told off by security.

The ceremony itself is quite fun, despite there being ten speeches. Afterwards there is a reception, and then we are taken down to the harbour. We board boats for a dinner cruise to watch the Fourth of July fireworks (the contestants going on a different boat to the Leaders, of course). As we sit down at our tables, all is calm, and we are able to chat, but as soon as the boat starts to move the cabaret on board start up a song, 'Let's get loud', that I suspect is now imprinted on the memory of every Leader. The decibel level makes talking impossible. Luckily, the singers soon go off to a different part of the boat, and we are left in peace.

July 5: The morning is spent is discussing the problems. We finally start to try to put together a paper. We have decided that one of the questions will be G1 or G2 (two of the geometry questions), and now we must decide which. The motion is 'that we adopt G1', and the votes cast are 30 for, 38 against and 11 abstentions. So this motion does not succeed. But it is becoming clear how silly our voting scheme is, as a similar motion for G2 would fail as well! There is much discussion. The Brazilian Leader suggests that the next motion should be 'that we take a vote on G1 versus G2, with the larger number winning'. This is clearly sensible, but now of course the lawyers amongst the Leaders have a field day, arguing about whether or not this amounts to changing our voting system, and whether we have the power to change our own voting system, and so on and so on. At one point the above idea is made into a motion, but then someone wants to 'table' (ie. put aside for a little while) this motion. Chaos ensues, as none of the foreigners (and not too many of the English-speakers) have any idea what to 'table' a motion means. Someone gets up to explain, but gets it exactly wrong. Eventually, we do vote on a motion to table the above motion, although it is clear that most of the Leaders have no idea what is going on. The motion to table the previous motion fails (there is a majority in favour, but not an absolute majority!). So we do vote on the non-tabled motion, and this gets through easily. So we proceed to the G1 versus G2 vote, with G2 narrowly winning 40-38. We break for lunch.

After lunch, the selection of the paper carries on. The combinatorics problem I had

liked so much, C6, is not chosen, but an equally good problem is chosen instead, so I do not mind so much.

Eventually, five of the six questions have been chosen, and we just have one number theory question left to choose to complete the exam. The possibilities are N2, N3, N5, and N6. Of these, many Leaders have spoken in favour of N3 and N6. The former is Kevin Buzzard's lovely question, and the latter is also fantastic: it is very hard (harder than N3, even), but hard in a non-technical way, so that it tests brain-power, not training. This is in contrast to N2 and N5: N2 is quite easy and N5 is hard, but in technical ways, so that a well-trained student will have a big advantage.

Several Leaders argue for N2, as they do like it, but it is clear they are in a minority. Some Leaders express worry that N3 and N6 are too hard in an exam that already looks difficult, but we try to reassure them that the elegance and non-technical nature of the problems makes up for this. After all, the way to make the exam accessible to a less well-trained student is surely to put in non-technical questions (and in a variety of areas, of course), rather that just to go by an absolute measure of 'how hard'. During all this, N5 is hardly mentioned, as no-one likes it (only one Leader has ever spoken in its favour), so there seems no need for a discussion of how unsuitable it is.

But now we vote by single-elimination: each Leader has one vote, with the least popular question being removed each time. First N2 goes, then N3, and now we are down to a vote between N5 and N6. Amazingly, N5 wins out 41-38. It seems that, in the heat of the moment, people worried about the difficulty of N6 had just voted for 'the other question left', forgetting how hard (and technical) it itself was. In the tea break, I talk to many Leaders who had voted for N5, and find that this is almost universally what has happened!

Once the exam has been fixed, we are told the countries of origin of the shortlisted questions – kept secret from us until now, to discourage 'political' voting. The South Koreans are congratulated for having two questions on the paper.

Now comes the preparation of the English-language version. The Leaders of the English-speaking countries go off to a separate room and try to couch each question in clear, unambiguous language. The most troublesome is Problem 3, with the boys and girls. Do we prefer 'boys and girls' or 'girls and boys'? Do we say 'at least one' or 'one'

or 'there exists one' or 'some' or 'a'? Do we prefer 'was' or 'is'? Eventually, everything is decided, and we bring it back to the Jury for their approval.

July 6: This is a chance to relax, while all the foreign-language versions are prepared. There is some discussion about whether or not it is traditional for the heading 'International Mathematical Olympiad' to be translated. There is also some question about how, in each language, it is to be made clear to the contestants that there might be more than one possible triangle in Problem 5, and at the same time let them know that they are free to assume that there does exist at least one solution (this latter point is rather odd, but the Jury have somehow got it into their heads that that is the way the question should be!).

Lunch is a cookout, with amazing smells from sizzling meats. Our last remaining Observer, Béla Bollobás, arrives. Béla was a contestant for Hungary at the first three IMOs, forty years ago, and we have prepared him that things may have changed quite a bit since then.

In the afternoon, the fifty or so translations are displayed, and we wander around the room gazing at them. Some of them are works of art. Then we sit down for the serious business of the afternoon: the mark-schemes. Each of the six Problems has a Senior Coordinator, who comes and tells us his proposed mark-scheme for that question. This goes well: we are very impressed with how thorough and sensible they are being. So everyone is in a good mood when we stop for dinner.

July 7: There are no meeting scheduled for today – we have done our job a day ahead of time! So off we go to walk to the White House and see the nearby sights. Washington is pretty hot at this time of the year, although it is not too humid, so it is actually quite pleasant to stroll around. Peter Taylor, one of the Australian Observers, is taking bets on the predicted cutoffs for medals, and how many perfect scores there will be, and everyone joins in with this.

July 8: This is the first day of the competition. For the first half-hour, contestants are allowed to ask questions. These must be submitted in writing, and are then faxed over to us. When a question arrives, the relevant Leader reads out the question and gives his proposed response. The Jury may agree, or may choose to alter or even completely change

the reply given. The most common reply is 'no comment'.

There is much hilarity at some of the questions. Contestants ask whether or not zero counts as 'strictly positive', and many ask for clarification of what 'for each boy and each girl' means. A contestant from Turkmenistan asks if he is allowed to have the Turkish translation, and of course the reply is 'yes'. But then it occurs to us that all that will happen is that the contestant will receive a response-sheet with the word 'yes' on it, so there is some frantic mobile-phoning to the invigilators to make sure the request is acted upon.

In the afternoon, we are taken to a reception at the National Academy of Sciences. This is in a very impressive building, with a huge statue of Einstein outside. The food consists of canapés, with the shrimp much in demand.

We have been told that the scripts from today will be getting to us at 6pm, so all the Leaders are back at the hotel in good time. The scripts have to be photocopied first, so that the Coordinators can have a look as well, and this is an enormous task. So, based on past experience, many old hands say 'We will be lucky to get the scripts by midnight'. But, to our amazement, at 6pm prompt the scripts arrive. It turns out that the Americans have employed a special firm to do this, who specialise in last-minute mass photocopying for lawyers!

I sit down with Geoff, Adam, Bill and Béla to open the envelopes, and what we see puts us in a very bad mood. The output is awful. David has done Problems 1 and 2, and Ed has solved Problem 1, but that is all. Three solutions out of a possible eighteen! Dinner promises to be miserable, but is cheered up by a quiz from Angela: we have to guess who the American public have just voted the top five pop duos of all time.

After dinner, we look more closely at the scripts. David's solution to Problem 2 stretches over many pages, involving both neat and rough work, and is full of comments like 'see page 13 for a proof of this'. At one point in the evening, we think his proof has collapsed, but five minutes later it lives again (he has done some calculations in a needlessly contorted way, but they are all correct). And we are also put in a slightly better mood by the fact that Hannah seems to have *almost* done Problem 1: she has reduced it to a trigonometric inequality, and stopped there, but luckily that inequality is not too hard to prove.

Meanwhile, Geoff is looking hard at Problem 3. There really is hardly anything there. He manages to find odd little bits that may, if the Coordinators are generous, be worth 1 point.

July 9: The second competition day. I am awake early, to look through the scripts some more. There may be 1 point for Nathan for Problem 2, but otherwise I am pretty sure that Problem 2 will get us four zeros. Hannah will get 4 or 5 or 6 for Problem 1, with the other three getting 0, and on Problem 3 all we can hope for is some single points!

The question-and-answer session this morning starts with embarrassment for the American Leader, as one of his contestants has asked a question (given how hopeless most of the questions are, it is deemed highly risible to receive a question from one's team). Unfortunately for those of us laughing, it turns out to be one of the very few sensible questions (just checking that $n!$ does indeed divide 0). But suddenly the UK Observers rush up and prod me in the back, as to my horror there is a question from a UK contestant. It is from Paul, and it is not a sensible question. It says 'Haven't I seen this problem before, or am I wrong?'. To much amusement, I propose the answer 'no comment' (although in fact Paul is on the right track, as a similar question *has* appeared recently in the problems journal *Crux Mathematicorum*, where Paul had remembered it).

This session is livened up by two things. Firstly, Tony Gardiner, who is the godfather of most of the UK maths competitions, arrives, and is soon being greeted by dozens of Leaders. And secondly the Wimbledon final is reaching its climax, so that every five minutes or so Peter Taylor comes in and tells us the most recent fifth-set score.

As we finish our meeting, the Deputies start to arrive at our hotel. They are impressed at the place. We meet up with Richard, who tells us that he has had a difficult time trying to cheer the team up after yesterday's performance. Then off we go by bus to the contestants' site, to be able to greet the team as they come out of the second exam. Things seem to have gone a little better. Nathan, Paul, David and Hannah have done Problem 4, and Ed has done Problem 5. We are impressed at Ed, who has done the two geometry problems and nothing else, as usually geometry is one of the UK's weaknesses!

We have lunch with the team, and then it is back to the hotel to read scripts. We must be fully prepared, as coordination is starting the next morning. Richard, we are

discovering, is an expert at losing his watch. He manages to lose it at least twice, until it turns up each time in some obvious place.

Geoff, Adam and Richard are working hard on the geometry problems. They re-read Ed's solution to Problem 1, liking it more and more each time (it really *is* a beautiful solution). Then they find a little bit of progress by Stuart and Paul and Hannah in Problem 5, and even more progress (although still far from a solution) from David.

Taking stock, it looks like David has done enough for a Silver, and Ed should be safe for a Bronze. The only other possible medal would be a Bronze for Hannah, if things go well at coordination.

July 10: We start the day with an early-morning coordination for Problem 6. Geoff comes along to watch (the rules are that one Observer may be present in coordination, although only the Leader and Deputy may speak). Everyone except Hannah has an obvious 0, so we quickly pass to Hannah's script. The key idea in (one attack on) this question is to interpret the given algebraic condition geometrically – in fact, when one has done so, and applied Ptolemy, it is pretty easy to finish off the solution. Now, Hannah *has* had the geometric brainwave, but has not thought of applying Ptolemy. This seems to me to be worth 2 or possibly even 3 points, as she has had the one non-routine idea. But the Coordinators offer only 1. We argue for the rest of our half-hour. Soon we are arguing with the Senior Coordinator for the Problem and all the other Coordinators as well. We decide to break, coming back a day later to sort it out (this is fairly common when a dispute cannot be resolved on the spot).

On our way out we spot the Poles, looking amused. They explain that they had prepared through the night for an 8am coordination, and arrived at the room only to find that they had misread '8pm'.

Soon it is time for Problem 3. None of the team have got anywhere, but the mark-scheme offers a very generous 1 point for a particular trivial observation, and Ed, Paul and David have made this observation. I tell the Coordinators that I am ashamed to claim these points, and they say 'Well, we are ashamed to offer them'.

On to Problem 4. Nathan, Paul and David get their 7s. Hannah has a misprint (she has misread her own handwriting), which should bring her 7 down to a 6 or a 5, but the

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Coordinators say that it is clearly a 6. Stuart has made a little progress, for which we propose 2 points, but the Coordinators say 'Here on his rough work is at attempt at an induction proof – if that can be made to work, it could be worth more'. So we thank them and go off to think – this is good evidence for the non-adversarial nature of the Coordinators! We are pretty sure that there is no possible induction proof. Béla, who is one of the world's most famous combinatorialists, volunteers to have a go, but even after spending an hour on it there is nothing! So we go back and agree to the 2 points.

Our last coordination of the day is at 10pm, on Problem 5. This passes with no surprises. We head up to the results boards to find out what the medal cutoff predictions are. Various people, most notably the Irish Deputy, keep track of every score as it is posted on the boards, and then try to predict things, based on the very incomplete and sketchy information before them. It looks like 30 or 31 for a Gold, 19 or 20 or 21 for a Silver, and 13 or 12 for a Bronze.

We retire upstairs to take stock. David has 19 points so far, with 7 to come on Problem 1, so that is a safe Silver. Ed has 9, with 7 to come, so a safe Bronze. Paul has 10, but there is no real progress in his Problem 1 work, so he looks like just missing out. And Hannah has either 9 or 10 (depending on what happens on Problem 6), with 4 or 5 or 6 to come. This is too close for comfort. We go back downstairs and ask the Irish Deputy 'How sure are you that 13 points will get a Bronze?', and he says 'Not very'.

So back to chat to the Problem 6 Coordinators. They tell me that the German Leader has a similar case, and has also postponed his discussion. So I go and talk to him, and we plan what to do the next day. Unfortunately, several Leaders with similar cases *have* signed (meaning that they formally accept the mark of 1), so it is looking as though an argument of 'Well, this is how we have treated similar cases' will carry the day.

July 11: Coordination for Problem 1 is right after breakfast. As usual, we arrange to meet outside the coordination rooms five minutes early, but Richard is nowhere to be seen. We rush off to find him, and there he is having a leisurely breakfast. He gets up from his seat, and Geoff and I say 'Good, come on', but it turns out he is only getting up for some more fruit. He does turn up, though, a full 10 seconds before our coordination is due to begin.

The Coordinators are very impressed with Ed's solution, which we have shown them first to get them in a good mood. When we get to Hannah, they offer 5, which we are happy to take, as it pushes her minimum possible score up to 14. The last script we look at is Paul's. He has done some immensely ugly coordinate work, rewriting the conditions and the conclusion in terms of coordinates but making no attempt to see why the conditions might imply the conclusion! Nevertheless, the Coordinators decide that he has 'engaged' with the problem, so they award him one point.

I check at the scoreboard that 14 is a guaranteed Bronze, and then go with the German Leader to see the Chief Coordinator about Problem 6. We tell him that we will accept whatever he says (in principle, it is possible to appeal to the Jury itself, but this is extremely rare). After much consultation, he decides that the 1 point will stand.

As the final results come in, it becomes clear that 12 will be the cutoff for Bronze. The rules state that 'no more than one half of all contestants shall normally receive a medal'. Now, half of all contestants is just over halfway down the scores of 11, but the Jury always interprets the rule strictly: there is no 'rounding down' (unless the borderline would include just one half of one person too much). We feel sorry for Paul, who will miss a medal by 1 point.

The final Jury meeting, to hear any appeals and decide the cutoffs, starts at 8pm. This is normally quite a short meeting, but this time there is some excitement, as the Coordinators have noticed that one of the countries (not a top one) has six more-or-less identical solutions to one of the harder questions. This is discussed and discussed, and eventually, at 2am, the matter is resolved. Now we must vote on the cutoffs. The motion is that 12 should be the cutoff for Bronze. There is a majority, but not an absolute majority! And people are now so tired that, when the next motion is for 11 to be the cutoff, it passes by a huge majority. So Paul has got his Bronze – we are delighted.

July 12: Today we have an all-day excursion to Baltimore harbour. We meet up with the team, and confirm to Paul that he has a medal (the 2am decision had come too late to be sent to the contestants' site). Everyone has a great time, visiting the science museum, with ultra-realistic flight simulator, and the aquarium, which has an incredible collection of sea-horses and some even stranger things called sea-dragons, looking like sea-horses with

bits of leaf stuck on to them. By dinner time we seem to have separated. Geoff and I decide we need a nice meal, so we head for a seafood restaurant and gorge ourselves on clam chowder and lobster.

July 13: A lazy morning, spent shopping. After lunch, we head off by coach to the Closing Ceremony. There is plenty of talk about the 'medalling ceremony', which causes predictable jokes about towns in Colombia. Andrew Wiles gives a speech, after a standing ovation that lasts for several minutes. There are other speeches, a video clip of greetings from George Bush, and then comes the actual presentation of medals. This is a raucous affair, with the South Americans cheering wildly whenever one of their contestants goes up on stage. The whole event is handled very well – the Americans have put clearly put a lot of thought into what the ceremony should be like, and the result is impressive.

Then we are off to the Closing Banquet. I can't remember much about this, but I am told it went very well. Adam is ceremonially handed the IMO flag, for next year.

July 14: Departure day. We set off mid-morning to the contestants' site. There we have lunch with the team, before taking a bus to the airport. We have plenty of time to kill, so the team go shopping. Well, they claim it is shopping, but in fact all they do is go to a bookshop and read science fiction there. We have been assigned seats that are far apart, but at the gate Richard does a brilliant job of cajoling the attendants into reseating us, and when we get on the plane we are indeed one large 'maths block' of seats.

The not-very-good films help the flight pass, and we arrive at Heathrow early the following morning. Stuart has to rush off, to join his school cricket team in mid-tour, and Nathan is flying to Bolivia on a school trip the next day, but the other four can relax.

IMO2001 was a pleasure from start to finish. The Americans had put together an event that ran very smoothly, which is a real credit to all of their organisers and volunteer helpers. Dealing with 83 teams, this is a pretty remarkable thing to achieve. We are very grateful to all of the Americans for this wonderful two weeks. It will be a hard act to follow next year!

Closer to home, I would like to thank

- all the pupils who took part in any stage of the UK competition
- all the teachers who encouraged them
- Peter Neumann, chairman of the UKMT, and Adam McBride, chairman of the BMOC
- Alan West and Brian Wilson, organisers of BMO1 and BMO2 respectively
- everyone involved with the BMO1 marking, particularly Christine Farmer and Brian Wilson
- Christine Farmer for help in the hectic period between BMO2 and the Training Session
- all those who helped with the Trinity Training Session, particularly Adrian Sanders
- all of our sponsors, particularly Trinity College, Cambridge for hosting the Training Session and the Royal Society for hosting our September celebration
- the DfEE for a grant covering travel to and from the IMO
- Ben Meisner for producing the BMO 2001 booklet, copies of which are now owned by practically every Leader and Deputy
- Richard Atkins for assistance with the Correspondence Course
- Richard Atkins, Béla Bollobás, Adam McBride, Bill Richardson and Geoff Smith for all their help in America.

As always, I would like to reserve my greatest thanks for Tony Gardiner and Adam McBride for continuing to give me advice on the role of Leader. Whatever I ask them, I always receive thoughtful and useful replies.

This just leaves the squad of nine. They are bright, enthusiastic, and a pleasure to work with. It was particularly enjoyable to have them all together at our late May 'extra' training session. It has been a privilege to deal with them.

Nathan Bowler and Paul Jefferys will still be around next year. Of the others, Ed Catmur is off to Jesus College, Oxford, to study Maths with Philosophy. The other six, Robert Backhouse, Hannah Burton, James Cranch, Andrew Fisher, Stuart Haring and David Loeffler, will read Maths at Trinity College, Cambridge. We wish them all the very best for the future.