The 26th Balkan Mathematical Olympiad
Leader's Report

The 26th Balkan Mathematical Olympiad was held from 28th April to 4th May 2009 in Kragujevac, Serbia. The 6 members of the UK team were

Chris Bellin	Queen Mary's Grammar School, Walsall
Luke Betts	Hills Road 6th Form College, Cambridge
Joseph Briggs	Kennet School, Berkshire
Nathan Brown	Camp Hill School for Boys, Birmingham
Ruth Franklin	Manchester High School
Sean Moss	Havering Sixth Form College

and the leaders were Vesna Kadelburg of MPW Cambridge and Robin Bhattacharyya of Loughborough Grammar School. All the students were 17 years old. The trip was kindly sponsored by Winton Capital Management.

As always, the Balkan Olympiad is a great experience for the students, most of whom have never participated in an international competition before. It is a friendly event, less formal than the International Mathematical Olympiad, although in recent years it has been expanding with new Balkan countries joining as official members, as well as an increasing number of guest teams. This year, there were twenty participating teams: member countries Albania, Bosnia and Herzegovina, Bulgaria, Cyprus, Greece, Macedonia, Moldova, Montenegro, Romania, Serbia and Turkey, and guest teams from Azerbaijan, the City of Brno from Czech Republic (twinned with the City of Kragujevac), France, Italy, Serbia B, Tajikistan, Turkmenistan, UK and Uzbekistan.

The exam last four and a half hours and consists of four problems, traditionally one in each of algebra, combinatorics, geometry and number theory. Here are the problems.

1. Solve the equation
$$3^x - 5^y = z^2$$
in positive integers.

2. Let MN be a line parallel to the side BC of triangle ABC, with M on the side AB and N on the side AC. The lines BN and CM meet at point P. The circumcircles of triangles BMP and CNP meet at two distinct points P and Q. Prove that $\angle BAQ = \angle CAP$.

3. A 9×12 rectangle is partitioned into unit squares. The centres of all the unit squares, except for the four corner squares and the eight squares sharing a common side with one of them, are coloured red. Is it possible to label these red centres $C_1, C_2, \ldots, C_{96}$ in such a way that the following two conditions are both fulfilled

(i) the distances $C_1C_2, C_2C_3, \ldots, C_{95}C_{96}, C_{96}C_1$ are all equal to $\sqrt{13}$,

(ii) the closed broken line $C_1C_2 \ldots C_{96}C_1$ has a centre of symmetry?

4. Denote by S the set of all positive integers. Find all functions $f: S \to S$ such that
$$f(f(m)^2 + 2f(n)^2) = m^2 + 2n^2, \quad \text{for all } m, n \in S.$$

<u>Tuesday, 28th April: Arrival day</u>
The trip started smoothly enough, with everyone arriving at Heathrow on time, despite delays on the London underground. At the check-in we are told that the flight has been delayed for 3 hours and are given £5 food vouchers each, so we spend the time having lunch and playing cards. After a

couple of hours we pass through security and are suddenly rushed to our departure gate. We had just enough time to get some duty-free presents for our guides. After some more confusion about the departure time, we finally take off just after 4pm and land at Belgrade airport some two and a half hours later. The organisers have kindly provided transport to Kragujevac, but we have to wait for the Turkish team to arrive. This means another two hours at the airport and more card games. We are finally ready to board the bus at 10pm, but there are further problems. The luggage compartment is too small, so most of our suitcases are stacked in the aisle (something that definitely did not feature on my risk assessment!). Then the bus wouldn't start, so the students get out to help push it down the slope. At this point I should mention that I originally come from Serbia, so none of this seems too strange. Robin promises to point out anything odd enough to mention in this diary.

We finally arrive in Kragujevac just before midnight (this is 11pm UK time). They have dinner waiting for us. I am then taken to another hotel, about half an hour drive away, and won't see the students again until after the exam. Unusually for an international olympiad, the leaders' hotel is not as nice as the students'. This seems to make sense – they will be the ones doing all the hard work on Thursday, and we will only be spending two nights here. It is nearly 1am, but I am supposed to look at the shortlisted problems in preparation for the jury meeting. It is a shame that I don't have more time to attempt all the problems properly, but after a couple of hours I decide that it might be a good idea to get some sleep.

Wednesday, 29th April: Jury meeting

Today is the main Jury meeting where the leaders discuss the shortlisted problems and select the four contest problems. As a guest team we do not get to vote, but can make comments and help with the formulation of the questions. I am pleased to see that I know several of the leaders, either from Serbia or from previous olympiads. The meeting is conducted in English, but the leaders speak among themselves in a variety of different South Slavic languages, with another group using Turkish and Russian. I note that Robin would be fascinated by the linguistic mix.

The problem selection committee inform us that they have received 26 proposals and have selected 19 for the shortlist. The problems have been split into four groups: Algebra, Combinatorics, Geometry and Number Theory, and the problem selection committee have designated each one as easy, medium or hard. The aim is to select one problem from each group, preferably with one easy, two medium and one hard. In the first part of the discussion several problems are rejected as known or relying on university-level knowledge. This leaves us with just one combinatorics problem which will therefore have to be selected. It is also proposed that no more than two problems should come from the same country. This may restrict the selection even further, although at this stage the identity of the proposers is supposed to remain unknown.

Next, an algebra problem is selected as problem 4. This should give us more flexibility in choosing a medium problem. After some discussion about geometry problems, a vote was taken and one of them was selected for problem 2. This left us with a relatively simple choice of an "easy" number theory for problem 1. The authors of the problems are revealed as Greece (problem 1), Moldova (problem 2) and Bulgaria (problems 3 and 4). General opinion seems to be that the paper is quite difficult, but the problems are nice.

We still need to agree the exact formulation of the problems, before they can be translated into all the different languages. Most students will get the official English version of the problem sheet, as well as one in their own language. This is my big moment – it is a tradition for the UK leader to help decide on the best wording. There is a lot of input from Dan Schwarz, a Romanian who used to live in Canada and is one of the problem selectors and coordinators here, as well as Claude Deschamps and Massimo Gobbino, the French and Italian leaders. The combinatorics problem is re-worded several times before everyone is happy with it. There is the apparently mandatory

discussion about how to denote the set of positive integers (and just why 'positive' in French and Romanian includes zero), resulting in the agreement that the official version will use the letter S, and everyone can use whichever letter their students are used to in their national versions.

Once the official version is agreed we can start translating. My work is not done however, as the statement of problem 3 talks about a 'centrally symmetric path', which I need to 'translate' as 'having rotational symmetry of order two'. To my knowledge, this is the first time that there will be a UK version different from the official English version. With this out of the way I join the Serbian leader in preparing his translation, which will also be used by Montenegro. Students from Bosnia and Herzegovina can choose from three possible languages, two of which are identical only written in different alphabets.

With all this work done we deserve a lunch break. Our hosts are taking their role very seriously and we are taken to a restaurant in a small traditional village. After the first three courses we walk up the hill to a local church. It turns out that the guide does not speak English, so my linguistic skills are put to a test as I attempt simultaneous translation. The guide is keen to take us all the way to the top of this volcanic hill, but most of us are not really up to it after the generous starters and local spirits. Under the excuse that we need to return to work soon, we go back to the village to complete our lunch with the spit-roasted lamb. There is some post-meal entertainment and to everyone's amusement I am made to demonstrate a traditional dance with the restaurant owner. Luckily the Kazakhstan leader is keen to join in.

The rest of the afternoon is spent checking and printing out the translations. After dinner we all gather in the conference room to watch football: Man United v Arsenal, Champions' League semi-final. People in the Balkans are well versed in all matters football-related and seem to be particular fans of the Premier League. The Bulgarian leader is even wearing an Arsenal top. As smoke fills the room I realise how quickly I have forgotten what pubs used to be like before the smoking ban. After the match (for the record, Man U won 1-0 – Robin will be pleased) I work on the problems some more in preparation for the marking.

Thursday, 30th April: Contest day

We leave the leaders' hotel and will check in with the students after the exam. The contest will be held in the First Kragujevac Grammar school, an imposing building dating from 1887. It was the centre of Serbian education in the early 19th century. The opening ceremony is quite short, involving only one government minister and no folk dancing (all that practice for nothing!). Students go straight into the exam and we prepare to answer questions. Even after all the debate about positive integers, several students ask whether zero is included – I guess it's better to be safe. We also receive a question from Joseph about function notation. On the way back to the hotel Robin and I decide to take a walk around the town. There is a children's football tournament going on in the main square with lots of people watching; once again this reminds us of the importance of sport in this part of the world.

After the exam Robin brings the students back to the hotel and we all have lunch together, discussing the problems. Later on we will talk to each student individually to find out how they think they have done. Students' comments can be very useful when marking the scripts. Several of the team find mistakes and holes in their solutions, and not many full solutions are claimed, but there are some good ideas all round. Everyone seems glad that the exam is over and they can get on with meeting the other contestants.

The third jury meeting is held in the afternoon. The coordinators have proposed marking schemes for each problem but there is still a lot of discussion about the details. Everyone seems to have a different opinion on how many marks should be taken off for not checking that a certain function

satisfies the given equation, and whether any marks should be awarded for spotting one solution to the number theory problem. After the amended mark schemes are voted through the students' scripts arrive and we are keen to get marking straight away. Each team's leaders mark the scripts according to the marking scheme. Coordinators for each question have photocopies of the scripts and will suggest marks. Over the next two days we will meet with the coordinators for each problem and agree the marks. Our job is to convince them that any omissions or lack of clarity are 'trivial', and possibly find alternative solutions which follow from our students' incomplete work.

We will only coordinate problem 2 on the first day, so we start by marking that. It is geometry, and generally considered quite difficult. None of our team claim to have solved it, but Chris and Joseph have done some useful angle calculations. As we are done with problem 2 quite quickly, we decide to have a go at two more problems, only leaving problem 1 for tomorrow. We keep being distracted by Dan Schwarz who has decided that we are working too hard. So we spend quite a lot of time listening to his stories from Romania and Canada and despairing over the state of education today, while passively smoking a packet of *Camels*. Nevertheless, we manage to mark problem 3 and most of problem 4 before deciding to go to sleep around 1am.

Friday, 1st May: Excursion and first coordination
In the morning Robin and the students go on an excursion while I stay at the hotel to look over the scripts some more. Coordination will start after lunch. We are not expecting many marks for problem 2, and we are a lot more excited about a football tournament the local students are organising. After lunch we make our way to the school where the coordination is taking place. The organisers have set up a live score board so we can see the latest scores as soon as the coordination is over. There is already a crowd looking at the first results.

While waiting for our coordination slot Robin and I decide to look around the school. The classrooms are quite bare, although we find some interesting science posters in one. Robin is surprised at the amount of graffiti everywhere. I reflect on my school days and the number of poems and random thoughts recorded on school desks. It is certainly very different from most schools in the UK.

The football tournament is taking place in the gym. Teams of six should be made up of students from two different countries. It turns out that only Nathan is keen for football, and the others go off to play frisbee. Robin and I are more than happy to make up the numbers in the UK-French team. Before the match starts however, we need to go off and coordinate problem 2. This was reasonably quick and more-or-less as expected. We return to the gym just in time for the game against the Azerbaijan/Turkmenistan team. Despite our best efforts and Nathan's goalkeeping skills we lose 4-0, but enjoy ourselves thoroughly.

We now have the rest of the day to finish marking the other three problems. Problem 4 should be reasonably straightforward. Nearly everyone has one mark for proving that the function is injective, but Chris has also come up with some interesting identities, so there should be a few more for him. Problem 1 also turns out to be reasonably clear cut, as everyone's solutions fit neatly within the marking scheme. Problem 3 is the most interesting. Luke and Nathan both thought they had solved it, but failed to consider the possibility that a symmetric path may consist of two self-symmetric parts. We spend quite a lot of the evening trying to decide how many marks to ask for, but eventually go to join the other leaders and coordinators in the bar. Meanwhile the students have made friends with everyone and are teaching the Italians how to play Mafia.

Saturday, 2nd May: Second coordination
Today we will coordinate the remaining three problems. The students will spend the day sightseeing and playing games. Problem 3 is probably the most interesting. We need to explain some of Luke's

and Nathan's arguments in more detail, but eventually they get the marks. Problem 4 coordinators are very impressed with Chris' ideas and award his script 3 points – the eighth highest score out of the 115 contestants. Problem 1 is our best overall: Luke solved it completely and everyone else gets substantial marks according to the marking scheme. The coordination is over reasonably quickly. To our slight disappointment, we had no exciting arguments with the coordinators, but it does mean that we have the whole afternoon free to go out for a walk. The scoreboard is taking its final shape, and it looks like we may get quite a few medals.

We have several hours to spare until the final jury meeting, so a group of leaders, deputies and coordinators decide to go for a walk around the memorial park. The park commemorates citizens of Kragujevac, including schoolchildren, who were killed in a Nazi revenge attack in the Second World War. The museum is fairly distressing, but the park is beautiful. At some point we bump into the students on their way to a lake several miles away. It's good to see that they are being sociable and having a good time.

The final jury meeting is the place to sort out any unresolved coordination issues and decide on the medal boundaries. There are no problems with the scores, and the results are officially approved. As predicted, the scores are quite low. The winner is a 16-year-old Serbian student with 35 out of 40. Serbia are also top of the unofficial ranking, closely followed by Turkey and Bulgaria. The discussion about the medal boundaries takes a while. Although the rules specify very clearly the proportion of contestants to be awarded each medal, some leaders are in favour of awarding slightly more, while others think that the numbers should be decreased. The organisers point out that there is only a certain number of actual medals. Eventually the boundaries are set slightly lower than expected, so we end up with six medals, one silver and five bronze. Robin and I can't wait to get back and tell the students. The jury also decides to award several special prizes to students who solved the difficult third and fourth problems.

The students are back in the hotel playing games and are delighted to find out that they all got medals. Our final scores are:

	P1	P2	P3	P4	total	medal
Chris Bellin	5	1	1	3	10	bronze
Luke Betts	10	0	4	1	15	silver
Joseph Briggs	8	1	0	1	10	bronze
Nathan Brown	6	0	4	1	11	bronze
Ruth Franklin	6	0	0	0	6	bronze
Sean Moss	6	0	0	1	7	bronze

Sunday, 3rd May: Excursion and the closing ceremony
Today is the day of the main excursion. In the morning we visit the Resava Cave, a large cave with some fascinating formations. It was only discovered in 1962, but quickly became one of the main tourist attractions in Serbia and a popular destination for school trips. We then drive a short distance to the Manasija Monastery, built in the early 15th century. Inside we find out that the art of graffiti dates back several centuries – we find names engraved on pillars, with dates like 1746! The next stop is a restaurant on a fishing lake. It starts to rain again. Luckily most of the seating is under a roof, but some of us have to hold an umbrella in one hand whilst eating the soup. The fresh fish is really excellent.

As the rain has caused a slight delay, we don't have much time to spare before the closing ceremony,

held in the school's impressive assembly hall. This is again quite brief, with a complete absence of politicians and folk dancing. Of course, it takes time to award 88 medals but it is great that all our students get to go up on the stage. Medals received, we all get on the bus and head for the closing dinner.

It is not often that one can see a group of young mathematicians dancing to excessively loud music. But most of our team are on their feet soon, led by the Serbian guides. Joseph's hat turns out to be a particular hit. Even Robin joins in for a bit. I use the time to chat to my Serbian friends. After the party is over we head back to the hotel to pack. We are leaving very early in the morning.

Monday, 4th May: Departure day

As we prepare to board the bus just before 6am I find out that the students had decided to stay up all night. They all seem slightly dazed, but the suitcases are ready. Well, at least the journey will pass more quickly if they are asleep. We arrive at the airport with plenty of time to check in, have breakfast and do some last minute souvenir shopping. I won't ever fully understand why it is important to buy spoons, but the students manage to find some, so everyone goes home happy.

Vesna Kadelburg
July 2009