

The 50th International Mathematical Olympiad

UK Leader's report

The 50th International Mathematical Olympiad was held in Bremen and Bremerhaven, Germany in July 2009. This is in the North-West corner of Germany, near its North Sea coast.

The actual competition consists of two papers, each containing three very hard questions drawn from algebra, combinatorics, geometry and number theory. Each question is marked out of 7, according to an agreed marking scheme. Half the competitors receive medals, and these are awarded in the ratio gold : silver : bronze = 1 : 2 : 3. This year 104 nations competed. Each country may send up to six students, and most do send the maximum number. Like the athletic olympic games, the competition is between individuals, not countries, but inevitably nations compete unofficially by comparing the sum of the marks obtained by their students. Populations of countries, educational standards and levels of preparation vary widely between nations. Some countries have specialist IMO schools which take talented young mathematicians out of the normal school system at a young age, and others have training regimes which involve weekly meeting of the most able students with their trainers. In the UK we do not allow IMO preparations to interfere with normal school life, and almost all our events take place during school vacations.

This year the team was led by Dr Geoff Smith, University of Bath. The Deputy was Dr Vesna Kadelburg of Mander Portman Woodward, Cambridge. Observer with leader was Mr James Cranch, University of Leicester, and Observer with students was Ms Jacqui Lewis of St Julian's International School, Carcavelos, Portugal. The UKMT executive director Mary Wimbury also made a short visit to study IMO procedures.

The UK team was:

Chris Bellin	Queen Mary's Grammar School, Walsall
Luke Betts	Hills Road Sixth Form College, Cambridge
Tim Hennock	Christ's Hospital, Horsham
Peter Leach	Monkton Combe School, Bath
Sean Moss	Havering Sixth Form College
Preeyan Parmar	Eton College

The three reserves were:

Nathan Brown	King Edward VI Camp Hill Boys School
Andrew Hyer	Westminster School
Craig Newbold	Whitley Bay High School

Each of the six questions is marked out of 7. The UK picked up marks on 5 of the 6 problems.

	<i>P1</i>	<i>P2</i>	<i>P3</i>	<i>P4</i>	<i>P5</i>	<i>P6</i>	Σ	Medal
UNK1 Chris Bellin	7	7	1	4	1	0	20	Bronze
UNK2 Luke Betts	7	7	7	7	3	0	31	Silver
UNK3 Tim Hennock	7	7	7	7	4	0	32	Gold
UNK4 Peter Leach	7	3	1	7	7	0	25	Silver
UNK5 Sean Moss	7	7	0	0	7	0	21	Bronze
UNK6 Preeyan Parmar	6	7	1	7	7	0	28	Silver
	41	38	17	32	29	0	157	1G, 3S, 2B

The cutoffs were for 32 for Gold, 24 for Silver and 14 for Bronze. The efficiency prize therefore goes to Tim Hennock who achieved his medal with no margin of safety whatever.

It was gratifying that all six British students received medals, and that Tim Hennock was awarded a gold. Luke Betts only missed a gold medal by 1 mark, and is determined to do even better next year. The rest of our team are ineligible because they are going to university in 2009. Our reserves Nathan Brown and Andrew Hyer will be available next year. From the point of view of a coach, I was particularly pleased that we scored nineteen 7s and only one 6. Our efforts to minimise the number of marks frittered away by careless writing up have been almost completely effective.

In terms of the rank order, the UK was in the peloton as usual, on equal 19th with both Hungary and Bulgaria. The detailed statistics of IMO 2009 and all previous IMOs can be found at the excellent

<http://www.imo-official.org/>

site. Details of previous British performances at the IMO can be found at Joseph Myers's IMO register:

<http://www.imo-register.org.uk/>

As for international achievement, the leading performances of the 104 countries participating were: 1 People's Republic of China (221), 2 Japan (212), 3 Russian Federation (203), 4 Republic of Korea (188), 5 Democratic People's Republic of Korea (183), 6 United States of America (182), 7 Thailand (181), 8 Turkey (177), 9 Germany (171), 10 Belarus (167), 11= Taiwan (165), 11=Italy (165), 13 Romania (163), 14 Ukraine (162), 15= Vietnam (161), 15= Islamic Republic of Iran (161), 17 Brazil (160), 18 Canada (158), 19= United Kingdom (157), 19= Bulgaria (157), 19= Hungary (157), 22 Serbia (153), 23 Australia (151), 24 Peru (144), 25= Georgia (140), 25= Poland (140). Here are some selected other performances of possible interest to Anglophones: 28 India (130), 29 Hong Kong (122), 30 Singapore (116), 31 France (112), 43 South Africa (84), 50 Sri Lanka (74), 58 Bangladesh (67), 66 New Zealand (53), 69 Cyprus (45), 75 Malaysia (31), 76 Trinidad and Tobago (28), 85 Pakistan (21), 89 Ireland (20), 90 Nigeria (17) and 100 Zimbabwe (5).

Of course China is to be congratulated as usual. This year's big surprise was the wonderful performance of the Japanese team. The two Koreas are establishing themselves as leading players. In the case of North Korea this is remarkable since they were absent from the IMO for many years. One expects Russia and the USA to be near the top, but this year there were excellent performances by Thailand and Turkey. After that comes a rush of nations on fairly similar scores, headed by Germany.

There were three exceptional individual performances, these being precisely the three students who solved problem 6 completely. They were: 1= Makoto Soejima, Japan (42), 1= Dongyi Wei, People's Republic of China (42) and 3 Lisa Sauermann, Germany (41). Of course the German media were very excited by Lisa's performance, but strangely enough she elected to have her medal presented to the German team's tiger mascot.

Another remarkable feature of this IMO was the participation of 11 year old Raúl Chávez Sarmiento from Peru. He won a bronze medal, and a prolonged standing ovation at the medal ceremony. Under the leadership of Emilio Gonzaga Ramírez, Peru have become quite a force at recent IMOs.

Our training partners Australia obtained two good golds through Andrew Price (37) and Sampson Wong (34). There were 59 girls participating at the 50th IMO, breaking through the 10% barrier. Here are the problems.

1. Let n be a positive integer and let a_1, \dots, a_k ($k \geq 2$) be distinct integers in the set $\{1, \dots, n\}$ such that n divides $a_i(a_{i+1} - 1)$ for $i = 1, \dots, k-1$. Prove that n does not divide $a_k(a_1 - 1)$.
2. Let ABC be a triangle with circumcentre O . The points P and Q are interior points of the sides CA and AB , respectively. Let K, L and M be the midpoints of the segments BP, CQ and PQ , respectively, and let Γ be the circle passing through K, L and M . Suppose that the line PQ is tangent to the circle Γ . Prove that $OP = OQ$.
3. Suppose that s_1, s_2, s_3, \dots is a strictly increasing sequence of positive integers such that the subsequences

$$s_{s_1}, s_{s_2}, s_{s_3}, \dots \quad \text{and} \quad s_{s_1+1}, s_{s_2+1}, s_{s_3+1}, \dots$$

- are both arithmetic progressions. Prove that the sequence s_1, s_2, s_3, \dots is itself an arithmetic progression.
4. Let ABC be a triangle with $AB = AC$. The angle bisectors of $\angle CAB$ and $\angle ABC$ meet the sides BC and CA at D and E , respectively. Let K be the incentre of triangle ADC . Suppose that $\angle BEK = 45^\circ$. Find all possible values of $\angle CAB$.
 5. Determine all functions f from the set of positive integers to the set of positive integers such that, for all positive integers a and b , there exists a non-degenerate triangle with sides of lengths

$$a, f(b) \text{ and } f(b + f(a) - 1).$$

(A triangle is *non-degenerate* if its vertices are not collinear.)

6. Let a_1, a_2, \dots, a_n be distinct positive integers and let M be a set of $n-1$ positive integers not containing $s = a_1 + a_2 + \dots + a_n$. A grasshopper is to jump along the real axis, starting at the point 0 and making n jumps to the right with lengths a_1, a_2, \dots, a_n in some order. Prove that the order can be chosen in such a way that the grasshopper never lands on any point in M .

The problems were submitted by Australia, Russia, the USA, Belgium, France and Russia respectively.

Leader's Diary

The IMO of 2009 was exceptionally well organized and a great success. Nonetheless, for comic effect, this diary will inevitably focus on the things which went awry. As usual, I hope that the organizers will forgive my impudence and gross exaggerations.

July 5–6 This year my IMO began on July 5th. We are having a joint pre-IMO camp with the Australian team, and they are due in at dawn at Heathrow. The clever thing to do is to pre-position myself in Oxford on the 5th, and use the excellent all-night bus service to Heathrow, catching the handy 04:03 service on July 6th. I purchase a ticket in advance, so when the driver pulls over to pick me up outside Queen's College, he welcomes me by name, a level of service that I do not normally expect on a bus. Congratulations to the Oxford Bus Company.

I have ordered a minibus taxi to pick up the Australians from Heathrow, and it arrives just before the team enters the arrivals hall. The team leader is Angelo di Pasquale, and his able deputy is Ivan Guo. The travellers all look rather battered.

The taxi gets to Cambridge just in time for us to race to the Great Hall of Trinity College for breakfast. The UK pastoral person Jacqui Lewis is already there, and assisted by the local organizer Lee Zhao and the UK deputy leader Vesna Kadelburg, we sort things out. The Australians spend a gentle day reconstructing their personalities. The UK team arrives in time for supper, and old friendships are renewed and the endless round of card games begins.

July 7 Today we have our first practice exam. It is held in the Junior Parlour. This room is only just big enough to accommodate both teams, and overlooks a busy street where noise is generated. Thus the venue is not quite ideal, but after discussions with Angelo, we decide that it is good enough and that we will not have to switch locations for subsequent exams.

July 8 Today is tourism day. After breakfast we catch a train to London, and use the tube to get to Green Park and walk across to catch most of the changing of the guard at Buckingham Palace. Angelo finds it hard to believe that the marching chocolate soldiers in the elaborate uniforms are actually fighting men, rather than unemployed actors earning a bit on the side by prancing about. We then stroll up The Mall, walk under Admiralty Arch, and then lounge around in Trafalgar Square for a while. Angelo asks if the

bloke on top of the tall pole is Napoleon Bonaparte, on the grounds that the statue has the right kind of hat. He uses a straight face, so there is no way to tell if this is a deliberate wind-up.

One of the lesser plinths in the square is given over to performance art, and someone is hurling an endless stream of high-performance paper darts to entertain the crowd. Eventually we tear ourselves away, and walk up Whitehall, past many ministries, Downing Street and at length to Parliament Square. We purchase bespoke sandwich lunches using the shiny new UKMT Debit Card, and eat in Parliament Square. Then we look at the Parliament Buildings, and stroll west through Victoria Tower Gardens and cross the river over Lambeth Bridge. We then turned left, walking past Lambeth Palace, the gaff of the Archbishop of Canterbury when he is in town, and back to the south end of Westminster Bridge. Here was the high point of the day, as we passed St Thomas's hospital, birthplace of the current United Kingdom IMO Leader. Making our way through the hordes of supplicants, all trying to kiss the hospital's sacred concrete in an attempt to cure scrofula, gout, halitosis, dandruff and so on, we arrived at an attraction called the London Eye, a Ferris wheel of some note. We took a flight, and then boarded a boat trip. This took us under several of the well-known bridges, including Tower Bridge, and back again. The commentary was grotesque.

At length we escaped, and made our way by tube to Leicester Square, and a Japanese restaurant for dinner. That really was very good, and we made it home to Cambridge for plenty of sleep before the next day's exam.

July 9 The Mathematics Ashes is the trophy of the annual contest between the British and Australian IMO teams. The burnt remains of the scripts of the first exam are stored in a funerary urn. Today's exam will decide the Ashes. This year the result was very satisfactory from a northern hemispherical point of view, and the UK will hold the urn for at least the next 12 months. In the evening we have a celebratory dinner in hall (shirts, ties, shoes, the works). We are joined by the UK Observer A, James Cranch, who will accompany me to the IMO the next day.

After the dinner I am feeling very relaxed, confident that tomorrow's journey to IMO Germany will begin after lunch. It comes as a slight shock when James Cranch points out that we are on the early morning Stansted to Bremen Ryanair flight. Whoops. I rapidly print out a boarding card and James orders a cab.

July 10 After midnight, I pack at speed, and grab a few hours sleep until my alarm fails to wake me. James goes to a party in Trumpington. He returns

at 4am to provide very valuable door thumping.

The flight to Bremen is uneventful, and takes only 100 minutes. Careful readers of this diary series may recall problems with being greeted at IMO Spain in 2008 (but no problems at all in Vietnam 2007 where there was a dedicated IMO passport control). The organizers of IMO 2009 have obviously decided to lower our expectations by making a severe hash of the arrivals process (if you can call it that).

James and I arrive in the Ryanair terminal. There is no-one to meet us. We rattle around in hope of finding someone familiar. After a while we locate another couple of leaders outside the terminal, scanning the horizon for signs of an IMO bus. I venture back inside, and at length discover the Pakistan leader and his party with a boy who claims to be from the IMO organization. This lad doesn't know very much, but tells us that a bus is coming, and it will be yellow. Another IMO lad turns up, but then they both disappear again very promptly.

By now other leaders are starting to arrive. We decide to make ourselves visible by sitting together. Some leaders monitor incoming arrivals, and usher leaders to our self-designated waiting area. The Romanian leadership is in charge of finding the yellow bus. A replacement local organizing lad turns up. We ask him to phone and find out about the bus. His phone is not working so I lend him mine. Eventually he gets a message that the bus will arrive at 10:15. It doesn't. The number of jurors in the arrivals hall is now quite healthy, and we consider holding a meeting. I phone the UKMT executive director Mary Wimbury in the UK, and she starts working the phones to the IMO organization. Soon she send me a text saying that she has got through to the IMO travel organizer, and that the bus is about to arrive outside the Ryanair terminal.

In short order an IMO bus turns up at this advertized spot. Unfortunately this IMO-designated parking spot is in the strictly-forbidden parking spot category as far as the airport authorities are concerned, and the bus is chased off to a place several hundred metres away across a car park. News spreads among the leaders and their deputies and observers A. Several fall to their knees, shouting praise for the great yellow bus of which they had long dreamed. Women and children sob for joy, and the refugee IMO leaders gather their wheeled suitcases, and head at speed for the yellow bus. When they cross the car park, a yellow bus has the doors to its luggage bay wide open, and enthusiastic leaders load their bags with great gusto. While they are still doing this, the yellow bus drives off with the bags inside, luggage

doors open, and no leaders on board. There is some consternation. Several leaders go so far as to express mild irritation.

As you may have surmised, it was the wrong bus. Happily it turns out that it was an IMO bus, but not the *right* IMO bus. The correct bus is there, and is not yellow. The driver of the yellow bus is contacted by phone. He is on his way to pick up more leaders from the train station. Those leaders who have just seen their luggage disappear are reassured that it will be delivered to their hotel. We board the correct IMO bus, and are driven to the jury site at Bremerhaven. En route, I use my newly acquired IMO travel organizer's telephone number to place a call urging her to get someone competent to the airport pronto. It will later turn out that the situation for those arriving at the train station was rather similar. However, by the time that the students started to arrive, these problems were sorted out.

After that start, things had to improve, and they did. We arrive at the Atlantic Hotel Sail City in Bremerhaven in time for lunch. I am based here and James is based nearby. We are given our copies of the shortlist, and set to work trying to solve some problems. The hotel is very luxurious, and by dinner most of the jury has arrived.

James and I decide to explore. My hotel is next to the estuary of the river Weser in the harbour area. There is a shopping mall which connects the hotel to the main street. Near the fancy hotel, the outlets are expensive perfumed palaces which attempt to flog lotions, handbags and sunglasses to footballers' wives and girlfriends. I know where I am not welcome, and scurry past. Next we pass some burger outlets, and now I am on more familiar ground. At length we arrive at the main drag. Here the shops are very familiar. Every third one is peddling mobile telephones. We pick up supplies and return to the comfort of the flash hotel.

My room is one level up from ground level (so I sleep on what I call the first floor), handily positioned so that no lift journey is required to get to the restaurant, a 20 metre run. The jury area is on the ground floor, but when you arrive at the hotel the only way to get to the jury area is via the first floor, so you climb up and then down. You can see where you want to go through glass walls, but it is not a completely trivial matter. At first I assume that this arrangement is designed to give us some welcome exercise, and it has that effect. However, as time goes by I begin to suspect that we are experimental animals in a giant transparent labyrinth, and that our movements are being monitored. There is a special room for smokers, again with glass walls.

James is not quite so lucky as me. His hotel is a 10 minute walk away, and he reports that it is splendid, but that it is slightly downmarket because of the marginal inferiority of the make-up on the receptionists. Of course I am horrified by the thought of interacting with anyone who does not have immaculate facial presentation, so I decide not to visit his hotel.

July 11 James and I spend the day working on the shortlist problems. At 4pm we pick up the version of the shortlist which contains solutions, and carry on working during the evening. It is difficult to think of anything to say which might be entertaining, except of course to confess that we convinced ourselves that the grasshopper question, later to become Problem 6, had an easy solution.

July 12 The jury is chaired by Prof. Dr. Hans-Dietrich Gronau. He has a bell to keep us in order, and is an enthusiast for keeping to time. This is good news. The wonderful quality of the academic side of the administration was a particular feature of this IMO. There are very helpful jury guides. The hard-to-please UK observer James Cranch is satisfied by the arrangements.

The jury decides to select the so-called easy problems first, numbers 1 and 4, because there is a shortage of very accessible problems. By now the procedure is finely honed. First the jury conducts a paper vote called the *beauty contest*, where problems are rated for difficulty and beauty. The results are published, and then inform the selection of problems. Problem 1 will be N1, and Problem 4 will be G1, the questions which the Problem Selection Committee rank easiest among the shortlist problems in Number Theory and Geometry respectively. Selection is done via a ‘devil take the hindmost’ process, where a large number of pairs of so-called easy questions are considered, and pairs are eliminated by a long sequence of votes. Oh yes, and there are speeches.

Next the jury selects the hard problems by similar means. The problems selected are A6 and C7. The first involves subsequences of a sequence, where you look only those terms indexed by terms of the sequence, or one more than the terms of the sequence. Just the sort of convoluted mind-bending stuff that sits well at problem 3. Problem 6 concerns the ability of a grasshopper to do combinatorics, and will be fully solved by only 3 students. Thus it is worthy.

By now the jury is in ferment, because it has selected four problems and only one of them concerns geometry. Like a vampire after dusk, the jury has a thirst that must be slaked. It lunges for G2, the next best thing to G1. It is not enough. There is an inequality question which mentions triangles,

and that is grabbed as a kind of *geometry lite* problem. We now have the medium problems. The jury is now in a blood-frenzy, and wants to select more geometry problems, but the chair points out that we are only allowed to choose six problems, and the jury reluctantly climbs back into its box.

In recent years it has become the usual practice for the English Language Committee to sort out both the English wording, and also the mathematical notation for the problems. This year the full jury works out the notation first, and then the ELC does its work on the wording.

In the afternoon I visit a submarine moored near to the hotel, an exhibition of what sailors had to endure in WW2. Thanks to my long-standing interest in limbo dancing, I found passing through the circular holes between the airtight chambers perfectly straightforward.

July 13 Today is student arrival day, but we are cut off from all that.

The Anglophones have some time to spare, as other language groups work on their exam papers. The co-ordinators draw up six marking schemes, and these are mostly accepted by the jury. There is a case where the jury feels that the proposed marking scheme is too loose, inviting lengthy performances by disputatious leaders. The jury advises the relevant Problem Captain to tighten up the scheme, and he does.

I and several colleagues have lunch with Hans van Duijn, Rector Magnificus of Technische Universiteit Eindhoven. He will play a major role in IMO 2011 in the Netherlands. Hans knows several of my colleagues in the maths department of the University of Bath, but even so, he seems a very nice bloke.

After lunch some of us decide to mount an expedition to a local attraction. Next to the shopping centre there is a museum of climate. It has recently been opened, so presumably it will be in excellent condition. A strange assortment of characters make their way round to its front. We really don't know what to expect. Once we get through the entrance there is a giant sealed indoor construction, a building within a building, stretching to the very high ceiling. This is clearly where they keep the climate.

Someone, it may well have been someone Swedish, suggests taking a lift to the top and walking down. This seems the idle way to proceed, so we all agree. The lift takes us to the roof, where we can gaze at the skyline. They really do have a lot of wind farms here. Next we try to go down. We find an unlocked door and enter. I feel safe because while Indra (leader of Trinidad and Tobago) is with us, nothing really bad can happen. The door closes and locks behind us. We are trapped in a fire escape. The only way

out is to return to ground level and start again. Down we plod, feeling rather embarrassed. At length we get back to where we started.

We adopt another plan, and decide to access this giant climate pod via its entrance. This works much better. At first the climate is pleasant. We are, after all, in Bremen, a suitable place for people to live comfortably. We start our journey by following signs, and are led on a winding path. Next comes Switzerland, and beautiful smells from the high meadows. So far it was all very nice. Then things start to go wrong, for we are exploring the world as it is for people who do not have air-conditioning. You may not have realised, but it turns out that much of the world is hot and some of it is hot and sticky. I quite like this when in a sauna or a Turkish bath, in a suitable state of *déshabillé*. I don't care for it at all when fully dressed and climbing. There are devices on the wall which you can feel in order to experience the environment. It is my age of course, but I prefer precise information written in lots of major world languages. There are many areas off the main route, so it is easy to slip away from the group. I have formed an escape committee. I power walk through the museum, pushing mothers to one side, tossing their children into exhibition pools and swamps. Every now and again there is a lift. These are clearly intended for disabled access, but I am nervous about using them following the unfortunate business of getting trapped in a fire escape.

I am well ahead of the group now, and hopeful that soon I shall enter a temperate zone. A cafeteria suddenly appears. It is still hot and sticky, but I cannot resist a coffee. John Webb, IMOAB secretary appears. He pretends to know what he is doing but he has obviously been stuck in here for several days. Then the Canadian leader Dorette Pronk comes in. She claims to have information on how to escape, but I am not persuaded. I leave and follow the main route. Soon there is a rapid improvement. I go through an ice tunnel and enter an Alaskan winter. All sorts of tourists are wrapping themselves up against the cold, but this is ideal weather for an Englishman in shorts. I rest and relax, bringing my core body temperature down to an acceptable level. At last I continue the journey, and escape back in to the outside world.

This museum has an imperfect business model. They should let you in for free, but charge to show you the way out.

The translations of the paper have all been prepared by now. This year there are 55 language versions. My efforts to persuade the Australian leader to construct his own version of the English language paper fall on deaf ears. The idea would be to replace the grasshopper by a kangaroo.

In the evening I discover to my surprise that the hotel has a bar, with particularly erudite staff. A learned discussion concerning the organic chemistry of beer naturally leads on to a debate concerning the statistical distribution of numbers of faces of bubbles in beer foam.

July 14 The opening ceremony. The jurors and students must not meet, so the logistics of this event are always challenging. The jury is driven to an industrial area of Bremen, and spills out of the coaches on to tarmac. We are led round a corner to what appears to be a giant industrial unit. We step inside and discover that it is a concert hall for rock music. Unusually, proceedings begin and end with a performance by a breakdancing troupe “Breakmathix”, with sonic accompaniment drawn from the beatbox genre, an aspect of soi-disant hip-hop culture. There are some well-judged short speeches by local and national politicians, a video address by Angela Merkel (who senses the mood rather better than George W. Bush did in 2001), and words of welcome from the Chair of the IMO advisory board, József Pelikán.

The teams parade across the stage. This year the UK students are not wearing Panama hats, but are carrying large numbers of UKMT frisbees which they hurl from the stage over the audience.

David Brindley, who produced the BBC film “Beautiful Young Minds” is at the ceremony with the writer James Graham. That film was a 90 minute documentary on the build-up to IMO 2006 in Slovenia, including material on the IMO itself. It had great critical acclaim, receiving nominations for both BAFTA and RTS awards (the two most prestigious TV awards in the UK). I have no idea what project David and James have in mind, but I spread the rumour that they plan an IMO-based soap opera, with love interest provided by the contestants (q.v. *Highschool Musical n*), and dramatic power struggles between jurors. Lisa Sauermann (GER6) would be played by Ashley Tisdale and József Pelikán by Brian Blessed. This family entertainment would involve a sequence of violent deaths, with various geometric instruments being used as murder weapons.

In the evening the hotel lays on a themed German dinner, with various types of *Wurst*. I have never seen Paul Vaderlind, the Swedish leader, so happy. It was very good indeed.

July 15 Day 1 of the exams. The jury meets at 09:00 to consider its answers to questions of clarification. All three problems give rise to questions. Problem 1 involves a positive integer n which is at least 2. The wording does not state that it is at least 2, but this follows immediately from other information in the question. Nervous candidates seek reassurance. Problem 2 is a

geometry problem which involves the interior point of a line segment. Various students want to know what this terminology means. Finally, problem 3 involves a subscripted subscript. This frightens the horses.

I am slightly baffled by the question from UNK3 who asks if he may use a geometrical instrument known as a set square (a transparent triangle) which is not marked with angles (so it is not a closet protractor). Protractors are banned from the IMO exam, along with squared paper, pocket calculating machines and similar instances of moral corruption. Set squares are standard items in British geometry sets, and their use has never been in question. I will return later to this important matter, the *Geodreieck scandal*, as it will henceforth be known.

In the evening the scripts arrive, and I am pleased that our students have written up their solutions very clearly, so that it is relatively easy to see what they have done. They have had a good day, especially Tim Hennock who has written a perfect paper, and walked out of the exam after just two and a half hours.

It is time for advisory board business. The old system of voting in IMOAB elections was not designed for a large electorate, and for some time it has been clear that it needed reform. A previous attempt had foundered on the fact that there are too many alternative systems from which to choose, and the debate collapsed when someone mentioned the word 'beach'. In IMO close season (the period between late July and early next July) when nothing much of interest usually happens, some leaders hatched a plot to implement electoral reform. The new system would have to possess two properties: (a) it would have to be a considerable improvement on the old system (otherwise, why change?) and (b) it would have to be almost indistinguishable from the old system (otherwise it would be rejected out of hand). We managed to find a system with the required properties, and nervously put it forward, fearing that it would be destroyed by a storm of constructive suggestions. Happily (in my view) the jury got the point, and almost everyone sat on their hands and bit their tongues or *vice versa*. So, contrary to expectations, the IMO jury is prepared to sanction change, provided that the change is in a good direction and has magnitude ε . That is a helpful lesson.

July 16 Day 2 of the exams. Once again, all three of the problems attract questions of clarification from the candidates. As ever, there are students who do not know the definition of an incentre, a point which appears in problem 1. In problem 2, the vexed matter of degenerate triangles arises again. I am tempted to say that a degenerate triangle is one which is drawn on squared

paper, or uses a protractor or a calculator. We give a more helpful answer involving collinear vertices. Problem 3 involves a grasshopper jumping to the right on the real line. Some students choose their real line to point vertically, so that jumping to the right takes the insect off the real line. The Netherlands leader successfully convinces me that it is Dutch national policy to have all real lines pointing vertically up, and that we have a real problem on our hands. Nice one Quintijn, a brilliant wind-up. My student UNK6 Preeyan Parmar asked the question “what is a grasshopper?” Since this was frivolous, following the sensible warning of Finnish leader Matti Lehtinen, I wait until all serious questions had been dealt with before proposing an answer. “A grasshopper is an insect of the suborder *Caelifera* in the order *Orthoptera*. It is modelled by a variable point on the real line.” The jury agreed that this answer be sent.

Later I was to discover that the *Geodreieck* scandal had burst open on Day 2. A Geodreieck is a set square. Each juror was equipped with a wooden voting stick in the shape of an arrow, the arrowhead taking the form of a set square decorated with protractor markings rendering it IMO illegal. In IMO morality, a protractor is deemed even worse than odious areal co-ordinates and multivariate calculus methods for solving three variable symmetric inequalities. The fact that this taboo instrument featured so prominently on the voting sticks foreshadowed other problems.

Someone handed out a gift of real illegal Geodreiecke (with protractor markings!) to all competitors at the start of the IMO. A better informed fragment of the IMO organization then swung into action, issuing a Ukase prohibiting the use of this evil instrument in the IMO. On Day 2 of the IMO, matters came to a head.

Chief Invigilator Dierk Schleicher found that UNK3 Tim Hennock was about to use a set square, and ripped the offending instrument from Hennock’s geometry set. Hennock lunged across the IMO examination hall, forcing Schleicher to the ground, and the two of them exchanged blows and traded insults concerning the legitimacy of set squares. The IMO candidates stopped attempting the problems, and formed a circle around the protagonists who continued to batter the living daylights out of one another. The predictable nationalities started to make bets on the outcome.

The previous paragraph is, alas, completely untrue. Prof. Dr. Schleicher queried UNK3’s use of his set square, and was satisfied when told that the jury had given explicit permission for it to be used. I think the previous version was better though.

After lunch the jury and their retinue transfers to the students' site, Jacobs University in Bremen. Rice University in Houston played a major role in creating Jacobs University. Leaders meet their teams and deputies, and listen eagerly to the students' sometimes fanciful tales of how they have performed on Day 2. We also get to meet the UK deputy Vesna Kadelburg, and our observer with students Jacqui Lewis. They recount their adventures, and give us a tutorial on living at Jacobs University. It all seems very straightforward. The way to find things out is to ask the guide of the Australian team. Now is the time to cut the students a little more slack, and this is very easy because the campus is such a safe environment. Jacqui co-ordinates by text messaging.

In the evening the scripts arrive, and we see that our team have gathered more marks. The performance is not so strong as that on Day 1, but it is not a disaster.

Walking into a crowded university cafeteria comes as quite a shock after the days of being spoiled at the Atlantic Hotel Sail City. The UK team stayed in Rice University, Houston on the way to IMO 2005 in Mexico. Jacobs University has some features which are improvements over Rice. For example in Jacobs University there are some signs. Rice has a shop but as far as I could see, Jacobs does not. Of course the Rice shop does not sell toothpaste, food or soap, but rather things that you might need like sweaters with the word Rice written on them. So, both institutions score well in the 'no useful shop' category.

There is a little café tucked round the side of the porters' lodge. It serves good coffee, rolls and beer. It opens and closes at random, but seems to be able to read my mind. When I want to use it, it is almost always open.

July 17 Today is the first co-ordination day. The co-ordinators are the IMO police, and they will examine our marking of our students' papers, and make sure that our suggested scores conform to the various marking schemes.

An area of campus is coned off, and becomes forbidden ground to competitors. Inside the security zone are two buildings; East Hall and West Hall. Each contains the co-ordinators for three of the problems.

We begin with problem 1, for which we have six solutions, including fair wear and tear. The co-ordinators are happy to give 7 to four of the scripts, but argue for reductions to 6 for the two remaining ones. We adjourn for a brief discussion, and go back in to face them again. We put our hands up and agree that the case against Preeyan Parmar UNK6 is proved, and that he must drop to 6, but we argue carefully why the imperfection in the other

script of Peter Leach UNK4 is much less, and ask for leniency. Now it is time for the co-ordinators to call a time out. They meet with their colleagues, and grant the 7. Well, 41/42 is a good start.

Next we have to co-ordinate problem 2. This is a geometry problem. We have five excellent and uncontroversial solutions, but Peter Leach UNK4 is in trouble. He has overlooked the straightforward solution, but has made a remark in pencil which indicates he sees how to clinch the argument. He has also killed time by performing an insane attempt on the problem by means of Cartesian co-ordinates. We go in ready to beg for 1 for this remark in rough. The co-ordinators, however, have different ideas. There is a new tradition (this is how a change of policy is described in IMO speak). We are apparently going to give non-trivial part marks for serious attempts at algebraic solutions. Now, brother Leach's progressively more sordid calculations sprawl over many pages, accurate in all respects until the algebra explodes in a mess of errors at the end. For some reason the co-ordinators insist on giving 3 marks for this nonsense. We accept under protest.

Next we must co-ordinate problem 5. Mary Wimbury, the UKMT executive director, has arrived on a fact-finding mission from HQ. She goes in to observe James Cranch and Vesna Kadelburg do the talking. I am left pacing the corridor in expectant father mode until I am called in to sign off the marks. The negotiations go well. Attempts by James to point out one or two weaknesses in the solutions have been firmly over-ruled by the co-ordinators, and we have an attractive collection of marks.

By now we have discovered that there is room near the leaders' hall where you can get coffee and cake, and watch partial IMO results projected on screens. There is also a timetable which allows you to read the co-ordination schedule. This is colour coded, so it tells you which co-ordinations are happening, which are pending, and shows you when extended co-ordination meetings have been rescheduled. This is all magnificent, but we rapidly discover that the information is false. It depends on humans updating the system in real time, and you have no way of knowing whether or not the relevant human has gone to lunch, fallen asleep or died. For example, we have a co-ordination scheduled at 18:00, but the display tells us that our table is queued up, and that there are a couple of nations in front of us. Vesna points out that the organizers said that they would keep to schedule. We walk 50 metres to see, and sure enough our co-ordinators are ready to start on time.

July 18 First thing, Vesna and James co-ordinate problem 3. It goes well.

We have two certain 7s, and some scraps. We are hoping for more credit than we get for these fragments, but the marks are fair.

In the middle of the day, we co-ordinate problem 6. Here we are claiming five 0s and one script which might be worth something if you believe in fairies. The co-ordinators do not. Nonetheless, they are so bored with handing out 0s that they keep us talking about the non-empty script. We sense that they are lonely and bored. It is therefore with a sense of guilt that we withdraw, leaving them to hand out more 0s and not see a smile for hours to come.

Finally we must co-ordinate problem 4, a geometry problem so I will lead. We have three uncontroversial 7s, one stone cold 0, and two scripts worth talking about. We begin with Chris Bellin's script. You are supposed to discover and verify two possible triangles. There is the main case, and a side case which using some methods is harder to spot. Now our UNK1 has turned the problem into trigonometry, and has then introduced a classic sign error but pressed on. His subsequent calculation involves the same bits of algebraic cunning as the correct solution, so it is clear that had he not made this algebraic slip, he would have found both solutions. Unfortunately he does not make a serious attempt to verify the solutions, so that loses him 2 marks. I can see what is coming. Using the new tradition of awarding significant credit to incorrect solutions, he is going to get a good score. The co-ordinators try to get me to say how much I think it is worth. 'Not a lot' is what I am thinking, but I persuade them to declare first. They say it is worth 4 points. I say that in my judgement it is worth 2 points, but agree that they have to be consistent, so we are willing to accept the 4 points.

July 19 We have the final jury meeting first thing in the morning. The leader of Pakistan makes a forlorn bid to challenge a mark awarded on one of his scripts, but unless the case is overwhelming, the jury always sides with the co-ordinators. The medal boundaries are quickly agreed, and there are no delicate judgements to make. Luke Betts UNK2 has fallen one mark short of a gold medal. Thus we have GSSSB. Our position is level with Bulgaria and Hungary in 19th place. As usual the rank statistic is very sensitive for those countries in the *peloton*. If one member of our team had solved an extra problem, we would be on 164 points and in 13th place, just behind Italy. On the other hand, if one of our students had solved one question fewer, then on 150 points the United Kingdom would have been in 23rd position, just behind Australia. We are second in the Commonwealth behind Canada. We are top in the list of countries which contain a G but not an A in their name.

The chief co-ordinator gives a report on the exams and co-ordination. He

reports unusual patterns of toilet cubicle use. Such matters were carefully monitored. Despite suspicions, no hard evidence has yet been found, and no country is mentioned by name. These days the IMO scripts are scanned and on permanent record, so if evidence of irregularities is found in the future, it will be possible to revisit and re-examine past scripts.

The IMO Advisory Board secretary John Webb announces the lists of names of people who have been nominated for various IMOAB positions. The actual elections will be held next year. He also explains that negotiations are in train for future IMOs. The next few are already set: 2010 Kazakhstan, 2011 Netherlands and 2012 Argentina. I will suppress further details since the information is not yet official, but it seems that we have excellent candidates to host IMOs in 2013, 2014 and 2015.

Our German hosts have inserted an extra day in the programme to celebrate the 50 years of the IMO. We are treated to a series of short lectures by luminaries from IMOs past. The full list of speakers is Béla Bollobás, Tim Gowers (both based in the UK), László Lovász, Stanislav Smirnov, Terence Tao and Jean-Christophe Yoccoz. The UK team know the charming young man László Lovász Jr. through our joint winter camps with the Hungarians. Father and son both have IMO Gold Medals.

Between lectures there are breaks, and the stars (including three Fields Medallists) sit on stools and chat to the students, many of whom are determined to gather autographs. I was reminded of the febrile atmosphere in IMO 2001 in Washington when the students treated Andrew Wiles as if he were Jagger in his pomp. I try sitting on a stool and it works. A queue forms. Now who shall I pretend to be?

Former UK leader and 2002 Jury Chair Adam McBride made a very welcome short visit to the IMO in the context of the anniversary celebrations. He went so far as to desert his partner to indulge in the marking of scripts. There are many old IMO friends who have made short visits, and it is a delight to see them all.

July 20 Today is the outing. I have a long standing and well grounded aversion to IMO excursions, especially those which involve boat trips. Who can forget the *Hell on the Potomac* at IMO 2001, or the *Waverley Steamer Ordeal* of 2002? For some strange reason (stupidity?) I decide to risk the boat trip to Wangerooge, a Frisian Island. These islands are close to the shore, and run from the Netherlands through Germany to Denmark. We have an early morning start, so I get some sleep on the bus. As we arrive at the coast, we see extraordinary numbers of wind turbines. The journey

out into the Wadden Sea is punctuated by an announcement that a child is missing. My heart sinks but soon the news comes through that she is safe. There is a significant quantity of rain.

On the trip it is clear that Charles Leytem, leader of Luxembourg, is trying to hang around with the leaders of proper countries. Ever since he began to bring a full team of six students to the IMO, he has started to get ideas above his station. We send him away and tell him to play with the leader of Liechtenstein.

As we land, the weather improves a little. Lunch is being served in a multi-purpose hall from giant saucepans. Enough said. I make my way into town and sit next to some statues of seals. I then adjourn to a bar and drink a glass or three. The team are having officially sanctioned fun elsewhere, with the organizers laying on a beach Sudoku treasure hunt. I sit for a while staring at the sea to acquire the Wangerooze sunburn effect. This is not associated with dangerous exposure to UV-radiation, but rather depends on sand-blasting, as the grains of silica are blown against your tender skin by robust winds.

July 21 One feature of this IMO is that events keep being held in different places. The closing ceremony includes some excellent Beethoven, and the usual medal ceremony. The German organizers introduce a new method of handing out the medals. This involves having exactly the same number of students on stage receiving medals as there are people presenting medals. I am not sure why this has never been tried before, but it seems to work.

The compere has an extraordinary talent for saying not quite the same thing over and over, congratulating people for existing and so on. He really does it very well, but there is a difficult moment when he mentions ‘The United Kingdom of England’, an object rather akin to ‘The United States of California’ or the ‘Federal Republic of Bremen’. Now, to an Englishman, this remark is of little consequence, but I know that Adam McBride, the Scottish national treasure, is in the hall. If he had any hair, it would be standing on end. I worry that we are about to witness a brutal scene from *Braveheart*.

It was sad that some competitors felt it appropriate to boo other students. Until now, the IMO has been free of negative political gestures, and I hope that this situation can be restored.

We returned to the Jacobs University Campus for a party in the evening. I am feeling rather exhausted, but stay around because of accurate rumours that I have won the *Microphone d’Or* again. This is the award for the most garrulous juror. This year the trophy is a fabulous construction. The award

is not the thing of course. Rather it is the absolutely pointless build up which is so entertaining. The guardians of the official IMO languages translate the details and rules of the competition with mock solemnity. Claude Deschamps of France is particularly impressive as he gathers himself up to his not inconsiderable height to become the physical embodiment of l'Académie Française as he attests to the dignity of the proceedings. This comic masterpiece is in the hands of Rafael Sanchez, leader of the Venezuelan Team. The Spanish speaking nations (including Israel and Sweden for some reason) are very prominent in the ceremony, performing the following variation of the IMO hymn (words supplied by Gordon Lessells, the Irish deputy).

IMO Hymn written by Lidia Roisman, adapted 2009

Volaremos por el cielo
recorreremos caminos
esto no tendra fronteras
sumando nuestros destinos.

Unidos en un anhelo
venimos de todas partes
a compartir la alegria
de juntar ciencia con arte.

Yo hablo e intervengo
callando muy mal me orientes
y muchos puntos obteigo
y el golden mic yo me llevo.

Sumamos, multiplicamos
y llegamos a un total
infinito es nuestro sueno
sin medida, de verdad.

Volveremos a encontrarnos
resolviendo los problemas
razonar es nuestro estilo
la amistad nuestro sistema!

Speaking and motion making
being silent is not my style
many points accumulating
just to win the golden mike.

Je parle, je fais des motions
sans parler, je me sens mal.
J'accumule beaucoup de points
pour gagner le golden mic

Ich spreche und ich vermitte
mir gefällt ganz nicht die Ruhe.
ich erlange viele punkten
und den golden Mic mitbringe.

July 22 The journey home was straightforward for the UK team. It is more amusing to recount the adventures of the Irish side. Their deputy Gordon Lessells wrote: “Cillian decided to lose his passport between getting through customs and the flight call. The crew refused to take him without a passport. I was all set to get off the plane with him with the prospect of reporting the lost passport to the German police and negotiating some alternative route home. While the crew were arranging to remove our bags, I decided on one last search of his body and bag. In a side pocket I found the missing object to the relief of all involved.” Those of you who went to the Slovenian IMO may recall Gordon (without a passport) talking his way into Italy and later back into Slovenia while brandishing a library card, so he is expert in these matters.

I can report a second pathetic attempt to leave the IMO. Ivan Guo, Australian deputy, writes: “The trip was reasonably uneventful, apart from a minor mishap on the train to Frankfurt Airport, where 3 of the 6 were too tired to drag their luggage off the train before the doors closed. Luckily we had an observer who got off and I was still on the train. And even more luckily the next stop, Frankfurt Hbf, is only 15 minutes away. Everything was fixed within 40 minutes or so.” I think the Australians should get out more, and perhaps place a greater emphasis on sport. Then they would be more alert and not get themselves into these scrapes.

There are hundreds of UKMT volunteers whose work, directly and indirectly, supports the UK effort at the IMO. There is also the contribution

of the administrators in our Leeds HQ, and our sponsors. We are assisted by various academic institutions, including Trinity College Cambridge, The University of Bath and Oundle School. We must also tip our hat to the Bolyai Society in Hungary and the Australian Mathematics Trust, sister organizations with which we run mathematics camps. Thanks to all concerned.

Geoff Smith, Bath, 27-vii-2009