Balkan Mathematical Olympiad 2010

UK leader's report

Chişinău, Moldova

The Balkan Mathematical Olympiad is a mathematics competition for children at secondary school. It is held annually, at a site which moves around the region. In recent years, a few non-Balkan countries, and countries of marginal Balkanity, have taken to participating unofficially, either as an end in itself, or as valuable practice for the International Mathematical Olympiad held each July.

The 27th competition was held in Chişinău, Moldova during May 2-May 8 2010. Moldova is the poorest country in Europe, but the warmth of the welcome, and the efficient organization of the competition made this hard to believe. I would like to mention all the main organizers by name, but with characteristic modesty, the website does not specify their identities.

The UK has a self-imposed rule that no student may participate in the Balkan Mathematical Olympiad more than once. This ensures that lots of students gain international experience, but it also means that we send teams which are weaker than they could be. Thus UK scores in this competition are usually quite low. However, the team of 2010 rose to the occasion, and put in a very creditable performance.

Here are the members of the UK team.

Benjamin Elliott	Godalming College
Richard Freeland	Winchester College
Sahl Khan	St Paul's School
Jordan Millar	Regent House School, Northern Ireland
Sergei Patiakin	Dame Alice Owen's School
Jack Smith	King's School, Grantham

Leader was Dr Geoff Smith of the University of Bath, and Deputy Leader was Jacqui Lewis of St Julian's School, Lisbon, Portugal. The involvement of the UK team in this competition is sponsored by Winton Capital Management.

The paper lasts 4 hours 30 minutes, and each of the four questions is marked out of 10 points. The medal boundaries are determined by the results of official participants, and this year the cut-offs were 12, 25 and 35. Problem 4 proved very taxing, and no student produced a complete solution to it during the exam. Here are the results of the British students.

Name	P1	P2	P3	P4	Total	Award
Benjamin Elliott	10	0	2	7	19	Bronze
Richard Freeland	10	10	10	0	30	Silver
Sahl Khan	10	1	0	0	11	Honourable Mention
Jordan Millar	10	0	1	0	11	Honourable Mention
Sergei Patiakin	10	10	5	2	27	Silver
Jack Smith	10	0	5	2	17	Bronze

More information concerning the event can be found at

`http://www.math.md/bmo2010/index.php`

Here is the paper, sat during 4 hours 30 minutes.

1. Let a, b and c be positive real numbers. Prove that

$$a^2b(b-c)/(a+b) + b^2c(c-a)/(b+c) + c^2a(a-b)/(c+a) \geq 0.$$

2. Let ABC be an acute triangle with orthocentre H. Let M be the midpoint of AC. Let C_1 on AB be the foot of the perpendicular from C, and let H_1 be the reflection of H in AB. Let the points P, Q and R be the orthogonal projections of C_1 onto the lines AH_1, AC and CB, respectively. Let M_1 be the point such that the circumcentre of triangle PQR is the midpoint of the segment MM_1.

   Prove that M_1 lies on BH_1.

3. A *strip* of width w is the set of points in the plane which are on, or between, two parallel lines distance w apart. Let S be a finite set of n

($n \geq 3$) points in the plane, such that any three different points from S can be covered by a strip of width 1.

Prove that S can be covered by a strip of width 2.

4. For each positive integer n ($n \geq 2$), let $f(n)$ denote the sum of all positive integers which are at most n and are not relatively prime to n. Show that $f(n+p) \neq f(n)$ for each such n and for every prime p.

Acknowledgements

I thank the small army of coaches and examiners who help to prepare our teams, the students who compete in our competitions, and the families whose plans are disrupted by mathematics competitions. This includes my own. The support of the Leeds Office of UKMT has been invaluable, and of course we are all very grateful for the continuing financial support of *Winton Capital Management* whose backing for UK participation in the Balkan Mathematical Olympiad and the Romanian Master of Mathematics competitions has been unwavering, despite recent vicissitudes in the finance industry.

Leader's Diary

The Leader's diary is inspired by reality, but events are subject to arbitrary exaggeration and distortion.

May 2 It is Sunday, and our plane is scheduled to leave Heathrow Terminal 4 at lunchtime. I plan to catch a train from Bath to Reading, and transfer to the airport by bus. All goes well for the first 3 minutes, until I notice that the train has turned off the main line, and is heading for Portsmouth. A jovial conductor informs me that this is a scheduled diversion, and that we should reach Reading on time. He reckons without a sequence of what are termed *engineering overruns.* We reach Reading via Newbury 45 minutes late. There is no choice, and I catch a cab to Terminal 4, the fare being two limbs.

I am the last of the party to arrive. We take an excellent and uneventful TAROM S.A. flight to Bucharest where we meet up with the French team in the transit lounge. The Italians have taken a direct flight to Moldova.

Our luggage is checked through to the Moldovan capital Chişinău. We use a 50 seat propeller driven plane to reach our destination.

The entrance formalities are light, and our bags arrive quickly. I read the customs regulations with interest, not wishing to get thrown into gaol. It turns out that you can bring in arbitrarily large finite amounts of foreign currency. The euros in my wallet are therefore legal.

We arrive to be warmly greeted by several representatives of the competition, including our guide Florentin. He has never visited an English speaking country, but has a wonderful facility for languages. He came second in a Moldovan national English competition. He is young, and very enthusiastic.

We are first driven to the Leogrand Hotel in the centre of town, so the team can check in. This is a posh place, with several uniformed doormen. It has an opulent foyer, and a sweeping staircase up to the mezzanine floor. I try to disguise my disappointment at the absence of an indoor waterfall, but we must somehow make do. We share the hotel with the teams from France, Italy, Kazakhstan and Saudi Arabia. I bid the team and Jacqui farewell, and accompanied by the French leader Claude Deschamps, I am driven to the secret jury site, east of Chişinău and just outside the city limits. We are among the last leaders to arrive, but we are just in time for a late supper. I look around, and see many friendly faces from previous IMOs. We receive copies of the problems shortlist, and repair to our rooms to study the options.

There is a conventional hotel building, but at the back there is a garden, complete with a swan pond and picturesque and romantic farming machinery. Of course it is dark, so I am not aware of all this yet, and am temporarily a sort of Moldovan Bishop Berkeley. Following a path made of decking, I am eventually led to my faux rustic cabin. This is excellent in almost all respects, but there are issues concerning illumination. There one large room, and a bathroom. The bathroom has a light in it, which is a blessing, and a heated wall. The sleeping end of the big room has no light at all. The other end has a light, but no visible light-switch (remember it is dark). To skip ahead, I will discover a switch behind a sofa next day, but for the moment I am stuffed. Well, I am very sleepy, so it doesn't matter.

May 3 After an excellent breakfast, we work on problems for a while, and then select the paper. There are some very good problems on the shortlist, but I really wish we had more time to think about the problems. Only the official Balkan Mathematical Olympiad countries can vote on the jury, but everyone is welcome to air their views. The jury chair is Valeriu Guţu, and he is very warm, businesslike and fair-minded.

We select a paper. See

`http://www.bmoc.maths.org/`

Problem 1 is a classic cyclically symmetric 3-variable inequality. The official solution is short and neat, but it will turn out to have very many solutions. This is a fairly easy problem by the standards of international mathematics competitions. Problem 2 is a geometry problem. The official solution makes the problem seem harder than it is. Also the way the problem is posed actually disguises what is really going on, and of course that is deliberate. Problem 3 is a combinatorial geometry problem. Later I will read a wonderful solution on Mathlinks (Art of Problem Solving) by *Stifler* from St Petersburg which involves a slick move which leads to many fewer technical details than the usual solution. Problem 4 is a hard number theory problem, and seems to require a solver to have several good ideas.

We jump into a bus and go to a theatre where the students have gathered for the opening ceremony. We are penned in the green room to prevent conversation with the students. The Secretary of State for Education drops in, as they do. Then we go onto the stage to get clapped by the students, a welcome inversion of the usual procedure. Then we step down into front row seats to enjoy some children engaging in folkloric dancing. After a couple of sets, both the minister and the jury depart, leaving the students to enjoy the entertainment.

Back at the jury site, I am buoyed by the discovery of a light switch deep behind the sofa, but am starting to fret about the fate of Crystal Palace FC. By now they have played their last match of the season against their rivals Sheffield Wednesday, but I have no way to discover the result. My phone doesn't function in Moldova, and the jury site does not have internet access. I scan the staff looking for fans of weak Championship sides, but none is evident. For both teams, defeat would mean relegation, and in the case of Palace, likely bankruptcy and oblivion. However, here in Moldova, the result of this match has the status of Schrödinger's cat.

We work on the language of the papers. We sort the English version out first, and I am drawn in as the local expert. As always, I argue for clarity rather than elegance of expression, and the jury is very helpful. In the evening there is a very late banquet, and the spirits flow, in both senses. Fortunately I have practised drinking vodka with my father-in-law, and so can employ the Russian method, the digital form of drinking where glasses are either full or empty at all times. This has the advantage that it is always clear whether

or not your glass needs to be refilled. I drink a large amount of bottled water before going to bed, for which I am truly grateful next morning.
May 4 We return to Chişinău immediately after breakfast in order to be able to answer questions of clarification during the first 30 minutes of the exam. Then we discuss the marking schemes, and it all goes very smoothly. I meet Jacqui and the students at the end of their exam, we have lunch and I check-in to the Leogrand hotel. I raise the matter of the Palace-Wednesday game, and as wave functions clatter to the floor all over the room, it turns out that the Glaziers have survived.

I skip the afternoon excursion, and have a long shower and rest. The students give me individual reports on what they think they have done on the paper.

In the early evening, the scripts become available. I walk with Fawzi Al-Thukair and other Saudi leaders to pick up our scripts. The celebrated Titu Andreescu is now part of the Saudi coaching set-up, and it is very heartening to see him again. I return to the Leogrand Hotel and start to mark scripts.
May 5 I have breakfast with Jacqui and the students. A welcome feature of the hotel is that a young lady harpist gives a recital throughout the meal. I find this very soothing, and will introduce daily recitals at home. What is the point of having children if they cannot play the harp? I spend the rest of the morning doing more marking, and in the afternoon co-ordination begins.

Our first two co-ordinations are for Problem 2 and 4. The team are safe with our guide at a bowling alley, so Jacqui pops in to observe. We have two correct geometry solutions, and a bonus mark for Sahl Khan for making a useful observation. In the number theory Problem 4, Ben Elliott has made significantly more progress than the other British students, but I expect him to get about 4/10. The co-ordinators spring a pleasant surprise, because they have a quick way to complete his argument to a full solution, and so they are more impressed by it than I am. He gets 7 points, and I try to control my involuntary eyebrow movement.

When we have finished co-ordinating the scripts of the the first three students, the co-ordinator looks at my notes, and thinks that I have written 2,2,2 next to the names of our last three candidates. In fact I have written smudge, 2, 2. She concedes 2,2,2 and we start to wrap up. As she is filling in the forms, I notice that my smudge is not a 2, and I look at the relevant envelope (belonging to Jordan). He has handed in no pages. Now, getting 2/10 for an empty script seems a big reward, even for someone with my negotiating skills. I interrupt her and point out the difficulty, and explain

that the UK is claiming 0 marks for this student. She produces a script, and sure enough it deserves 2/10. I explain that I have not seen this script before. Perhaps there was a mistake in the photocopying room, but there is still the mystery as to why the envelope was marked as containing no pages.

I tax Jordan over dinner. He is pretty sure that he handed nothing in, and is very impressed with a score of 2/10. I ask him if he did some work in rough, and perhaps that was somehow inserted in the envelope? He thinks not, but now he is in a confused state. After dinner, all is explained, and Jordan loses his marks. It turns out that a student from another country had not labelled his script, had elected to write in English, and had handwriting very like that of Jordan Millar. His page had become incorrectly associated with Jordan. Well, these things happen, and fortunately the student concerned got his correct marks in the end.

May 6 Today Jacqui is staying with the students, and perhaps this is a good thing because co-ordination is considerably more eventful. We begin with Problem 1, for which I am asking full marks for everyone. The authorities accept that this is the correct mark for five of the six students, but they are concerned that Jordan Millar's solution might be wrong. This comes as a surprise to me, so I ask them (with trepidation) to point out the error. He has a cyclically symmetric expression in three variables, which he first simplifies, and then he elects to bust the symmetry in a manner which is 'without loss of generality'. There is no doubt that this move is vulgar, but the co-ordinators suspect that it is illegal. After some conversation, the co-ordinators accept that this sordid trick is allowed. At this point I expect them to fold, and hand over 10 marks, but to my horror they then proceed to show concern about another matter. Later on, Jordan has indulged his appetite for mathematical thuggery even further, and has deliberately broken the homogeneity of the expression, splitting the problem into two cases. The co-ordinators think that what he has done is not logically correct. They are so adamant that I begin to think I am mad, so I ask for a time-out and go upstairs. As soon as I reflect, I realise that Jordan is right, and I write out a formal justification of his move. I rush back, but the leader of Kazakhstan has begun his co-ordination. I hand over the note containing my argument. Later in the morning we have another meeting, and the co-ordinators now agree that Jordan's move is legitimate, and he gets his 10/10. I am delighted about this because he only has 1 mark on the other problems.

Problem 3 is relatively straightforward. Richard Freeland has a solution almost identical to the official solution and gets 10/10. Sergei Patiakin has

handed in half a solution. Unfortunately there is a non-obvious fact which he has simply stated as obvious, and the absence of any justification costs him 5 marks and so he gets 5/10. Jack Smith, on the other hand, has supplied precisely that part of the argument which Sergei's script lacked, and nothing else. It makes perfect sense that Jack gets 5/10 marks.

In the evening there is the final jury meeting. I mentioned the jury chair Valeriu Guţu before. There are other people who played important roles in organizing this event, including Valeriu Baltag. The two Valerius have an interesting discussion about the medal boundaries which goes on for a considerable time. Eventually the jury decides to follow its own rules, and the cut-offs are selected. These are determined by the performances of students from countries which are official participants. The bronze cut-off at 12 means that neither Jordan nor Sahl get a medal, which seems very harsh, but those are the rules. They do both get Honourable Mentions for solving a whole problem.

Ben Elliott and Jack Smith both have well deserved bronze medals. If ever Ben varies his strategy of going *misère* on geometry problems, he will be a force in mathematics competitions. Jack famously won BMO2 this year, and we bid him a fond farewell as he goes to university next year. Richard Freeland and Sergei Patiakin have safe silver medals, so this is easily the best performance by a UK team in this competition.

May 7 I put my alarm on for 04:00, both as practice for the early start next day when we will come home, and also so that I can watch the election results at the expected crucial time (02:00 UK). As you probably know, at that moment nothing was clear, so I went back to sleep. As I got up for breakfast, the BBC called the result as a hung parliament.

We visit a monastery in the morning, and have the closing ceremony in the late afternoon. There is high drama as Jack becomes queezy on stage. There is the possibility of projectile activity (q.v. *The Exorcist*) but he makes it off stage in the middle of the presentation, and Jacqui takes him back to the hotel.

This happens just as I am called up on stage to present some bronze medals. By chance one of the students to whom I present a medal is Matthew Fitch, who plays for France despite his name. After the ceremony Sergei Patiakin gets a text from Jacqui to the effect that Jack is in bed and resting, and that she will stay at the hotel with him.

We go to the closing banquet. We leave quite promptly because of the early start next day. Our plane will leave at 06:40. The travel organizers

want to have a coach pick up the UK and France at 04:10 and take us to the airport. Since the airport is a 20 minute drive from the hotel, and the airport small with very limited traffic, Claude and I do not agree. We reschedule the bus for 04:45.

May 8 At 04:10 I get a call from Valeriu Baltag who is downstairs with a bus and is wondering where to find the teams of France and the UK. Unfortunately the message about the rescheduled time has not got through. Valeriu is accompanied by the quiet man. He speaks no English, but shakes hands at all possible opportunities, and is in charge of conference cash. These poor chaps have kindly come down to see us off in the middle of the night, and most of us are still in bed. The guides of the UK and France are also there, and they will even come to the airport to see us off.

The UK guide, Florentin, deserves a report all to himself. He has been fantastically helpful at all times, and has made a tremendous effort to befriend our students. There are comic aspects to the situation, but everyone likes Florentin. He does follow organizational instructions with excessive zeal. He must have moved us between the hotel and the exam site about a dozen times. This is a 15 minute walk through a small safe town with low traffic density. To Florentin, however, Chişinău was more like Baghdad in 2005. He would constantly scan the horizon for threats, and direct the traffic at junctions.

The hotel serves coffee and croissants from 04:30, so we are able to charge up before beginning the journey. We inspect Jack, and decide that he is just fit to travel, being a paler shade of green than the previous evening. It seems that after I went to bed, Kazakh students had organized a rather good party of some sort, which bodes well for IMO 2010. At the airport we say farewell to Florentin, and are surprised to find that we will travel to Bucharest in a jet plane.

The journey home is uneventful, and we are met by many happy family members. Jack now has a pink tinge. Certificates are handed out in Terminal 4, and there is much clapping. I transfer to the main bus station with Jordan, which is next to his terminal for Belfast, and the game is over for another year.