International Mathematical Olympiad 2010

UK leader's report

Almaty and Astana, Kazakhstan

This year the annual world championship of secondary school mathematics was held at various sites in Kazakhstan during July 5 – July 14th. The six students sit two exams, each of duration four hours 30 minutes under stictly controlled conditions. This year the UK side consisted of six boys, and unusually five of them are being educated in the maintained sector.

The team was selected by competitive examination, and consisted of the following six people.

UNK1 Luke Betts	Hills Road 6th, Cambridge
UNK2 Nathan Brown	King Edward's Camp Hill, Birmingham
UNK3 Andrew Carlotti	Sir Roger Manwood's School, Kent
UNK4 Richard Freeland	Winchester College, Hampshire
UNK5 Sergei Patiakin	Dame Alice Owen's School, Hertfordshire
UNK6 Aled Walker	King Edward's Camp Hill, Birmingham

The reserves were Andrew Hyer of Westminster School, London, and Jordan Millar of Regent House School, Northern Ireland. The provenance of the team and reserves by chance includes all four countries of the United Kingdom. The team were accompanied by Dr Geoff Smith, University of Bath as leader; Dr James Cranch, University of Leicester as deputy; Dr Joseph Myers, CodeSourcery as observer with leader; Dr Ceri Fiddes, Millfield School as observer with students. For the pre-IMO phase, when Dr Fiddes was unavailable, Miss Jacqui Lewis of St Julian's School, Lisbon acted as observer with students.

The results of our team members were as follows.

	P1	P2	P3	P4	P5	P6	Σ	award
UNK1 Luke Betts	7	1	1	7	1	0	17	bronze
UNK2 Nathan Brown	7	0	1	7	7	0	22	silver
UNK3 Andrew Carlotti	7	0	0	7	0	6	20	bronze
UNK4 Richard Freeland	7	0	0	7	0	0	14	honourable mention
UNK5 Sergei Patiakin	7	0	6	7	7	0	27	gold
UNK6 Aled Walker	7	0	0	7	0	0	14	honourable mention
totals	42	1	8	42	15	6	114	

The medal cut-offs were 15 for bronze, 21 for silver and 27 for gold.

This result had us ranked 25th in terms of points scored, at the back of the peloton. A dramatic underperformance on Problem 2, a geometry question, served to undermine some excellent work on the far more demanding problems 3, 5 and 6. The scores of the leading countries were as follows: 1 People's Republic of China 197; 2 Russian Federation 169; 3 United States of America 168; 4 Republic of Korea 156; 5= Kazakhstan 148; 5= Thailand 148; 7 Japan 141; 8 Turkey 139; 9 Germany 138; 10 Serbia 135; 11= Vietnam 133; 11= Italy 133; 13= Canada 129; 13= Hungary 129; 15 Australia 128; 16= Romania 127; 16= Islamic Republic of Iran 127; 18 Peru 124; 19 Taiwan 123; 20 Hong Kong 121; 21 Bulgaria 118; 22= Ukraine 117; 22= Singapore 117; 24 Poland 116; 25 United Kingdom 114; 26 Uzbekistan 112; 27 Belarus 110; 28 Azerbaijan 109; 29 New Zealand 106; 30 France 105.

As usual, the performance of China was excellent, and they had the only student with a perfect score. Australia performed very well this year, and the results of New Zealand are truly exceptional, given their modest population base and their performances in the past. I should also draw your attention to the performance of Saudi Arabia. The Kingdom is making a great training effort, and this year secured 55 marks and two bronze medals. This is more than double the number of marks they obtained in all five previous participations combined.

As for the UK, if we had performed well on Problem 2, and say scored 21 points, then that would have given an overall score of 134. As I have noted before, the rank statistic is extremely sensitive to small perturbations in performance for those teams which finish in the range 10th–30th. Two of our students have that rarest of distinctions, a double honourable mention. In most years, 14 points is enough to get a bronze medal, and they can count themselves unlucky to have lost out.

Congratulations are due to all of our students, and one must mention the special performances of Nathan and Sergei in obtaining silver and gold medals respectively.

Detailed results and statistics can be found at the official IMO site:

http://www.imo-official.org

It is with sorrow that I must report that the jury disqualified the team of North Korea following allegations that some of their students had supplied solutions which were exceptionally close to those in the official solutions booklet.

Here are the six problems of the 51st Mathematical Olympiad. The problems should be taken in two groups of three, and for each three questions the time limit is 4 hours 30 minutes.

1. Determine all functions $f\colon \mathbb{R} \to \mathbb{R}$ such that the equality

$$f(\lfloor x \rfloor y) = f(x)\lfloor f(y) \rfloor$$

holds for all $x, y \in \mathbb{R}$. (Here $\lfloor z \rfloor$ denotes the greatest integer less than or equal to z.)

2. Let I be the incentre of triangle ABC and let Γ be its circumcircle. Let the line AI intersect Γ again at D. Let E be a point on the arc BDC and F a point on the side BC such that

$$\angle BAF = \angle CAE < \frac{1}{2}\angle BAC.$$

Finally, let G be the midpoint of the segment IF. Prove that the lines DG and EI intersect on Γ.

3. Let $\mathbb{N}$ be the set of positive integers. Determine all functions $g\colon \mathbb{N} \to \mathbb{N}$ such that

$$(g(m) + n)(m + g(n))$$

is a perfect square for all $m, n \in \mathbb{N}$.

4. Let P be a point inside the triangle ABC. The lines AP, BP and CP intersect the circumcircle Γ of triangle ABC again at the points K, L and M respectively. The tangent to Γ at C intersects the line AB at S. Suppose that $SC = SP$. Prove that $MK = ML$.

5. In each of six boxes $B_1, B_2, B_3, B_4, B_5, B_6$ there is initially one coin. There are two types of operation allowed:

   Type 1: Choose a nonempty box B_j with $1 \leq j \leq 5$. Remove one coin from B_j and add two coins to B_{j+1}.

   Type 2: Choose a nonempty box B_k with $1 \leq k \leq 4$. Remove one coin from B_k and exchange the contents of (possibly empty) boxes B_{k+1} and B_{k+2}.

   Determine whether there is a finite sequence of such operations that results in boxes B_1, B_2, B_3, B_4, B_5 being empty and box B_6 containing exactly $2010^{2010^{2010}}$ coins. (Note that $a^{b^c} = a^{(b^c)}$.)

6. Let $a_1, a_2, a_3, \ldots$ be a sequence of positive real numbers. Suppose that for some positive integer s, we have

$$a_n = \max\{a_k + a_{n-k} \mid 1 \leq k \leq n-1\}$$

   for all $n > s$. Prove that there exist positive integers ℓ and N, with $\ell \leq s$ and such that $a_n = a_\ell + a_{n-\ell}$ for all $n \geq N$.

These problems were submitted by France, Hong Kong, USA, Poland, Netherlands and Iran respectively.

Leader's Diary

Readers are warned that sometimes this diary is more accurate than seems likely. It is intended to entertain and inform, and I trust that no-one will take offence.

This year the IMO produced its own official diary, and various editions can be downloaded from http://www.imo2010org.kz using the top right tab labelled "Diary of Olympiad". These are large files, for the diary is in glorious colour, and has many pictures. I urge the reader to examine these documents as an accompaniment to my more subversive commentary. The first edition has some impressive pictures of the architecture of Astana, and much else besides.

June 30 I make a very early start to get to Heathrow Airport very early. I am carrying almost all the passports, and the consequences of being late are too horrible to contemplate. My bags are very heavy because they are

stuffed with IMO detritus for the team. I reach Paddington on time and reverse direction using the Heathrow Express, arriving at Terminal 1 before 09:00.

I am first to arrive because pre-IMO pastoral supremo Jacqui Lewis is having trouble finding a Hotel Hoppa bus. Joseph 'Combinatorics' Myers is first to show up, wearing a UK Panama hat lovingly preserved from IMO Vietnam. Then Jacqui arrives, and the team start to trickle in. There is much swapping of goods to be done, so we open our bags. The sadly absent Vesna Kadelburg has sorted out some natty team uniforms, and these are distributed. I pass out Union Flags for the ceremonies. The Panama hats for 2010 have not arrived, and I suspect sabotage, but say nothing. The last parent says good-bye, and we are ready to face the IMO.

Things are going very well, until we try to check in. We are booked on Transaero, a Russian independent airline. Think Virginski. While two of our four landings will be a touch heavy, Transaero will come out rather well in this report. They messed up some of the vegetarian meal bookings, but overall we were treated rather well. The same cannot be said of BMI, the company which was supposed to handle the check-in process at Heathrow. I will not go into great detail because I understand that deputy James Cranch is planning a monograph on the topic. However, here is a brief summary.

First I should explain what a sensible airline does when a large party (10 people) turns up to check-in together. The party is told to ignore the machines, and is ushered to a check-in desk or two, so that they can be processed in a systematic way. Suitable seating arrangements are worked out, and there are no weight issues because so many students carry light suitcases. Not BMI. They have a young lady who gives instructions which we do not wish to follow, and advice of no quality. The parrot wants us all to check-in separately. She has no idea what a bad idea this would be. We have several students who cannot be trusted to handle their own passports, and who could not possibly navigate through well-designed check-in software (let alone the maze which is BMI's entry to the world). Such a strategy would have the party seated all over the plane.

We ask for advice, and she gives a long complicated answer which can be summarized as 'I cannot help you'. At this point I have to let off steam. I explain to her what her answer means, and that I would be perfectly happy with it if it were so expressed, but that dressing it up as if it were a helpful explanation is pointless and annoying.

Jacqui and James go to work, and find it is an uphill battle. The interface

is awful and despite deploying considerable intellectual power, it takes an age. They are admonished for not obeying the parrot's instructions. Joseph Myers and I check-in separately, and that proves easier. Whenever I get stuck I ask 'what button should I press Joseph', and being a professional computer wizard, he tells me, often correctly. I saw no sign of foreign language options, but in fact an inability to understand English might have made the process easier.

When we finally get our computer-generated boarding cards, we go to hand in our luggage, and the nice check-in people explain that it doesn't matter which seats you select using the check-in machines, because they have the power to reassign seats. If the parrot had mentioned this, then a lot of nonsense could have been avoided.

We have managed to arrange the party in a loose clump on the packed aeroplane. We have an uneventful flight to Moscow. There we need to enter the transit area, which means going through a security check. The relevant chappie is not to be found, so we wait for while, but eventually we are admitted to the Moscow Domodedovo playground. This is not a bad place, but has a parochial policy on cash. They take roubles and nothing else. Neither dollars nor euros work, though credit cards will do nicely. We find an Italian outfit where the team can grind their way through pizzas, until at midnight we board a Transaero flight to Astana, Kazakhstan.

I spot the Vietnamese leader Hà Huy Khoái, but he is presumably en route to Almaty whereas we are going to Astana.

July 1 We arrive in Astana very early in the morning. There is no hurry, and we find ourselves at the back of the immigration queues. At this point a whirlwind called Madina arrives. She is a local organizer, and marches us to the front of the queue. An IMO 2010 sign is then slapped on an immigration portal and we are whisked through at speed. There is a friendly lady from the Ministry of Education and Science standing there to oversee the process. The immigration officer is genial, and discards my immigration card as being unnecessary. I have doubts about this, because I fear that the lady from the Ministry will not be there on the way out, and I may meet a less generous border guard who doesn't happen to know how important I am, and may feel that I ought to be brandishing a doubly stamped immigration card. In fact I am being quite prescient. Read on.

Then the IMO officials and the lady from the Ministry attempt to march us through the green channel of customs. I explain that I have read the Kazakh customs regulations, and that I wish to declare a large sum of foreign

currency (for fear of piano-wire, gouges, thumb-screws and so on, but I leave out that bit). Once again I am informed that the regulations do not apply, and we are marched through customs. I have a sinking feeling. The local organizers are clearly making a big effort, and this bodes well for the rest of the IMO. Since we are four days earlier than most of the teams, they are practising the art of greeting on us. Well done, and thank you.

The airport is much like the rest of what we see in Astana. It is an excellent building, but is not supported by the usual clutter of infrastructure – hotels, car hire firms, giant car parks and petrol stations are absent. This is an airport surrounded by empty space, an asset with which Kazakhstan is richly endowed. I knew about this in advance, having failed to find anything when searching the internet using the keywords: Astana Airport Hotel. I then tried a map tool and discovered that with Astana Airport, you get what it says on the tin. Nothing else.

We have booked a driver to transfer us to the Imperia G hotel. I have been informed that the driver will carry an 'Imperia G' sign. We don't see him immediately because 'Imperia G' is written in the Cyrillic alphabet. The organizers promise to come to see us soon, and we are driven into town.

The Imperia G is in the old city, the naturally occurring place called Akmola which has been extended by Presidential fiat into the new city called Astana.

At first sight, Astana seems to be the capital of Mars. It is a showcase of contemporary architecture. The shapes are a festival of geometry. Stately pleasure domes erupt from the steppe at startling angles. Postmodern shopping centres, apartment blocks and sports arenas do not jostle at all. Giant administrative buildings curve and glint, and are rarely simply connected. In non-mathematical terms, think Gruyère cheese, or the work of Henry Moore, or polo mints. Kisho Kurokawa and Lord Foster have been set loose. Norman Foster has supplied the Khan Shatyr Entertainment Centre, the Palace of Peace and Reconciliation and the Pyramid of Peace.

We check-in, and have breakfast. The Australians are staying at the same hotel, and we run into them shortly before going upstairs for showers and bed.

We get up at lunchtime, to try to force ourselves into the local timezone. By evening, the Australians have yet to return, so we explore. A nearby market allows us to change some cash into the local currency (1 euro = lots of tenge). We find a Turkish restaurant, and let the team loose on the menu. It is very satisfactory, including kebabs, ice-cream and other health foods.

The host doesn't quite understand about giving clients space, and proceeds to join us at our table. I think that we are so far East that Brits remind him of Turkey.

Next day we hook up with the Australians over breakfast. Angelo and Ivan are their leaders. We plan to have practice exams together. The hotel quoted us a silly price for the hire of appropriate facilities, but our excellent IMO guides have obtained the use of a local school. Joseph Myers and I must transfer to the jury site today, so we bid farewell to the students as they leave for their first training exam.

The IMO guides persuade us that it is pointless and expensive to take a taxi to the airport, and a schoolteacher escorts me, Joseph, Angelo and our bags on the necessary pair of bus journeys. As a grotesque foreigner, I am a source of entertainment for local toddlers, and I spend quite a bit of time with my fingers clenched in tiny Kazakh hands. The Australian leader Angelo is with us, and given his unmarried and relatively youthful state, seems to be making excellent progress with the young women of Central Asia. His success is probably down to his experience working the Melbourne trams.

The flight to Almaty is uneventful. There is a lot of cloud, but when we do get to see the ground we observe that it is empty grassland, steppe in all directions. We are on Air Astana, the only local airline without an explicit five-skulls health warning at the UK Foreign Office website. Air Astana are excellent. We are met at Almaty airport, and the local organizers put the three of us, and all our bags, into one medium sized car. This is not actually possible, but it happened. The driver clearly had ambitions in Formula One, but was inhibited by being placed in rush hour traffic. Nonetheless, he managed several inappropriate and entertaining changes of lane.

After a little more than an hour we reached the sanatorium where the jury was based. I have some experience of hotels in the former Soviet Union, and this place is top of the range. The food is not the universal hotel diet, but is all the better for that. I have a bedroom on the 7th floor, and to my delight I discover that I have a floor lady, just as it was in the USSR. A woman sits in charge of each floor all the time. They have shifts, but she is always there. She never does anything. The system was part of the full-employment economics of Soviet la-la-land, and presumably the ladies could also report if counter-revolution was breaking out on their floor. Why are they still doing this, twenty years on? Presumably there was no moment at which it seemed appropriate to change this bizarre arrangement (don't knock it, this is the basis of marriage).

Although the food is wholesome, and I am attuned to the former Soviet obsession with buckwheat, some people clearly find the diet a little strange. Personally I thought it fine, and in fact providing such an array of meals must have taken a great deal of effort and thought. Kazakhs are hippophagists, and when I see an unusual meat on a cocktail stick, I cannot resist adding another tick to my list: frog, camel, goat, crocodile etc. While it is absolutely fine, I wouldn't go out of my way to eat dobbin again, for it is a little dry. My attempts to find *kumys*, fermented mare's milk, end in failure. There was also a tuck shop, where dissolute sanatorium clients could purchase chocolate, vodka, brandy and beer, or so I was told.

This is one of the happiest times of the IMO, when you get to see old friends again, and engage in ruthless teasing and mockery until settling down to see what events may bring.

After some effort, we get our hands on copies of the short-list, the proposals from which the problems on the papers will likely be chosen, and we go to work. I am concerned about $G1$, the easiest geometry problem, because I can solve it immediately. It turns out to be a lovely exercise by my UK colleague Christopher Bradley, but it is a little too easy, even for a first question. If you work forwards and backwards at the same time, from where you start and also from what you want to be, you quickly meet in the middle by the use of cyclic quadrilaterals and chasing angles. Fortunately $G2$ looks about right for a geometry problem. It will yield easily if you recall that if two points are mutually inverse with respect to a circle, then that circle is a circle of Apollonius with respect to those points.

There are very many excellent medium level combinatorics problems. We are blessed with an excellent choice this year. There are a few monstrous questions too. Other than a shortage of easy questions, it is a good selection, but there is no "grasshopper problem" as memorable as the one used in 2009.

July 3 We continue to work on the problems. I have set up camp on my balcony. The view is magnificent. We are on the edge of the Zailisky Alatau Mountains, and sufficiently high that insects are not a problem. The air is clean, and towards the end of the day the Kazakh brandy is delicious.

As we work on the problems, we confront the fact that the IMO is happening in parallel with the World Cup. Late at night, the jury room is darkened, and we can watch matches on a giant screen. Relevant leaders take a keen interest in proceedings.

A jury meeting excludes $G3$ and $N3$ because of prior art.

July 4 UK Observer A Joseph Myers has sunk $C7$, the most difficult of the

combinatorics proposals. He supplies chapter and verse of a paper published by the American Mathematical Society which includes the result. I ask permission for Joseph to address the jury, and this is granted. I have taken out all of his batteries, and only given the clock-work mechanism a quarter-turn. Joseph normally speaks at the same rate that he thinks, and for most people this causes serious bandwidth problems, especially people who are not native speakers of English. Joseph manages to slow down enough to get his point across, and unfortunately a very attractive problem is removed.

Eventually we start to select the papers. This is done in the order easy, hard and finally medium. We choose a functional equation with an unusual solution set and a nice geometry problem for the easy problems. For the hard problems we find an intriguing question which is, in a way, another functional equation, but it has a peculiar method of solution. We also select a hard algebra problem.

Next we turn to the two medium problems. Since we have not yet chosen a combinatorics problem, and we have a raft of excellent combinatorics problems of this level, we do have the option of choosing both medium problems to be combinatorics, but of course there is the geometry lobby. They have their way, and we select $G4$. Next we agonize over the combinatorics question. In the end, when it comes down to a choice between two problems, and is clear that support for the problems is equally divided, we make the choice on the basis that one problem is likely to much easier to co-ordinate than the other.

There is a notation phase, and eventually the English Language Committee meetings to sort out the wording. We produce rival versions of questions for the jury's perusal. In fact we offer to recast Problem 3 in terms of sequences, because in its current formulation, it is a little too close to being another functional equation problem. However, the jury decides to go with the original formulation.

July 5 Individual language groups proceed with the translations of their papers, with the official languages done first.

There are elections to the advisory board. Nazar Agakhanov of Russia will be the new IMOAB chair following two terms by the excellent József Pelikán. There are eight candidates for the two remaining vacancies on the board. There is a two-stage electoral process for the two seats, and I had played a significant role in the design of this system last year. Joseph Myers seemed very amused when I forgot the details of the process and accidentally spoilt my ballot on the first round. Myung-Hwan Kim of South Korea is re-elected,

and I am elected for the first time. IMOAB elections are a little strange, because you generally want everyone to win, especially your opponents.

July 6 The principal value of travelling the world is to meet and try to understand radically diverse cultures. As much of western and central Europe is converging, Kazakh priorities are very different. This is clear from the amazing effort that they put into ceremonial activities. The DVD of the IMO will have a few brief shots of young people doing mathematics or playing sport, but it is dominated by formal public events; parades, medal ceremonies and middle-aged people making speeches.

It is this attitude to the role of public ceremony which had Aztec children given as sacrifices to the gods, prisoners fed to wild animals in Roman circuses and IMO leaders and Observers A transported from Almaty to Astana in the middle of the night. This was to allow us to attend the IMO opening ceremony. Personally, if I were running an IMO, I would have sought another solution. These people have flown in from all over the world, most of them suffering from jet-lag. What do they need after a couple of days in Kazakhstan? Well, obviously, they need to get up in the middle of the night in order to attend an opening ceremony at the other end of the country.

We woke in the night at 02:30 in order to be on the buses at 03:00. Samson the Nigerian leader is nervous that he will oversleep and asks me to wake him. After loading our luggage we engage in ritual sitting. After a while, we were told we were on the wrong bus. The resulting transfer involved getting off the bus, digging out suitcases from its innards, and moving to another bus. You might ask why this was necessary, given the law of conservation of seats, but let it pass. I am reminded of IMO 2003 in Japan when the guides tried to persuade leaders to sit on buses in alphabetical order of country. We have a police escort (which in Kazakhstan means that you go more slowly), but still arrive at the airport in time to catch one of the early flights to Astana.

Our buses pass through the Martian metropolis to the Independence Palace. We walk past what seems to be an honour guard of ladies in blue, through to a magnificent breakfast spread and lots of tables on which to eat, and the empty set of chairs. I quaff some keffir, the local dairy drink. After this, it seems that some lounging around on soft furnishings would be a good idea, but none is to be seen.

Now an IMO guide, a lady in yellow, tells us that we can go upstairs to relax. I quickly scoot off to get first go at the upholstery. No chance. The ladies in blue forbid us to go upstairs. I go back and report this to the lady in yellow. She looks astonished, and a stand off ensues. The issue was resolved

when yellow triumphed over blue, and we were allowed upstairs. Once again I was quick off the mark, and at the top of the stairs there were sofas and a remarkable picture.

I plonked myself down to gaze on this curious and fascinating piece of work. I hesitate to make judgements on works of art, and will confine myself to a simple description. The focus of the painting is President Nursultan Nazarbayev. He is wearing what is presumably the emblem of a chivalric order round his neck, and is walking towards the observer. On each side of him are arranged many of the celebrated world leaders from about 10 years ago, mostly looking on the President with warm admiration. The applause is clearly being lead by Jacques Chirac. While most people (Berlusconi, Yeltsin, George W. Bush, Jiang Zemin etc) have faces indicating great pleasure in the moment, two figures stand out by showing darker emotions. I detect shrewd admiration from Vladimir Putin and definite jealousy in the face of Tony Blair.

President Nazarbayev has an extensive collection of honours from all over the world. My own Queen Elizabeth II has awarded him a knighthood. I often make reference to the Luxembourg leader, Charles Leytem, and here is another opportunity. The Grand Duke has awarded President Nazarbayev a *grand-croix de la Couronne de chêne* and by co-incidence, Charles is in the same order, for he has the *croix de chêne* (the Oak Cross). I look up chivalric orders on the internet, to see what is available. While some might favour the UK's *Order of the Bath*, I am attracted to Denmark's *Order of the Elephant* and Brittany's *Order of the Stoat and the Ear*.

At length the ceremony begins. We leaders are at the front and the students are at the back. The students parade across the stage carrying flags, and there is a significant amout of folkloric activity. This involves formal dancing, and girls wearing remarkable hats resembling those tall thin desserts with a vertical axis of symmetry which, in 1960 at least, were forever dusted with dessicated coconut and sat on a wafer base. At one point hundreds of children fill the stage and each performs on the traditional Kazakh two-string lute.

There was also some dramatic drumming. Kazakh percussionists dress in shiny olive green clothes, big boots, and hats with giant ear flaps. Let us be honest, they are camp. Presumably the truth is that these flaps cover great wads of cotton wool which are taped to their ears in order to help them survive the sonic firestorm of their performance. The programme gently morphs into a traditional Kazakh heavy-metal set by *Ulytau*. There is an

associated video involving nomadic horsepersons, wolves and eagles. It all makes perfect sense, the Kazakh folk-myth is of Hell's Angels before the invention of the motor-cycle. I blame the Germans for introducing break-dancing and strippers to the IMO opening ceremony. The Kazakhs have simply carried the idea a stage further. I hesitate to suggest what the Dutch might do next year, in case they take it seriously.

At length the Minister for Education and Science makes a splendid speech of welcome, and the ceremony comes to a close.

We adjourn to our hotel about lunchtime, and there is complete chaos. Hundreds of people all want to check-in at the same time, and bless 'em, the Kazakhs have retained Soviet era stuff about filling in cards, handing in passports and so on. The hotel have laid on no extra staff to assist with this process, nor has anyone in the IMO organization given any thought as to how to make this business easier. The rooms are filled with extra beds, because the students had been jammed into the same hotel the previous night. There are no extra staff, so the hotel is having difficulty reconfiguring to normal patterns of bedroom use. Also many bedrooms have no desks, because they have been blagged for the jury room.

In such circumstances, I normally head for the bar, and wait for the foyer to clear. However, after the 2:30am start, I cannot face a drink. I scout around and find upstairs a nice big sofa outside the hotel manager's swish office. I reason that this person is partly responsible for the turmoil below, and that he or she would probably be annoyed if a slob of a guest set up camp outside their office. I remove my shoes and enjoy a prolonged kip.

After five hours I wander downstairs to find the queue has finally disappeared. Apparently one of the guides had eventually taken charge of the situation. By handing out registration cards, and getting people to fill them in while in the queue, she had dramatically sped up the process. She deserves a gold medal. I check-in, and discover (at 18:30) that my room is yet to be made up. I don't care. The staff are onto it quickly enough, and at length I have an extremely comfortable room on the fourth floor, not far from Joseph Myers. The TV carries lots of channels, including BBC World, and Channel 3 which has a strong focus on the presidential calendar.

July 7 This is the day of the first exam. The jury sits awaiting the first message from the students. The students are allowed to send questions of clarification to the jury during the first 30 minutes of the exam. IMO internet Tsar Matjaž Željko is trying to look serene, but the Kazakh 'just in time' philosophy of communications management is not completely attuned to his

more cautious approach. Matjaž is clearly doubtful that communications between the sites will work smoothly. The first student message is from an Australian student. It is a complaint about the conditions under which they are living. After that there are some sensible mathematical questions.

After the question phase is over, text messages start to filter in from the deputies. At this stage, it would normally not be appropriate for informal communications to take place between the two sites, but clearly some of the deputies and observers C are very concerned. They are not at the same site as the students, and there is as yet no means of the deputies getting to the students. This was bad enough for deputies, but for observers C this was outrageous. The designated purpose of observers C is to stay with the students. This is particularly important for countries such as the UK where parents have the expectation that their offspring will be protected and cherished by a designated person from their own country. Our observer C was Ceri Fiddes. She had travelled at great expense a quarter of the way round the world in order to look after our students, and now she was denied access.

I took comfort in the fact that I knew that Ceri had access to funds, and if necessary could simply pull the team out and take them home. Since I was not there, I cannot incorporate events at the student site into this diary, but I hope that Ceri and James will make a public report. I will recount one horror story. Ceri opened a public area first-aid box at the students' site, and found it to be full of homeopathic quack remedies (I deliberately use non-pejorative terminology). For those of us preferring science-based medicine, this is a little disturbing. Parents have no fear, for Ceri also had access to a UKMT first-aid kit containing strange things such as antiseptic ointments, bandages, plasters and so on.

In the evening Joseph and I get access to the UK solution scripts, and go to work. I take the geometry problem 2, and the scripts are simply dreadful. Joseph reports, by way of contrast, that they all seem to have solved problem 1. Sergei seems to have done problem 3, and a couple of students each have some useful scraps which should be worth a mark. I wonder if other teams experienced the same dismal performance at problem 2, and asking around, it seems that almost every team has done better than us. Our old geometric weakness has resurfaced. Shudder.

July 8 The second exam begins, and there are a decent collection of questions from the students. The set-square issue comes up again, left over from 2009. After the exam starts, it becomes completely legitimate for deputies

and observers at or near the student site to communicate with us, and text messages pour in.

The UK has won the Mathematical Ashes again. This is a private competition held just before the IMO between the UK and Australia. The urn can stay in the northern hemisphere for another year. We find out that it is now possible for deputies and observers to get access to the student site, but that there are security guards trying to discourage this process. I rest easier, since it seems that now we probably won't need to make an early departure.

I reflect and observe that the excessive zeal of the local organizers in trying to isolate students from the world actually generated a security hazard. Unofficial communication between the two sites during the exam period is forbidden, but inevitably people who are *in loco parentis* will put the safety of the students above all else, and if they are not allowed access to the students, this is sure to cause massive problems. Imagine you went on an organized holiday, and you had arranged for your family to stay to together in adjacent rooms, and paid extra to ensure that this happened. How would you react when it turned out that parents and children had accommodation in different places, and that there were security guards trying to limit access to the children by their parents?

In the afternoon we hear that an emergency jury meeting is being called in the evening, but no agenda is supplied. Rumours of an irregularity allegation circulate. My heart sinks.

For several days I have been waging a campaign to get hold of a large number of boxes posted to Astana from UKMT. These will contain all sorts of presents for IMO people, including the much sought after BMO solutions booklet by James Cranch. There should also be a plentiful supply of David Monk's master-work "New Problems in Euclidean Geometry". David has been supporting the UK IMO effort since 1968, and he is the author of more IMO problems than anyone else. Now he has produced this lovely book, full of really interesting problems. The boxes will no doubt also contain whatever trinkets are lying around the UKMT office. I have asked repeatedly, and been told every time that these boxes have not arrived.

Prompted by the ever-helpful Mary Wimbury in the UKMT office, we get the courier company to confirm that the materials were received, and supply the details as to where and when the delivery took place. Then I present this information to the friendly Kazakh IMO leader Almaz Kungozhin. He goes into overdrive, with the help of IMO organizer Evgeniya Kalchevskaya they track down the consignment. Soon a van appears at the hotel entrance, and

a dozen boxes are taken to my room. When I open them, I discover that the booklets and books are all present, and that UKMT had tidied its office by clearing out pens and key-fobs. I set about distributing the goodies to the rest of the IMO.

In the early evening the deputy leaders turn up after a long journey. They have moved to our hotel to assist with co-ordination. They tell stories which are quite disturbing, but also reassuring. When you have very limited information, you tend to worry that conditions might be simply awful. The reality is rarely quite so bad, and even James Cranch's story about getting into the students' site by pushing his way through security guards sounds quite normal. The team can rely on Ceri now that James is here. Gordon Lessells, the Irish deputy, had hired a van and driver to provide a bus service for other deputies while there was no IMO one provided. Well done Gordon!

I don't propose to give a blow-by-blow account of the jury meeting which concerned the allegations against DPRK. The regulations were followed, and North Korea were disqualified for this year. On reflection, the procedures could have been more transparent, and I hope that this unpleasant experience will lead to the introduction of a more refined process to deal with allegations of impropriety.

What has gone wrong? I have attended every IMO since 2001, and I know of several allegations of irregular conduct, with evidence of varying degrees of credibility. A pattern has become established, and the leaders of many nations clearly believe, rightly or wrongly, that some nations engage in systematic cheating. This suspicion is corrosive, because it leads otherwise good people to behave in a tawdry fashion.

I recall a recent conversation with someone who should know better. He is a leader of a country about which I have never heard any allegations. He told me that he always asks for one mark more than his scripts are worth, as part of the negotiating process. I was horrified, and asked him why he did this. He told me that 'everyone else does it'. Well, of course this is not true. There are plenty of honest leaders who would never ask for an undeserved mark. However, it seems that the atmosphere of suspicion is leading some people astray, and causing some otherwise moral leaders to step off the path of virtue. This is depressing, because it undermines the IMO spirit of which József Pelikán speaks so fondly.

I fear that something will have to be done. Current IMO procedures must change, and opportunities for improper behaviour must be minimized. Only then will the IMO spirit be restored to full health. Three things spring to

my mind, and to the minds of many other people I am sure.

1. The long process of exam paper construction involves the problems being seen by a hundred leaders, and plenty of co-ordinators, members of the problem selection committee and observers A. All this happens over a lengthy period, so that any corrupt people have ample time to transmit problems, and to prepare alternative model answers. Maybe it is not really happening, but in fact sufficiently many leaders believe that this sometimes happens that the atmosphere is soured. We must develop a streamlined method of paper construction which involves only a very few trusted, experienced and expert eyes seeing the paper in advance.

2. There must be sanctions against leaders who exaggerate the ingenuity and accuracy of a student's script. It is a relatively easy thing to give a slightly false account, especially if student has written in a language which is not widely understood. I would favour an ethics committee which would examine the scans of scripts during the period between IMOs. If a student has secured high marks on a script which is worth little, then the leader of that country can be asked to explain how this happened to the jury of a subsequent IMO.

3. Host countries must take their pastoral responsibilities seriously. Deputies, and especially observers C, must have ready access to the students. When this happens, there will be no need for any unsupervised communications at inappropriate times.

July 9–10 These are the co-ordination days. We divide the work between us, and I take all the geometry. I will address the problems in numerical order, though that is not how they were scheduled. Sometimes we called for breaks and had sessions re-scheduled.

I was to lead on Problem 1, though Joseph was also very familiar with the scripts. The co-ordinators conceded three 7s immediately, but wanted to talk about the other papers. In the time available we were able to go through two of the remaining scripts line by line, and collected two more 7s. There remained the script of Nathan 'seven lemmas' Brown. We knew his script to be correct, and he had broken the argument up into many small pieces, and had correctly explained how to put the parts together. The difficulty was that he had written his solution in tiny handwriting, and it was very difficult

to read, especially for co-ordinators who were used to Cyrillic script. I volunteered to write it all out again, with associated notes of explanation, and so we adjourned for a second meeting. When we resumed, the co-ordinators first convinced themselves that my version was a fair representation of the original, and then Joseph Myers took them through it, correcting a couple of gratuitous errors that I had inserted. The co-ordinators finally agreed and we had six 7s.

Problem 2 was the geometry problem on which we had bombed badly. Luke Betts got a single mark for accidentally observing a correct triangle similarity which was relevant to what had become known as the 'Iranian Solution'.

Problem 3 was of more interesting. Two candidates got a mark for proving that the function must be injective. Sergei Patiakin had solved the problem, but lost a mark for giving a dodgy justification of one of the steps.

Problem 4 was the second geometry problem, and so I led. We agreed four 7s straight away. The disputes were that I wanted a 7 for Luke Betts, and the co-ordinators were only offering 3 because they thought that there was a hole in his argument. On the other hand, they wanted to give Andrew Carlotti a 7 but I was only prepared to take 3. I explained to them the weakness in Carlotti's argument, and they looked a little surprised. I asked them about the alleged weakness in Betts's script, and I couldn't understand what they were worried about. I suggested an overnight adjournment so that we could both make detailed preparations, and we met again the next day.

When we met again, I explained that Luke had quoted the result that when two points are inverse with respect to a circle, then that circle is an Apollonius circle with respect to these points. Fortunately, when the marking scheme was being designed, I spoke in the jury and asked if an educated student quoted this result (or an equivalent one about orthogonal circles), whether it would be allowed. I was worried that a student might do this since I had explained it in a geometry session a couple of months before. The problem captain had accepted that this was allowed. In our second meeting, the co-ordinators rapidly agreed that Luke was right.

Finally we came to Andrew Carlotti's script. He had used an inversion map which, unfortunately, did not exist. If it existed, all would be well, and he had supplied a persuasive argument that it ought to exist. My difficulty was that the map didn't exist. However, since our previous meeting I had realised that you could dodge the problem by composing with a homothety with scale factor -1 about the attempted centre of inversion, and that would

fix the problem. I now thought the script was worth about 5 or 6. The co-ordinators had found the fix as well, but insisted that this manœuvre was simply a 'negative inversion', and that other scripts had got full marks for this argument, and so we had to accept full marks too. I put up some resistance, but finally we conceded for the sake of the IMO.

James Cranch led on Problem 5. When we walked into co-ordination we were pleased to see Géza from Hungary who is an excellent mathematician and speaks wonderful English. James quickly agreed two 7s for Sergei and Nathan. He then went into grovelling mode in an attempt to get a mark for Luke Betts. In the course of trying to prove the wrong thing, Luke had accidentally invented the Ackermann function. James thought that this was such a wonderful thing to do that it might be worth a mark. Indeed, it turned out to be in the mark scheme, and the mark was banked.

For Problem 6, only Andrew Carlotti had solved the problem. James began by remarking that he had translated the first part of Andrew Carlotti's problem 6 script "into English from Idiot", to which the Kazakh coordinators did not react but which caused Ilya Bogdanov at the next table to burst out laughing. Eventually James and Joseph managed to extract the correct score of 6 marks for Andrew.

We have given UKMT presents to all the co-ordinators, and now an interesting thing happens. Word has got out about David Monk's book, and we receive a stream of supplicants asking for extra copies. Fortunately we have enough copies to keep everyone happy.

July 11 There are various excursions today, but I decide to rest. For me the high point of the day will be a meeting concerning the launch in 2012 of a European Girls' Event. Murray Edwards College Cambridge (formerly New Hall) and UKMT are trying to construct an annual event starting in Easter 2012.

The original plan was to have a Girls' Mathematical Olympiad, just as they do in China. Indeed Ceri Fiddes and Alison Zhu will lead a UK team to the China Girls' Mathematical Olympiad in August 2010, in order to gain experience. The USA IMO leader Zuming Feng has been encouraging us to do this for a couple years, and the Americans now send a couple of teams to this event. In the UK, the USA and the Commonwealth, separate girls' education is seen as quite normal. However, in continental Europe it is not widespread. I have had informal discussions with a few Europeans, and the reaction is mixed. Some people do not like the idea of a separate competition at all, seeing it as insulting. Others are in favour, thinking that it might help

develop girls' interest in mathematics.

Having grown up during the early phase of the feminist movement, I am well used to being denounced for male shortcomings, hegemony and oppression, usually while I am making the tea. I can't face any more of that, and so I am hoping to hide behind some sturdy females. Unfortunately local arrangements mean that Ceri won't be around until the very end of the IMO, and my expected female deputy Vesna has had to withdraw from this IMO because, happily, she plans to bifurcate.

It seems that rebranding the event, and incorporating an education programme might make it more palatable. However, I am not altogether convinced that this is a good idea. Anyway, I presented the proposal in a vague and incoherent way to a group of countries, and there is some support (and some skepticism). There is clearly enough support to proceed, and I look forward to passing on the discussion to women mathematicians.

One interesting feature of the meeting is that Morocco, Nigeria and South Africa are all in attendance, which is stretching the definition of Europe a bit. However, they are all share time-zones with Europe, so this is not such a strange arrangement after all.

After dinner we have the final jury meeting, and the medal boundaries are determined. There are fulsome tributes to the work of József Pelikán, and he gets a standing ovation.

The incoming IMOAB chair Nazar Agakhanov makes a short speech in which he proposes some IMO reform. The decisions must be taken next year, and we have 12 months to reflect on the best way forward. Secretary John Webb explains that there is a problem in 2013, and we currently have no host country for that year. This will obviously be the first item on the agenda of the incoming Advisory Board.

That night, the World Cup Final was on a large screen in the leaders' hotel. Now that Germany was out, Dieter was no longer wearing full fan uniform. As you know, the Dutch tried to kick the Spanish off the park, but in the end the had to start behaving better when they had 11 yellow cards. Then a Dutchman was sent off, and the Spanish finally pressed home. It seems that the people in Spain were quite happy about this.

July 12 Today is the day that we get to see the students again, but the procedure is to go on an excursion in order to meet them. The main event is an equine festival out of town. Remember, a horse isn't just for dinner. It is also a means of transport.

We are so keen to see the students again, that I agree to go on the

excursion. We sit on a bus at the appointed time, and wait for 30 minutes. Then the bus drives us to the back of the hotel and we get out again, because this is where the first stage of the excursion takes place. This is a model of Kazakhstan, with all the most important mausoleums, power stations, palaces and railway stations so that you could experience the treasures of the country without having to visit it. As the universe is to a planetarium, as the ocean is to an aquarium, as the Old Course at St Andrews is to a mini-golf hole, Kazakhstan is to this display.

At last we paid a lingering farewell to this homage to Brouwer's fixed-point theorem, and made our way to the buses. Less than an hour later, we were at the equine sports centre. We sat in the stadium and after a while the students arrived from the other site, after a long and tiring journey. It was delightful to see Ceri and the team again, but for reasons connected with who was supposed to eat where, student delegations and leaders were ordered to sit in different parts of the stadium.

We watched some excellent Kazakh horsepersonship, including some 'riding two horses at once', which takes some doing, and some magnificent 'pick something up from the ground while riding at full pelt' which was also impressive. Then there was an event of dubious political correctness, where a Kazakh girl on horseback was chased down by a male rider who wrestled her from her horse, and pulled her to the ground in a rather robust form of kiss-chase. The fact that the programme insisted that this event was called "come up your girl" was intriguing. By now the music on the tannoy was getting very loud indeed, and I started to sneak out when they started a game of the traditional Kazakh polo. It is quite like the Afghan game where you use a dead goat instead of a ball. In Kazakhstan you use a calf, beheaded so that it rolls better. In fact they sanitized this event and used a bag instead of a calf.

Soon everyone poured out, and the local organizers tried to usher leaders into luxurious yurts (tents) for lunch. I resisted at first, because I was far more interested in talking to the team than having lunch. It was delightful to chat to them again after our long separation, and to hear edited highlights of their adventures.

At length the UK leaders adjourned to a yurt. This had been magnificently prepared by primary school children. We were serenaded by musicians, and we shown the unveiling part of the pre-nuptual ceremony. This was delightful. We ate salads for a while, and then a dish of rice and various sheep parts was presented. I was given the head. We shared food and had a won-

derful time. We thanked the artists for all the trouble that they had taken, and asked them to pass on our thanks to the schoolchildren.

The students would now move their base to Astana city, and stay in a hotel near to where the leaders are based.

July 13 The closing ceremony is held mid-afternoon. As I am now nearly on the IMOAB, I get to go in the van with the grown-ups (József Pelikán, Secretary John Webb, Slovenian power-broker Gregor Dolinar etc).

There is a vast pedestrian concourse in front of the Palace of Independence, and a long red carpet runs out of the Palace doors, down the steps, and all the way across the concourse to the road. There is a guard of honour on each side of the carpet, and musicians are waiting outside the palace.

In a wonderful ceremonial gesture, it turns out that the red carpet is for the teams. Each of them marches along behind a guide carrying their country's name. The students carry flags, and as they get close to the palace, the incessant Kazakh music becomes overwhelming. The jaunty firemen have giant trumpets, in the spirit of a vuvuzela, but three times the length. Stacks of speakers bring the music to the threshold of pain.

I and my new friends sit in the front row. We have no idea if we will be asked to assist in the medal ceremony, so we sit there and hope to be left alone. The teams parade across the stage once more, in a reprise of the opening ceremony. Then Prime Minister Karim Massimov comes in, and presents a prize to the only student who acheived a perfect score, Zipei Nie of China.

The astonishing Lisa Sauermann of Germany scored 36/42, and so earned another gold medal. She now has one silver and three golds, and will soon surpass the German deputy Christian Reiher who retired from competition in 2003 with one bronze and four golds. I think Lisa has a couple more IMO chances, and no doubt she will enjoy climbing above Christian.

Past Kazakh IMO competitors present the bronze medals. Various academic luminaries hand out the silver ones, and the Minister of Education and Science bestows the gold medals. These are accompanied by laptops, thanks to Exxon Mobil, one of the principal sponsors of this IMO. At this point UK gold medallist Sergei Patiakin is holding his Union Jack inverted, a traditional distress signal in the Royal Navy. Sergei is in trouble, and I surmise that his crew have yellow fever or perhaps he has been captured by Barbary pirates. I gesture to him, and he quickly rectifies the situation. False alarm.

József Pelikán made his farewell speech as chair, and made very gracious remarks wishing good luck to the new Advisory Board. We owe József an

enormous debt of gratitude.

After the ceremony we return to the leaders' hotel. The closing banquet is held in a large room round the back of the complex. It is a splendid affair, and adults even get a little wine. There is the presentation of the Golden Microphone. This has been won by the leader of the Netherlands, Johan Konter. As usual the comical overblown ceremony goes very well.

We go to bed early because we have an early flight the next day. The IMO organization has laid on a bus for us at 03:00. They wanted to pick up our students first at 02:00 but Ceri and James refused, and arranged a taxi for 03:30 instead, and told the organization about this. Nonetheless, an attempt was made to drag our team out of bed for a bus, and poor Sergei Patiakin had to mediate because of his Russian language skills. Sergei is a very well-mannered young man, not versed in the art of telling people, very firmly, to go away.

Joseph, James and I go by bus from our hotel, and meet up with the main party at the airport. As expected, the border guard takes a very dim view of my lack of an immigration card stamped in two places until I mutter the magic words 'International Mathematical Olympiad', and he lets me out. Joseph Myers had a similar problem.

The flight to Moscow is fine, and we have only a couple of hours in transit in Domodedovo. We have a light breakfast, and then find that we are very fortunate for our final leg. We have a giant plane with very few passengers. We spread out, and doze.

Arrival is easy enough, and we are met by friendly families.

Conclusion

A great many people worked very hard to create IMO 2010, and I would like to single out the guides for special praise. It seems that they were often acting without full information, and were under pressure to control the activities of the teams. Of course it would have been more sensible if the guides had been better informed, and then they could have taken on their proper role as facilitators and assistants with greater ease. Even so, all reports I had were that the guides were trying their best to be helpful and kind.

The Kazakh organization DARYN put in an immense effort to run this event, a sustained project which lasts many years of course, and considerable resources were expended. The Kazakh state was clearly committed to the

project, and the warm and wise words of two Ministers were very welcome.

Thanks to the students, their families, fellow UKMT volunteeers and employees who have been involved this year. We now have a rich cycle of events, involving more and more students in various national and international competitions.

As I step back from the task of leading the UK IMO team, I would like to make a couple of points. First I must thank my successor James Cranch for agreeing to take on the role, and the various deputies, observers and trainers who have helped with IMO participation in recent years. I would explicitly like to thank Vesna Kadelburg. She wisely decided not to go to Kazakhstan while her health was vulnerable, but continued to support the enterprise in many ways.

UKMT continues to prosper. It is a large and multi-faceted organization, engaging in various aspects of secondary school mathematics enrichment. All of these activities are successful, and moreover the financial structure which underpins UKMT and the British Mathematical Olympiad continues to function well. BMOS welcomes financial support from elsewhere, for example the generous support from Winton Capital Management, but we are not dependent upon it, but we do rely on the continuing goodwill of UKMT and BMOS volunteers.

Mathematics in the UK is enjoying a boom both in terms of secondary school participation, and in terms of recruitment to undergraduate mathematics courses. It would be gross hubris to attribute this marvellous circumstance solely to the activities of UKMT, but surely the complex and sustained mathematics enrichment programme that has been built under the UKMT flag in recent years has played at least a part in the revival of interest in mathematics in this country.

We must stay together, working for the good of the subject, and nurturing each generation of mathematical talent as it passes through. We must build UKMT, both in terms of volunteer effort and financial clout, until it sits as a permanent and powerful force behind mathematics at school level, ready to protect mathematics education of quality against the never ending threat posed by fools and charlatans, and those who would seek to divide us.