

# **ROMANIAN MASTER OF MATHEMATICS 2010**

(NB THIS YEAR THE MATHEMATICS COMPETITION WAS [THE MAIN] PART OF A WHOLE EVENT CALLED THE 'ROMANIAN MASTER OF SCIENCES 2010')

**AT THE 'TUDOR VIANU' NATIONAL HIGH SCHOOL OF COMPUTER SCIENCE,  
BUCHAREST (FEBRUARY 24 – MARCH 1, 2010)**

UK Report, written by Robin Bhattacharyya

## **INTRODUCTION**

I had been to Romania for a mathematics contest once before, in 2005 for the Balkan Mathematical Olympiad in Iași. By coincidence it was later that same year that some of the Romanian delegation at the International Mathematical Olympiad (IMO), held that year in Mexico, first had the idea of an elite competition for some of the stronger teams from the IMO. They were inspired by the end of season 'Tennis Masters Cup' (for just the very best tennis players), hence the name 'Master of Mathematics'.

In 2008 the competition was held for the first time, and the country with the highest team score that year was the United Kingdom. The UK returned to Bucharest in 2009, but on that occasion it was China who had the highest score. Now this year it was the third RMM (Romanian Master of Mathematics) contest, and my first. For the first time there were two exams – the IMO format – with three questions on each.

One Saturday in early February, nine of the people who help out in the UK with such competitions gathered in London to mark all of the students' scripts from Round 2 of the British Mathematical Olympiad. I was particularly keen to be there, because as soon as we had the results, we would be able to select the UK team of six for the trip to Romania. The following team was chosen :

Luke Betts	(18)	Hills Road Sixth Form College, Cambridge
Andrew Carlotti	(15)	Sir Roger Manwood's School, Kent
Richard Freeland	(17)	Winchester College
Andrew Hyer	(17)	Westminster School
Jack Smith	(18)	King's School, Grantham
Aled Walker	(17)	King Edward VI Camp Hill School for Boys, Birmingham

The Leader would be Robin Bhattacharyya of Loughborough Grammar School, and the Deputy Leader would be Dr James Cranch of the University of Leicester.

Luke Betts and Andrew Hyer had also been in the UK team for the RMM in 2009 (as well as one or two other mathematical contests abroad) but the other four students had not been to any international competition before, so it was quite an inexperienced team. I knew Luke quite well as we had both been to the Balkan Mathematical Olympiad in 2009 in Serbia, and I'd known James Cranch for several years, but at that stage I didn't know any of the others too well.

## DAY ONE - GETTING TO BUCHAREST

To fly to Romania we all have to get to London first, on the Wednesday morning. Unfortunately, four days before we have to leave, a train derailed near Market Harborough, on exactly the route that James and I would have to take to London (though thankfully this wasn't going to affect any of our students). Happily, just two hours before I set out, the line is cleared. The train is mercifully quiet, allowing me to catch up on some lost sleep. James has beaten me to St. Pancras station in London, and I quickly find him there. He appears to have even less luggage than I have, even though he is bringing his laptop computer; I am impressed. We take the underground to Heathrow Airport, an hour's journey. James and I reach the check in area over three hours before our flight to Bucharest, but Aled is already there, having been driven down by his father. James hands him his royal blue team polo shirt; James has brought all the shirts with him and this is my first sight of them – I approve of the colour. By 10am we are all there, except for Richard who is finding his way to the terminal from the Heathrow bus station. Thankfully everybody has remembered their passport !

James and I search for a flag, for the students to use at the opening ceremony. Surprisingly it isn't too easy to find one, but eventually, in a shop with 'Britain' in the title, we are successful. Two students have forgotten toothpaste, so more purchases are made. We board our Tarom plane shortly after midday, but then spend a whole hour travelling around the airport and then sitting in a queue of planes, waiting to take off. The students have a whole row to themselves. Unfortunately Europe is covered in cloud, and there is little of interest to see out of the window until we near Bucharest, just under three hours after taking off. There is clearly some snow remaining on the ground, among the fields near the airport. It is just after 6pm in Romania (two hours ahead of the UK) when we land, and we are through the airport rather quickly. I change some money at one of the booths in the baggage reclaim area, even though the guide books warn about the extortionate rate; it's not too bad, and we only lose about 12% from the real exchange rate.

I don't know exactly which countries will be attending the mathematical competition, although I do know who has been invited. I am therefore surprised to meet a contingent from Kazakhstan who will be sharing our bus into town, as Kazakhstan was not on my provisional list. Their Leader, Shamil, having introduced himself as coming from 'the land of Borat', tells me that he is here for a Physics competition, but it's all part of the same overall event as the Mathematics competition. We wait for a short while for the Italians. I have met their Leader, Roberto, before, and he is with the Italian Observer, Federico, and some of their students. However, Ludovico, the Deputy Leader, and the rest of the Italian students will arrive at Bucharest's other airport, as some of the Italians are flying from Milan, and the rest from Rome.

The bus that the organizers have provided takes us past the Romanian version of the Arc de Triomphe and through some busy traffic into the northern end of the centre of the city, which with over two million people is by far the largest host to such a contest that I have attended as Leader or Deputy. It turns out that we are staying in accommodation of the Academy of Economics of Bucharest. James and I will be sharing a room on the second floor of our block, while the students will be in two rooms, one floor below. We ask them to decide who shares with whom, and they show more decisiveness than some teams I've travelled with in the past, and quickly divide into Luke, Richard and Andrew H in one room, and Andrew C, Jack and Aled next door. We head off to dinner, in the nearby canteen block.

I had an early start, leaving home at 5.30am, and tiredness is starting to hit me now. I chat with the Italians (Ludovico and the students from the other airport having arrived now) and also with Shamil. I have been to the Balkan Mathematical Olympiad three times, and on each occasion the charismatic Romanian, Dan Schwarz, has also been there. Now in his home city he is in charge of the problems for the exams. He hands me the provisional list, and other information about the contest, on a computer memory stick. After eating, James and I return to our room to consider the list of questions – a preliminary set of six problems to be used if possible (three for each exam) and then a list of eighteen reserve questions, in case any changes need to be made.

A few weeks ago it was minus twenty degrees in Bucharest, but mercifully it's now warm enough that we need the window of our room open in the evening. There are many questions to look at, and we haven't the time to attempt them all ourselves, so we also read the official solutions. The six questions for the exams are very fine looking problems, natural questions to ask, and I am impressed. However, they are not easy to solve, with even the first steps of the solutions not being obvious. This is a competition for strong teams, and the customary couple of relatively straightforward problems that the IMO has is missing here. Question 4 looks the most approachable to us, and James quickly works out how to solve it. Only one of the six problems is on geometry, not a favourite topic for James or me, and traditionally an area of comparative weakness for the UK. We stop working on the questions at midnight.

## **DAY TWO – LEADERS DECIDE THE QUESTIONS**

We're up at 7.30am on Thursday, but a temporary lack of water makes a shower impossible. Many teams are at breakfast. One of the Chinese contingent is trying to fill a flask with hot water for a drink for later, but there are language difficulties; 'apa calda' or something similar seems to be the required phrase, and things work out. Water has now returned in the rooms, and James nips back for a shower. I hear about Brazil's difficulties in arriving, which make my Market Harborough worries seem insignificant. They changed from Lufthansa to Air France at the last minute because of a Lufthansa strike, but then there was an air traffic control strike in France. Much delayed, they arrived in Bucharest at 2am, and half of their number have had their luggage lost somewhere on the way, by airline or airport.

Deputies are to accompany Leaders rather than students, so James and I set off together, with our counterparts, on the twenty or twenty-five minute walk to the Tudor Vianu high school. We pass some quite grand looking buildings, in a part of Bucharest I haven't been to before. The temperature is pleasant for me, but less so for the Brazilians, coming straight from the Sao Paulo summer. We compare notes on, for example, selection procedures, and find out that it is much harder in Brazil to get many students together in the same place at the same time for training, because of the vast size of the country.

One of the sponsors of the competition has provided each of the leaders (as the Leaders and Deputies are almost always together, I shall often refer to them simply as 'leaders') with a smart shirt and tie. There is also an official T shirt of the competition, and a small rucksack. Some very nice pastries have been set out and we tuck in. Dan Schwarz is chairing the mathematical discussions. He mentions that Turkey, Ukraine and Iran have been invited but couldn't make it this year; hopefully next year they will be able to compete. There are twelve teams in the Mathematics contest, with two from the host school, two from the rest of Romania and one from each of eight other countries : Brazil, Bulgaria, China, Italy, Russia, Serbia,

the UK, and the USA. Of the 71 student competitors, only 5 are girls; of these, only one is from outside Romania (she is from Brazil). None of the Jury members (the Jury consisting of the Leaders and Deputies gathered together to make decisions) is female, the first time that I have seen this happen. Boards and posters around the hall of the school show the names of Tudor Vianu students who have represented Romania in international academic competitions in the last twenty years; it is an impressive list, and usually there seems to be at least one student in the IMO team. No school in the UK can rival this.

We begin to talk about the exam questions. Question 2 is rejected because it is known – someone has seen it before (I think it had been used in a past competition in Argentina). A few of the reserve questions are also eliminated because they (or very similar questions) have also been seen somewhere, in a competition or collection of problems used before at some stage. Ilya and Pavel from Russia are very useful in pointing out where questions have appeared before. People seem happy with Questions 1, 4, 5 and 6, but some feel that Question 3 may be too easy to be a Question 3. It is a geometry question, quite an appealing one, but James and I are perhaps not the best judges of the difficulty of such questions.

There is a wi-fi connection in the school, and James is able to access the internet. Neither the UK's male curlers nor its female bobsleigh team have won medals at the Winter Olympics in Vancouver, so it looks as though Amy Williams will be our only medal winner of the games. Cricket's first ever double century in a one day international has been achieved, by Sachin Tendulkar; this is exciting news for me, but I don't suppose that anyone else in the room is particularly interested. James is soon busy with e-mail.

Dan Schwarz stops for a break, and the smokers (Dan included) light up. Afterwards proposals are made about the competition questions, to be voted on. Romania is allowed just one vote, between all of its teams, so in all there are nine voters. The first proposal is to move the geometry question, currently Question 3, into the now empty Question 2 slot, because some feel that it is too easy to be the last question on one of the exams. There are three votes for this, not enough to make the change. So a replacement question must be found, from the reserve questions, to become the new Question 2. The US suggests an appealing combinatorial number theory question from the list, Russia puts forward a very attractive looking inequality, and James and I suggest an unusual looking question about functions. The function question makes it, by five votes to four in the run-off. So the exam questions are now all decided, and Dan Schwarz declares that it is a nice, but also a difficult, paper. I think he is right on both counts.

The wording of the questions now has to be decided, to be as clear and correct as possible, but also to enable straightforward translation into other languages – English is the language of all business at the competition, but the students will sit the exams in their own languages. Dan invites the US and UK to discuss the wording, and the US Leader Po-Shen and Deputy Yi join James and me to go over every word, comma and mathematical symbol. All four of us contribute, and there are many, if minor, changes. Other teams inspect our work from time to time, and Russia points out that there may be some ambiguity in the geometry question; we address this. Russia also notes that a question similar to the new Question 2 appeared in a 2003 competition in Russia, but the Jury then votes to keep the question, with four votes for and three against. By a quarter past one we think that we are basically finished.

There is now discussion of the new wording by the whole Jury. Following suggestions from various teams we have prepared two different versions of Question 1 – they both look fine to me, and one of them

is selected by the Jury. A few other slight alterations are made, including one to make the position of a question mark appear more natural. With everything decided, we all walk back to the accommodation centre to have lunch.

I am interested to discover that the US team is from all over that country – Seattle, San Francisco, Boston, Washington DC and Detroit. Po-Shen spent a year in the UK and actually attended the same lectures in Functional Analysis that James did, though they don't remember each other. Yi has news that Thomas Barnet-Lamb, a former UK IMO team member, and friend of James's and mine, is now married (and living in the US). After lunch James and I buy bottled water for the team, and then set off through the rain back to the school. I find that the Brazilian leader, Carlos, is unsurprisingly a football fan, and Palmeiras is his team. Po-Shen and Yi tell us about the Chinese names that appear in the US mathematics teams quite often – generally these students are brought up in the US, but while young have often been taught by their parents considerably beyond the syllabus of their elementary schools.

When we arrive at the hall of the school, it is filling up ready for the Opening Ceremony. Our students are already there, having been taken to the school by their four guides, students at the high school. The students are in good spirits, and all in their official team polo shirts. There is a television crew filming everything. The leaders are shown a computer room, where students of the school show us their project work. Then the Opening Ceremony begins. In the schedule, two hours are given for the ceremony, but it only actually takes forty minutes. After speeches by the Principal of the school, and a representative of the Minister of Education, we have words from a Physics professor, from Radu Gologan, who is very involved in organizing the Mathematics contest, and from another Mathematics professor who talks about how later in life students will experience a different type of mathematics, which is not just about problems to be solved in a few hours – indeed not all problems that mathematicians attempt will be solved at all.

The Physics teams, of three students each, parade on stage. We have Kazakhstan, Moldova, Bulgaria and teams from several high schools in Romania. Then it is the turn of the Mathematics teams. The Leaders have to say a few words, and we find different ways to say that we are happy to be here, and wish good luck to all of the participants. The UK team is a sight on stage, in their blue polo shirts and with a large Union flag. The biggest cheers are unsurprisingly for the home teams from the Tudor Vianu school itself.

After the ceremony it is time for the translations of the exam papers to be made; teams have brought laptop computers so that they can type up their translations. James and I read the proposed mark schemes instead of translating. There is a query from Russia about the clarity of Question 5 as it stands, and there is a suggestion from China to interchange Questions 3 and 5, to level out the difficulty of the two exams, but there is not enough support for this change to be made. With our work done for the day we head off back to the accommodation block, and have dinner.

Ilya, the Russian Leader, mentions that he is a member of the 'Seventh Man Club' since he was the first reserve for the IMO team of six when he was a student; James and I understand Ilya's position exactly. We hear about School 239 in St. Petersburg, which seems to produce more IMO members than any other school in Russia, and usually at least one each year; however, two members of this Russian team for the RMM are from near Orenburg, and from Tolyatti, so not all of the Russians are from the big cities. The Bulgarian deputy here, Ivan, was the IMO Leader for most of the 1980s and was one of the founders of

the Balkan Mathematical Olympiad (and also later the Junior Balkan Mathematical Olympiad). There is plenty of wine, and James and I end up in a room with Romanians and Italians, sharing stories and toasts.

### **DAY THREE – THE FIRST EXAM**

After a refreshing breakfast of tomato, cucumber, ham and sausage, with fruit tea, we make our way on Friday morning to the high school. The team are getting ready for the first exam paper; they are all fine. Andrew C is doing the Rubik's cube that he often takes around with him. The students split up to enter their different exam rooms, and James and I get ready to answer any written queries they might have about the questions in the exam. There is one question from Andrew C about the definition of the functions in Question 2 (to which I reply 'Yes') and there are several queries from other teams. After the half hour period allowed for questions at the start of the exam has expired, the Jury moves on to the mark schemes. Provisional versions have been supplied, and we argue about the relative weightings of the different parts of each question. After the discussion, and some close voting, most of the original mark scheme for the first exam remains. We move on to the mark scheme for the second exam. After discussion and agreement on Questions 4 and 5 there is a (cigarette) break at eleven o'clock. There is plenty of debate about the final question, particularly because it is felt that this question could decide who gets Gold medals and who does not. Four separate possible splits of the marks are voted on, and by a narrow margin it is decided that there should be one mark for a particular lemma, three for a more difficult lemma and then three marks for using the two lemmas to solve the problem. We have had a busy day and a half of work, but now we are to see a bit of the city.

We walk north along Kiseleff Avenue, a grand tree lined road, past some important buildings including political headquarters and some embassies (I think). A policeman tells me to delete one of the photos that I have taken; I'm not sure why. We see the Arcul de Triumf, based on the one in Paris, and built in 1935-6 to remember the fallen of the First World War, and to celebrate the earlier event of Romania's independence. We are taken to a beautiful church, predominantly white on the outside, but with high domes in green, and with much gold on the inside, and pictures of the saints of the Romanian (Orthodox) Church. We walk through Herastrau Park, and see the partly frozen Lake Herastrau, and across it the former government propaganda building, in the Stalinist style. The grey weather has been interrupted for a few hours and we are enjoying the February sunshine. We arrive at a restaurant whose name means 'Seagull'. A fine banquet has been laid out for us, with wine and țuica (Romanian plum brandy). There are many courses, and more food than we can possibly eat, including many, many vegetables, and a dessert of a type of Romanian doughnut. Roberto and Dan remember their own IMO experiences, forty years ago. Most of the teams here have a Leader or Deputy (or both) who attended the IMO as a student. There is also a discussion about darts, cricket and croquet; Dan is fascinated by peculiarly British games and pastimes. We walk back to the school, past the large old houses of north Bucharest, some of them now in a poor state of repair.

The US has drafted a new version of Question 5 to address the concerns of some of the leaders; James and I also contributed slightly, but it is mainly the work of Po-Shen and Yi. Now the Jury has a look at the new wording, which is substantially different from the earlier version, and accepts it. Of course this will now have to be translated into the various languages. Here is the final set of questions :

**Question 1** For a *finite* non-empty set of primes  $P$ , let  $m(P)$  be the largest possible number of consecutive positive integers, each of which is divisible by at least one member of  $P$ .

- (i) Show that  $|P| \leq m(P)$ , with equality if and only if  $\min(P) > |P|$ ;
- (ii) Show that  $m(P) < (|P| + 1)(2^{|P|} - 1)$ .

(The number  $|P|$  is the size of the set  $P$ .)

(By Dan Schwarz, Romania)

**Question 2** For each positive integer  $n$ , find the largest real number  $C_n$  with the following property.

Given any  $n$  real-valued functions  $f_1(x), f_2(x), \dots, f_n(x)$  defined on the closed interval  $0 \leq x \leq 1$ , one can find numbers  $x_1, x_2, \dots, x_n$ , such that  $0 \leq x_i \leq 1$ , satisfying

$$|f_1(x_1) + f_2(x_2) + \dots + f_n(x_n) - x_1 x_2 \dots x_n| \geq C_n.$$

(By Marko Radovanović, Serbia)

**Question 3** Let  $A_1 A_2 A_3 A_4$  be a convex quadrilateral with no pair of parallel sides. For each  $i = 1, 2, 3, 4$ , define  $\omega_i$  to be the circle touching the quadrilateral externally, and which is tangent to the lines  $A_{i-1} A_i$ ,  $A_i A_{i+1}$  and  $A_{i+1} A_{i+2}$  (indices are considered modulo 4, so  $A_0 = A_4$ ,  $A_5 = A_1$  and  $A_6 = A_2$ ). Let  $T_i$  be the point of tangency of  $\omega_i$  with the side  $A_i A_{i+1}$ . Prove that the lines  $A_1 A_2$ ,  $A_3 A_4$  and  $T_2 T_4$  are concurrent if and only if the lines  $A_2 A_3$ ,  $A_4 A_1$  and  $T_1 T_3$  are concurrent.

(By Pavel Kozhevnikov, Russia)

**Question 4** Determine whether there exist a polynomial  $f(x_1, x_2)$  in two variables, with integer coefficients, and two points  $A = (a_1, a_2)$  and  $B = (b_1, b_2)$  in the plane, satisfying all the following conditions:

- (i)  $A$  is an integer point (i.e.,  $a_1$  and  $a_2$  are integers);
- (ii)  $|a_1 - b_1| + |a_2 - b_2| = 2010$ ;
- (iii)  $f(n_1, n_2) > f(a_1, a_2)$ , for all integer points  $(n_1, n_2)$  in the plane other than  $A$ ;
- (iv)  $f(x_1, x_2) > f(b_1, b_2)$ , for all points  $(x_1, x_2)$  in the plane other than  $B$ .

(By Massimo Gobbino, Italy)

**Question 5** Let  $n$  be a given positive integer. Say that a set  $K$  of points with integer coordinates in the plane is *connected* if for every pair of points  $R, S \in K$ , there exist a positive integer  $l$  and a sequence  $R = T_0, T_1, \dots, T_l = S$  of points in  $K$ , where each  $T_i$  is distance 1 away from  $T_{i+1}$ . For such a set  $K$ , we define the set of vectors

$$\Delta(K) = \{ \overrightarrow{RS} \mid R, S \in K \}.$$

What is the maximum value of  $|\Delta(K)|$  over all connected sets  $K$  of  $2n + 1$  points with integer coordinates in the plane?

(By Grigory Chelnokov, Russia)

**Question 6** Given a polynomial  $f(x)$  with rational coefficients, of degree  $d \geq 2$ , we define the sequence of sets  $f^0(\mathbb{Q}), f^1(\mathbb{Q}), \dots$  by  $f^0(\mathbb{Q}) = \mathbb{Q}$  and  $f^{n+1}(\mathbb{Q}) = f(f^n(\mathbb{Q}))$  for  $n \geq 0$ . (Given a set  $S$ , we write  $f(S)$  for the set  $\{f(x) \mid x \in S\}$ .)

Let  $f^\omega(\mathbb{Q}) = \bigcap_{n=0}^{\infty} f^n(\mathbb{Q})$  be the set of numbers that are in all of the sets  $f^n(\mathbb{Q})$ . Prove that  $f^\omega(\mathbb{Q})$  is a finite set.

(By Dan Schwarz, Romania)

I should point out that the first exam (today's paper) consisted of Questions 1, 2 and 3, and lasted for four and a half hours. Tomorrow's exam will have Questions 4, 5 and 6, and will also be of four and a half hours in length, just as in the IMO. Each question for each student will be marked out of seven marks, with marks only available for significant progress; complete solutions gain much more credit than partial solutions, and observations that do not lead to a solution gain no credit.

Just before five o'clock we receive our students' scripts, these having already been photocopied so that the Romanian co-ordinators can also work through our students' solutions. James and I head off back to our accommodation, stopping to buy some toilet rolls, as the students are running out. We meet the students and ask them how the exam has gone. Luke has solved Question 1, and has some progress on Question 2, but there is not too much else of note, other than on the first part of Question 1, which encouragingly everyone seems to have dealt with successfully. This overall situation is not too surprising – the questions are difficult, and we are not geometry experts. The team are not disheartened, but anyway James encourages them, telling them that tomorrow is a new day with a new exam. James and I buy some water and chocolate for the students. The chocolate is certainly well received.

We look at the students' work through the evening, stopping only for dinner, and the occasional distraction from the Winter Olympics on television, for example Bjoerndalen's eleventh Olympic biathlon medal. There seem to be two channels on Romanian television that show only folk music and dancing, but we are more interested in Eurosport. We find that Luke's Question 1 is very clear, and very correct. He has good ideas on Question 2, but not that much has actually been proved, rather than merely conjectured. Andrew C has some nontrivial progress on Question 2, and Richard also has some on Question 3. Other results have been produced by the students, but if they don't help to solve the problems they will not lead to marks. We retire at a relatively sensible hour.

## **DAY FOUR – THE SECOND EXAM, AND FIRST CO-ORDINATION**

I am getting used to the mile or so between the accommodation centre (called Moxa) and the high school. We pass many concrete blocks near Moxa, but closer to the high school it is a pleasant walk, partly past or through parks; we are fortunate to have on our route some of the finest museums in Bucharest, including the Natural History Museum, the Geological Museum, the Museum of the Romanian Peasant and the George Enescu Museum of Music, all very interesting buildings, in different styles.

We arrive at the school early enough on Saturday to answer any queries for the second exam. There are many of these, despite all our efforts with the wording of the questions. Aled asks about Questions 4 and 5, and Andrew C about Question 6. Many students from other countries ask about vectors, and whether parallel vectors of the same length should be considered to be 'equivalent' or 'equal' (even though one interpretation would make the question absolutely trivial). After the first half hour, it is time for co-ordination. James and I have a final look at our students' work before we meet the Romanian co-ordinators to finalize the marks for the first exam.

The marks are pretty much as we expect them to be. Richard has not been careful enough to say exactly what the Chinese Remainder Theorem states in Question 1, so loses a mark from what we expected, but on the same question Jack is rewarded by an extra mark for an observation of his. On Question 3 Richard has a diagram, and a correct argument that an unlikely looking triangle is isosceles; this could be worth three marks, but he hasn't justified exactly why his diagram is correct. I am confident that he could immediately explain the reasons, but they are certainly not on the page, and only one mark is given. He should be pleased with his effort anyway. The co-ordinators do not have photocopies of all the rough work, so we have to show them a couple of things from the rough, during co-ordination; it can be hard to know what will end up being important observations while attempting problems in rough, but I think that in the future our students should be careful to put anything that they think could be useful in neat.

After lunch we speak to the students about how they have done today. It is a more promising picture, with four of the team having solutions or near solutions to Question 4. There may be some useful work on Questions 5 and 6, but it is too early to say. James and I return to the high school to collect the scripts for today's exam, at four o'clock. We walk with the Brazilians; two of them, including Carlos (the Leader) still have no idea where their luggage is. At the Tudor Vianu school they use the computers to sort out their flights home. I reflect on how much easier our journey has been than that of most of the teams. Even the Bulgarians had a seven hour bus journey from Sofia. The Serbians have travelled by train, a sixteen hour journey, longer than it might have been because of flooding in Romania. The US had also been worried about the Lufthansa strike, as they were travelling via Germany; indeed some of their team had a very long journey, from the Pacific coast of the US to Washington DC, and then on to Germany, and eventually Bucharest.

Back in our room we read our students' efforts. Jack and Luke have very clearly solved Question 4, and written their solutions up very well. Not everybody's work is as easy to follow. Andrew C's solution is vague in a couple of places, but is essentially correct. Andrew H has not read the question correctly, and has missed the importance of integers (and rational numbers) in the question. His notation is also confusing. However, he has the outline of a correct solution. Jack has the wrong bound for Question 5, but the rest of team should pick up a couple of marks each for a first step towards a solution to that question. Richard's argument for the remaining steps of the solution unfortunately contains a flaw, and it doesn't look as though it can be fixed. Andrew C's attempt at the last question depends on a lemma that he has no proof for; James and I don't think this is a standard fact, and are unable to furnish a proof ourselves. We are rather tired, and stop at about half past eleven.

## DAY FIVE – FINAL RESULTS

With no exam on the Sunday, co-ordination can begin earlier. Our first fifteen minute slot is at a quarter to ten, and is for Question 6. The co-ordinators have looked at Andrew C's script and decided that it depends on such a strong result that it is not worth any marks. There is nothing deemed to be of value from any of the other students on this question, so we are finished quickly. Next is Question 4. The co-ordinators immediately agree on perfect scores of 7 for Jack and Luke. We discuss the work of the two Andrews. Carlotti has not proved an explicit value for a constant, though it is fairly clear that such a constant exists; there are also concerns about how he has treated integer points in the new picture, after he transforms the problem. We explain exactly how to fix any tiny gaps of explanation. A mark of 5 is agreed. Hyer has used the square root of 2 in his work, even though it is really a question about rational numbers and integers, and his notation is mysterious, with a key function  $k$  presumably equal to a quantity  $q^2$  from the previous page. We have read the script very carefully and explain how it can be turned into a completely correct solution. The co-ordinators are not that impressed with the write-up, and he gets 3 out of 7 for the question. Even though these two students have lost some marks, Question 4 has been good for us. James and I have a final read through of our scripts for Question 5 before co-ordination for this question, which is supposed to be at 11.45am. However a queue has developed because some students from other countries have unusual solutions, and it is taking a lot longer than fifteen minutes for their leaders to explain their methods to the co-ordinators. It is clear that we will be late starting.

Italy takes a long time, and Russia is not quick either. With Brazil, the UK and Serbia still to start co-ordinating this question, and the US wanting to clear up some details, local organizer Sever decides to break for a 'speed lunch'. He wastes no time as he drives the UK and Serbian leaders to the Trattoria Roma where we are to eat. On the way we pass through the heart of the city, past the Atheneum, the former Royal Palace, the former Communist Party Headquarters, some very nice churches and some imposing buildings, not all of them finished. It is an excellent lunch, with wine. An hour and a half after leaving the school, we are back. Our co-ordination is pretty quick. The co-ordinators say that Richard's flawed method cannot be fixed because it is a local 'solution' and only global methods work. We will get 2 marks for each of the students (except Jack) for the easier part of the question, but our efforts on the harder part do not help towards a solution, and are therefore unrewarded. We now have all of our marks, but we don't yet know about any medals. By 4pm all of the co-ordination is finished, and with students arriving for the Closing Ceremony, there is not too long for the final Jury meeting, to formally agree the scores, and decide on the medals. The organizers have suggested that the cut-off scores for Gold, Silver and Bronze are 31, 21 and 11 respectively. The Jury thinks that this is fair. This means that Luke has a high Bronze medal, and Andrew C and Jack have also gained Bronze medals. Our scores are as follows :

	Q1	Q2	Q3	Q4	Q5	Q6	TOTAL	
Luke Betts	7	2	0	7	2	0	18	Bronze Medal
Andrew Carlotti	3	1	0	5	2	0	11	Bronze Medal
Richard Freeland	2	0	1	0	2	0	5	
Andrew Hyer	3	0	0	3	2	0	8	
Jack Smith	4	0	0	7	0	0	11	Bronze Medal
Aled Walker	3	0	0	0	2	0	5	

We are certainly pleased with our three Bronze medals. James gives the students the good news.

One Chinese student, Zipei Nie, has a perfect score of 42/42, which impresses me a great deal. This is a tough competition, and the very young Romanian Ömer Cerrahoğlu, who remarkably won a Gold at the 2009 IMO (and could get another five Golds over the next few years), has just a Bronze here, one mark behind Luke (who only just missed a Gold himself at the 2009 IMO).

A special prize is awarded to a Romanian student who has proved a better bound than that given in Question 1. There is not time to read the full solution now, but I find it later on the internet – the student has shown that  $m(P) < 2^{|P|}$ . The team competition counts only the best three scores in each team. So for example the UK score is  $18 + 11 + 11 = 40$ . There are prizes for the top three teams, and perhaps not surprisingly they are Russia, China and USA. The team results are given below :

Russia	101
China	98
USA	85
Serbia	67
Bulgaria	64
Italy	63
Tudor Vianu	54
Romania	51
Romania B	46
United Kingdom	40
Brazil	39
Tudor Vianu B	19

In fact while Russia has sent some of its best students to this competition, it has sent others to a competition in China; it looks like a strong year for the Russian squad. Also, I hear that ‘China’ at this competition is actually a team from the Shanghai area; no doubt (the whole of) China will have another formidable team at the IMO this summer. I don’t know how the other countries have selected their teams for the RMM (and I hear of one complication for the US), but presumably with this being a tough competition most have brought all of their very best students.

The average score of each candidate on each question, and the number of perfect scores on each question, are given below.

	Q1	Q2	Q3	Q4	Q5	Q6
Mean Score	3.62	1.69	3.35	2.77	2.27	1.13
Number of Perfect Scores	10	14	30	26	8	6

The marks schemes for Questions 1 and 5 made it relatively easy to gain some marks, but the number of perfect scores show what difficult questions these were. No-one from the winning Russian team was able to solve Question 1, so Luke did very well here. On the other hand, all five of the Russians solved

Question 3 (with five different methods); the US team found Question 1 much easier, but Question 3 much harder. Clearly, different parts of the world have strengths in different areas of Olympiad mathematics.

We have the fifth highest total on Question 4, and only the US has more than two marks more than we have on Question 1. However, we suffered on the geometry question which was neither easier enough for us to solve, nor hard enough for other teams not to solve. We finish in 10<sup>th</sup> place, out of the 12 teams.

The main business of the Closing Ceremony is the awarding of the medals. The winning team in the Physics competition is the home team, Tudor Vianu school. Radu presents the medals for the Mathematics contest. Luke is the first of our team up on stage, and then later Andrew C and Jack go up together. Medals have been awarded in roughly the ratio of 1:2:3 for Golds, Silvers and Bronzes, with between a half and two thirds of all competitors winning a medal of some colour.

There is a team trophy that the UK provided for the RMM, after winning the first team competition in 2008. The Chinese Leader hands it over on stage to the Russian Leader, and then Ilya holds it high above his head for some time, looking proud of his team. Dan Schwarz gives a speech that is both highly amusing and also deep. He loves mathematics, and in particular combinatorics. After mentioning Gauss's quote about mathematics being the queen of the sciences (and adding a similar quote about physics) Dan speaks about how mathematics is not part of the 'tragic nature of the human condition' but instead it is for man 'an abstract construction that serves his spiritual and aesthetic needs'.

With the ceremony over, the teams have a chance to wind down. The US team goes into the outdoor sports area and plays table tennis on one of the all weather tables there. The UK also heads in this direction, but for a team photo with our four guides (two boys and two girls from the Tudor Vianu school). During some of the time that James and I had been working on the questions, and the co-ordination, the guides have shown the team some of Bucharest, including the massive Palace of Parliament, and Revolution Square. Our students found the Village Museum particularly interesting, seeing traditional houses with thatched roofs, and workshops or other buildings containing contraptions, for example for washing or drying clothes, most of which seemed to use giant hammers of various sorts.

James and I chat with the students for a while about their experiences, back at the accommodation centre, but soon it is time for the Farewell Dinner. As with the other meals, the leaders are in one room, and the students in the next room. On our table there are huge piles of meat, of various kinds, and it is delicious. I'm not sure that vegetarian Luke will have been looking at the meat with as much joy - in the Balkans it isn't always easy to be vegetarian. I chat with the urbane Italians and learn about their Olympiad organization's involvement with two television shows, the Italian versions of 'Deal or No Deal' and 'Beauty and the Geek'. Both shows have contacted the Olympiad people about possible appearances. I'm not sure if anyone has tried to get on the latter show, but a former Italian IMO team member did appear as an expert on the former, to discuss the odds of success when people pick various boxes (the IMO deputy having refused on the grounds that it is entirely random). Meanwhile James is talking to some of the teachers of the Tudor Vianu school, who have been invited to the dinner.

Just before going to bed, on my last night here, I see the end of the Winter Olympics on television, with the men's ice hockey final. Canada wins in overtime, so it is yet more success for the home fans there.

## **DAY SIX – RETURNING HOME**

Our team is up at 6.40am, so that we are on time for our return flight. Sever has arranged for two taxis to take us to the airport, and he comes himself to make sure that everything is OK. James and I are grateful for his efforts throughout the competition on the organizational front. There have been no problems for us. We say goodbye, and it takes us about half an hour to reach Otopeni airport. We spend the last of our lei on some sandwiches and drinks. James is disappointed that there are no bottles of țuica on sale in the airport. The flight is on time; once again the students occupy the whole of one row. After three and a quarter hours, James's longest flight ever, we land at Heathrow. The screens on the plane say 'Welcome to Romania'. As I am walking inside the airport building, shortly after leaving the plane, Andrew Hyer runs past in the opposite direction; he goes back on to the plane to collect his house keys. After some final photos, Richard and Andrew C are collected by parents, and Andrew H leaves to get his bus home. The rest of us head for the underground into the centre of London, and then make our separate journeys home.

With the competition over, I shall miss the camaraderie among the leaders that we had during the competition. It is certainly a fascinating experience to meet people from such varied countries. As always it is also a pleasure to see people taking such enjoyment from mathematics. I find that the Olympiad communities around the world are not so isolated – for example I am told that our own British Mathematical Olympiad website was a source of very useful information for at least one of the other teams when planning their trip to the RMM, some weeks before the contest (so thank you Joseph!).

I would like to thank our team for being fine ambassadors for their country (and for the many card and word games that they have now spread to other countries!). They had a positive attitude throughout, even Andrew Hyer despite losing most of his battles with the unfamiliar shower (every day there was an update of the form 'Andrew 0, shower 4'!). James was excellent company, and of great help in the marking; also his computer skills were invaluable. Of course there are many people who have helped in the team's mathematical development over the years, and I thank all of them too, parents, teachers, mentors and trainers. I am grateful to my colleagues in the Mathematics Department at Loughborough Grammar School for covering my lessons for four days while I was away, without which I could not have gone.

I hope that the RMM has great success in the future; it is a tough competition designed for strong students, and there is a place for such an event. Of course my thanks go to all of the organizers, coordinators, guides, etc. My final words should be about the questions themselves. Of course it is to some extent a matter of opinion as to what makes a good or attractive question, but as far as I am concerned, I don't remember seeing a finer collection of beautiful problems. Reading the questions for the first time, I was impressed, and now writing some time after the competition, having considered the questions and their solutions in some detail, I am no less impressed. My heartfelt thanks to the composers of what I think are wonderful problems, and I encourage the reader to attempt some of the questions, if he/she has not done so yet.