

Balkan Mathematical Olympiad XXVIII

Leader's Report by Geoff Smith

May 4–8 2011, Iași , Romania

This year the twenty eighth edition of the Balkan Mathematical Olympiad was held in Iași, Romania. The name of this town is pronounced “Yash”. Iași is a major city in Romanian terms, and is situated close to the border with Moldova. However, the old region of Moldavia spreads across the frontier, and Iași is in it.

The UK selects a team of six students to attend each BaMO, but this year Sherry Jiang was unable to attend because of paperwork issues. The Romanian authorities were a model of helpfulness, and they were not the source of the problem.

The team which travelled was as follows.

GBR1 James Aaronson	St. Paul's School, London
GBR2 Sam Cappleman-Lynes	Shebbear College, Devon
GBR3 Martin Chan	Westminster School, London
GBR5 John (Hyunjik) Kim	Hampton School, London
GBR6 Joshua (Yeuk) Lam	The Leys School, Cambridge

Note that we were designated GBR rather than UNK by the organizers. Future organizers please note that we have a marginal preference for UNK, but this is not a major issue. We adhered to our longstanding policy of not allowing any student to participate in the Balkan competition more than once. This ensures that lots of students get experience in an international mathematics competition. GBR1 and GBR6 are in the current group of eight students contesting the six places on the IMO team in Amsterdam. Thus it is not a complete surprise that the marks in Iași fell as they did. This was our best ever team performance at the Balkan MO. Moreover, GBR1 James Aaronson was placed first in the competition, and that is clearly our best individual performance in this event.

Name	P1	P2	P3	P4	Total	Medal
James Aaronson	10	4	10	10	34	Gold
Sam Cappleman-Lynes	10	0	4	0	14	Bronze
Martin Chan	10	0	5	0	15	Bronze
John Hyunjik Kim	10	0	4	0	14	Bronze
Joshua Lam	10	1	10	1	22	Silver

I am Geoff Smith, University of Bath, and acted as Leader in Iași. The Deputy Leader, which for us is a pastoral post, was Rosaleen “Rosie” Wiltshire of Wootton

Bassett School. It was Rosie's first trip to an overseas maths competition. She did a great job, and seemed to enjoy the experience very much. Indeed, the students were also a happy band, and we had a marvellous time. There seemed to be a significant overlap between the people who organized the Romanian Masters of Mathematics in February, and the people who ran the Balkan Competition in May. They did first-rate work. Certain key figures are internationally known including Sever Moldoveanu and Radu Gologan, but there is a small army of Romanian volunteers who make these competitions possible, and we all owe them a great debt of gratitude.

There is another country, the United Kingdom, which is also blessed with a tremendous volunteer base, and a few dedicated professionals, who work to make our participation in international competitions possible. They also deserve virtual applause.

The Problems

Problem 1 Let $ABCD$ be a cyclic quadrilateral which is not a trapezoid and whose diagonals meet at E . The midpoints of AB and CD are F and G respectively, and l is the line through G parallel to AB . The feet of the perpendiculars from E onto the lines l and CD are H and K , respectively. Prove that the lines EF and HK are perpendicular.

Problem 2 Given real numbers x, y, z such that $x + y + z = 0$, show that

$$\frac{x(x+2)}{2x^2+1} + \frac{y(y+2)}{2y^2+1} + \frac{z(z+2)}{2z^2+1} \geq 0.$$

When does equality hold?

Problem 3 Let S be a finite set of positive integers which has the following property: if x is a member of S , then so are all positive divisors of x . A non-empty subset T of S is good if whenever $x, y \in T$ and $x < y$, the ratio y/x is a power of a prime number. A non-empty subset T of S is bad if whenever $x, y \in T$ and $x < y$, the ratio y/x is not a power of a prime number. We agree that a singleton subset of S is both good and bad. Let k be the largest possible size of a good subset of S . Prove that k is also the smallest number of pairwise-disjoint bad subsets whose union is S .

Problem 4 Let $ABCDEF$ be a convex hexagon of area 1, whose opposite sides are parallel. The lines AB, CD and EF meet in pairs to determine the vertices of a triangle. Similarly, the lines BC, DE and FA meet in pairs to determine the vertices of another triangle. Show that the area of at least one of these two triangles is at least $3/2$.

The time allowed was 4 hours 30 minutes, and each Problem was worth 10 marks. Problems 1 and 3 are good questions. Problem 2 proved very hard under exam con-

ditions. Problem 4 is even more attractive than it appears at first sight, and admits many lovely solutions. Problem 1 is by David Monk of Edinburgh. Problem 2 is from Greece, and the other two problems have Bulgarian provenance.

Leader's Diary

May 3

We gather in Terminal 5 at Heathrow Airport. We have only five students because, unfortunately, there are technical difficulties concerning Sherry Jiang's travel documents.

Rosie Wiltshire, who sensibly lives in the eponymous county, is in charge of discipline and good order. I arrive late, but laden with fetching royal blue UKMT Balkan MO polo shirts. We quickly verify that the students are not carrying geometry sets in their hand luggage. Taking the passports, I attempt to use a machine to generate boarding cards. However, the device quickly senses my limited grasp of its interface, gives up with good grace, and helpfully suggests that I go to a check-in desk. This works well, and we soon find ourselves in the departure lounge.

We are scheduled to arrive late at night, and an airline meal will not necessarily take the edge off the appetite of the students, so we decide to eat. We quickly agree to use an oriental restaurant. A couple of us go for bowls of excellent ramen. GBR1 James Aaronson has serious food issues, exemplified by his order of chicken curry with no curry. As most of us tuck in to our meals, GBR3 Martin Chan quietly points out that he has been to the restaurant before. We ask him if he enjoyed it, and he replied that he and his family all had food poisoning. I took the bait, and asked Martin why he did not point this out when we were choosing a restaurant. He replied that he expected to be outvoted, so did not offer any advice. In fact the food was excellent and wholesome (this time).

At length we boarded the flight, and travelled across the Netherlands, Germany, the Czech Republic, Slovakia, Hungary and finally Romania. We are met at the airport by a hotel bus, and transfer to the RIN 4. There are two RIN hotels near Henry Coanda (Otopeni) Airport, including the juxtaposed RIN 3 and RIN 4. The hotels have correspondingly many stars, so the arrangement serves to humiliate users of RIN 3.

At the check-in, late at night and tired, there is confusion over whether or not the rooms have been paid for in advance. The chappie gets quite excited until I produce the UKMT credit card, and then he soon calms down. Rosie ushers the students to their rooms, and I sneak off to mine.

Thursday May 4

This is contest arrival day, and we have a mid-morning flight from the same airport. At the gate we meet the Italian delegation and a small party from Indonesia led by Anton Wardaja. It turns out that the Italians did not diligently book early, as we had, and several of them have been forced to travel business class on this internal flight from Bucharest to Iași. Alas this will not involve reclining seats, champagne, quail, caviar

and truffles, but rather there is a curtain which gets pulled across, and you pay two and half times more. I am pained by the humiliation, and resolve that never again will we be upstaged by grandstanding Italians, and that UKMT treasure must be squandered to this end. The airplane has propellers.

Iași International Airport has a provincial feel. There is class of primary schoolchildren at the edge of the tarmac, and for a moment I fear that the folkloric dancing is about to begin. Happily it turns out that they are there to see the flying machines.

We walk into the airport terminal, and wait for the luggage to be handed in by a side door. After a few moments we gather our bags, and step outside. There are buses and personal cars to meet us. I wave farewell, and am driven to my hotel. This is the *Ciric*, an airport hotel in fact, so the journey takes about 5 minutes. The hotel is beautifully located next to a lake. There is virtually no noise from the airport since the planes are infrequent, small, and are not jets.

My room is on the first floor (that is the second floor in some cultures). I have a balcony from which you can view the lake, albeit over a large roof. Having unpacked, I pop downstairs in search of leaders, lunch and the shortlist. Leaders and organizers are easy to find. The first leader I meet gives me the traditional greeting, “Are you József Pelikán?”

There is a long narrow room which contains computers, coffee and bananas, and I settle in and chat with colleagues. At length someone suggests lunch, and I realise that the bananas are merely a prelude.

During lunch, many more people arrive, including Fawzi al Thukair from Saudi Arabia, and the Saudi coach Titu Andreescu. There is much shaking of hands and many warm smiles. There has been considerable confusion over the Italian Observer Maria Colombo. Maria was a contestant in IMOs 2005–7 (BGS). There were a few language issues at the airport, and she had been taken, incorrectly, to the students’ site. Now she arrived. She is assigned the use of the second bed in the Italian leader’s room. The Italian leader Massimo Gobbino is a respectable man of a certain age, and such an arrangement violates taboos (even Italian ones apparently). The hotel quickly sorts out a more appropriate arrangement, but Massimo and Maria have to endure serious teasing.

On return to the UK, forensic investigation of our records will reveal that Maria has form. She was originally assigned to be involved in domestic arrangements suitable for a *ménage à trois* with British students Alison Zhu and Dominic Yeo in the Balkan Mathematical Olympiad of 2007.

In the late afternoon we get conference bags and shortlists, and I retire to study the proposed problems. Many of the problems seem harder than last year, but then I get more stupid monotonically.

In the evening there is a banquet in an outbuilding at the lakeside. A flight of swallows is also enjoying its menu, swooping low over the surface to take insects. I am seated opposite Azimbay Kamila and Anton Wardaja, respective leaders of Kazakhstan and Indonesia. We introduce ourselves, and of course I say that I am “Geoff from Britain”. I am asked if I know “Smith”. I am tempted to answer, correctly, “no, but I do know Pelikán”, but there are limits to how much confusion a conversation can bear.

The handful of leaders who drink alcohol are served the local spirit from recycled

plastic water flagons. After a while, I feel very ecological, and go to my room to view the shortlist from a more philosophical point of view.

Friday May 5

I get up early to work on the shortlist some more. The story is that we have only a couple of number theory problems, one of which appeals to quadratic reciprocity in a way which seems both unavoidable and unacceptable. There are some very nice combinatorics problems, but again, not that many. If you screw up your eyes you can pretend that one of them is a number theory problem. Another one looks suspiciously like geometry, but the Selection Committee has obviously been forced to make some slightly *outré* classifications in attempting to foster the illusion that we have a full spread of problems.

We have some very attractive geometry problems of a wide range of difficulty. There are also lots of inequalities. I have neither talent for, nor interest in, inequalities. This lot all look impossible.

First thing after breakfast we get in a bus to go to the opening ceremony. As we wait, one of our hosts casually mentions that there will be politicians present. I reach into my inside pocket to retrieve the tie I keep for such situations. We are sat at the front of a hall, with luminaries on stage, and the students and deputies behind. In fact the deputies get a little mixed up with the leaders. At the IMO, sirens would sound, and special forces would come crashing through the walls to make sure that such a thing could never happen. However, the Balkan Mathematical Olympiad is a more relaxed event. The compere is the chief inspector of schools for the region. She power dresses, has power hair, and hands out chocolate bars recklessly. I can see that she must have a substantial fan base. She gave especially large amounts of chocolate to Indonesia, since that country had made the longest journey.

Leaders are each asked to give a short speech, under a minute in length. Massimo Gobino of Italy is hilarious. He starts by promising to speak for no more than 15 minutes. After the opening ceremony we are even allowed to meet the students, but I make sure that this happens in public surrounded by other leaders, so as to avoid giving the impression that any information about the shortlist is being transferred. This makes me uncomfortable, and I am pleased to slip away.

Back at the jury hotel, the jury meets, and the familiar “beauty contest” form is handed out. I counsel the jury that it would be wise to delay filling in this form as long as possible. This is because we want well-informed leaders to contribute to this survey. For this to happen, people need time both to study the problems, and to discuss the merits of the various proposals. We agree not to hand in the survey until the afternoon. We break up into small groups, but some of us prefer to work alone.

After lunch, I feel that my opinions have at least positive weight, and the survey now makes sense. There is a delay while the data is processed, and finally we get to see, in statistical form, the mind of the jury. There is agreement that there are very few easy problems.

It will eventually turn out that the geometry problems labelled $G1$ and $G2$ are both submissions from the UK. Problem $G1$ is chosen. Its author is the prolific IMO problem

composer David Monk of Edinburgh. We select a Greek Problem 2, an inequality with an official solution which looks like a sequence of magical moves, with the inequality neatly hidden at the end of the rainbow. I wonder how many students will be able to follow the Yellow Brick Road. Some people take the view that the students are smarter than we are, which is certainly true. However, I fear that this will end badly.

For Problem 3 we select a combinatorics problem which has a whiff of number theory. The first half is not that hard, but the second half requires a more cunning idea. This problem will clearly generate lots of part marks.

The combinatorial geometry question which attracts interest for Problem 4 is provoking an arms race. The official solution is very clever, but it does not seem that beautiful. On the other hand, the problem itself is very attractive. Everyone in the room can see that we must be missing a sweet solution. Massimo gets in first by suggesting the use of an affine transformation to generate a couple of right-angles. He then finishes it off with some professional algebra.

We choose the problem, but then I start peddling my solution, which involves stealing Massimo's ideas but using them differently. I propose using an affine transformation to generate a pair of oppositely oriented equilateral triangles, possibly of different sizes. You then move their centroids together in a way which preserves the integrity of the problem, and which I leave the reader to rediscover. Finally you deploy some *origami*. I am pretty pleased with this, but my initial solution had dodgy parts. A dialectic exchange over dinner with the Serbian leader Milos Stojakovic rendered the argument robust. Still, GBR1 James Aaronson will put me to shame, and come up with the "proof from the book".

There is a translation phase. A few of us form the English Language Committee, including the French leader Claude Deschamps. We are under pressure to produce the wording at speed. We decide to adopt an anthropomorphic attitude to hexagons, so that we can say "whose diagonals" rather than "with diagonals". The Saudi leader Fawzi quietly questions this usage, and I agree with him that we are stretching things a bit. However, a maths exam paper is not a piece of fine prose. It should be as economical with language as is consistent with complete clarity.

Saturday May 6

It is the morning of the exam. We have a very hurried breakfast, and scamper into town. We were warned that traffic would be heavy. Such advice could only come from someone more familiar with congestion in Iași, rather than, say, London. We arrived very quickly. The Hotel Astoria is opposite a sex-shop, which sells a wide range of implausible items. The Astoria contains a "London Pub", with the traditional Shamrock logo and the word "Irish" written on top. The corridor which leads to the pub is called the "London Boulevard" which lends a Gallic air to this traditional Irish London Paris pub. It is clear that the word "London" bestows mystery, just as the word "continental" is used in the UK, to denote something other-worldly, exotic and intriguing (such as having a croissant for breakfast).

The Astoria is internally linked to a second hotel, the Traian, where we will have meals. Opposite the Traian is "Cowboys and Angels: English Gentleman's Club". It

is pleasing that my country's reputation has spread so far, but perhaps we had better not explore these issues further.

We quickly move to the exam site, a school along the road. The jury assembles at 9:30 just as the exam starts. The deputies are allowed to join us, and this gives me the chance to catch up with Rosie. She reports that the students are well cared for, and there have been no unpleasant incidents. Apparently we have three excellent guides and they get on very well with the team.

For the first half-hour, students are allowed to ask written questions of clarification. Each leader who is asked a question proposes an answer to the jury. They discuss it, and approve a specific wording for the answer. Quite often a student is simply confused, and we answer "read the question again". A student sends a question which asks if he or she has understood the question correctly, rephrasing it for our benefit. The student has a perfect understanding of the problem. This is a tricky one, since we don't want to give uneven assistance. I suggest "do not read the question again", but the jury settles for a more sensible response.

A British student asked if a prime number was a prime power. The jury accepted most of my proposed answer, but struck out the word "idiot". Another British student, Martin Chan, had a question. Remember him? He was the one who tried to poison us all in Heathrow. Problem 1 concerned a cyclic quadrilateral which was not a trapezoid. His question concerned the difference between American and British usage of the word "trapezoid", and the related word "trapezium". The jury seems reluctant to engage in an etymological dispute, and we send him the relevant definition of a trapezoid.

When the questions finish, we have a discussion with the chief co-ordinator about the marking scheme. Calin is a Romanian topologist with an accent like John Gielgud's. He picked it up listening to BBC Radio. His beautifully modulated vowels put mine to shame, and I adopt an appropriate forelock-tugging frame of mind. The marking scheme is rather good, but with the hint of decadence that often surfaces at the Balkan MO. For example, there is considerable laxity concerning diagram dependency in Problem 1.

We now jump on a bus and go on the leader's outing. This goes to an orthodox monastery on one of the hills which surround Iași. We get a monk (Brian Blessed with laryngitis) who shows us icons, artifacts and giant barrels of dry white wine.

We return to our complex of hotels. Time is getting on, so I go for lunch before checking-in. After an excellent meal, I am asked for my room number. I explain that I have not yet checked in, and this causes consternation. I offer to leave a deposit and return with my room number, and honour is satisfied.

After lunch I have a chance to meet the UK students for a short while, and they tell me what they think that they have achieved. It sounds good, but at this stage I am circumspect. Dan Schwarz, the Romanian-born problems specialist, has invited me for a beer in the London Pub. He is also Canadian, so we need to catch up on the Royal Wedding and similar things.

A rumour enters the pub and speeds its way to our table. The scripts have arrived. I go looking for them, and discover them in an envelope behind the reception desk. I dash upstairs and take them in sequence.

Everyone seems to have a perfect solutions to Problem 1, although John Kim has resorted to a page of trigonometry to demonstrate that two triangles are similar, ig-

noring the one line geometric justification which lesser candidates used. For this he earns the *stylo d'or*, the award for the British student who produced the most impressive convoluted but correct justification of a statement which admits a straightforward proof.

Problem 2 is stony ground, and marks will be scarce there. Both James Aaronson and Josh Lam have full solutions to Problem 3, and everyone has done the first part. Martin tentatively introduces the necessary ideas for solving the second part, but undermines his credibility with asides such as “I can’t prove it”. Only James Aaronson has a full solution to Problem 4, and his argument is lovely.

Now is the time to indulge my passion for social media, and I tweet vague good news about the UK scripts from **@GeoffBath** using the hashtag **#BalkanMO**. I am looking forward to tweeting from the Amsterdam IMO in July, giving a running commentary on IMOAB meetings, especially during confidential and delicate discussions.

Sunday May 7

We will try to complete the competition today, because five nations need to leave early next morning. Co-ordination is moved, and starts at 8:30am. This begins with Problem 4. I had not sat down before the co-ordinators were passing on congratulations for James Aaronson’s wonderful solution.

Next came Problem 2. GBR6 Josh Lam gets a mark for dealing with a special case. Then the fun begins. I show them some of Sam Cappleman-Lynes rough work which comes within a whisker of doing what Josh did. It is true that Sam has not completely nailed it, but I have high hopes. The co-ordinators stroke their chins, and then one of them says that the work falls short of getting a mark. “If only there was a special reason that we should give the mark. For example, is the student a girl?” I start to flare up in outrage, when I suddenly realise that the co-ordinator is having fun by quoting from my IMO 2004 report, when a co-ordinator actually said this and meant it! We all laugh, and I say to them that they must have established policy on this matter, and they should simply implement it. They do, and unfortunately Sam collects a 0.

Then the co-ordinators pull a surprise. I knew that James Aaronson’s script was worth a mark for a good early move. However, he then plunged into a lengthy attempt to slog out the solution by brute force. It seemed unlikely to me that this would be worth anything, but they revealed that in fact the particular path he had chosen could be made to work, he had hacked a significant way through the jungle, and they were offering a total of 4/10 for his effort. They then teased me by suggesting that it would have been helpful if he knew the AM-GM inequality, since then he could have pressed further forward.

This is all excellent news. I scan the partial reports of other countries’ scores on display in the jury room. Each of our students is sure to get a medal, Josh will surely get a silver medal, and James should be a safe gold. There are only four or five students who can possibly beat him.

We have arranged that the UK team and guides will go out for lunch. We meet amongst the pigeons, and I give the students my forecasts of their medals, and in fact my assessments will prove completely accurate. We board a tram, which is quite exciting

for most Brits. After 15 minutes we arrive at a shopping mall and are taken to an Italian restaurant. I return to the school fairly quickly, because the final jury meeting is scheduled for 2:30. Well, I know this is not going to happen, since there is always a brutal co-ordination which holds things up, but I can't help my British programming, and return on time.

It turns out that Bulgaria are involved in a difficult negotiation over Problem 2. We wait and wait, until the room starts to steam up. At four o'clock we get permission from the jury chair Radu Gologan to adjourn for an hour. This enables us to change the air in the room, and to take showers.

When we return, the Bulgarian co-ordination is finished, and Rosie is already in the jury room. She is rather excited, because the rank list of performances has been distributed, and GBR1 James Aaronson is in first place.

It is time to begin. We stand for a moment to remember Jacqui Lewis, the UK observer with students and deputy leader at very many international competitions, including last year's Balkan Mathematical Olympiad. She was popular both with students and staff, and is badly missed.

Then we get down to business. The long wait has not improved the general mood, and various heartfelt grievances are aired. These involve issues of national self-respect, and so it is no wonder that some people are very engaged in the discussion. The mischievous leader of Turkmenistan, Erol Aslan, is sitting on my right, and urges me to get involved but even I am not that stupid. I decline the chalice and Erol gives a knowing smile. Jury chairman Radu Gologan steers us through the politics and back to our core concern, the administration of a successful competition.

At length the jury addresses the medal boundaries. The cut-offs (the thresholds) are determined by the performances of official Balkan participants. This gives rise to all sorts of complications, because this procedure only allows rather loose control of the numbers of medals being won. Truncation issues arise. The mood of the meeting switches to a more co-operative and generous spirit, and the chair negotiates a difficult path which results in decisions which are a fair approximation to justice.

We let our team know about the preciousness of their gongs, but we don't reveal James's first place. Later that evening, everything becomes public at the medal ceremony. Romanian politicians get themselves photographed next to James Aaronson enjoying what is probably his first Warholian time-slot.

The students and guides depart for a farewell disco, and the adults make their way to a restaurant for a final banquet. There is an array of impressive cutlery, but after one course, cups of coffee are brought round. This is quite disconcerting, and we wonder if the budget has run out. However, after the coffee the banquet resumes its expected course. Rosie and I leave at midnight, and I go back to send some Tweets and post a message on Ask nRich. At last the competition is over, so the obligations of complete jury secrecy are now relaxed.

Monday May 8

We catch a lunchtime flight from Iași to Bucharest, and then an afternoon BA flight back to Heathrow. GBR3 Martin Chan is playing Luddite tetris. He has a random number

generator which he hits every 5 seconds to select the shape of the next tetromino, and then he quickly draws it in using a pencil. Apparently it is best played as a competition between two players, presumably using the same sequence of tetrominoes, to see who can create the larger number of full lines.

GBR2 Sam Cappleman-Lynes has a plan to stay alone at Heathrow until past 10pm, and then catch a coach to Exeter, arriving in the middle of the night. We overrule this, and get him home at a sensible time by using a train.

Next Year

At the closing ceremony, the leader of Turkey, Sahin Emrah, invited us all to the next Balkan Mathematical Olympiad to be held in Antalya, Turkey. This is a very big place, population 2m, a holiday town on the Mediterranean coast with many buildings from the Ottoman era, and even a few from Roman times. Happily there are daily flights between Stansted and Antalya using Pegasus Airlines.