UK Leader's report, Balkan MO, 2012

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The UK team in the Balkan Mathematical Olympiad 2012 in Antalya (http://bmo2012.tubitak.gov.tr/) consisted of six boys. They were:

Robin Elliott	Reading School
Gabriel Gendler	Queen Elizabeth's School, Barnet
Daniel Hu	City of London School
Matthew Jasper	St Crispin's School, Wokingham
Matei Mandache	Loughborough Grammar School
Harry Metrebian	Winchester College

The team was led by Dr Geoff Smith of the University of Bath, with deputy Dr Gerry Leversha, formerly of St Paul's School, Barnes. The event began on April 26 and ended on May 2.

The problems were as follows. The language and notation has been slightly modified.

Problem 1 Let A, B and C be points lying on a circle Γ with centre O. Assume that $\angle ABC > 90°$. Let D be the point of intersection of the line AB with the line perpendicular to AC at C. Let l be the line through D which is perpendicular to AO. Let E be the point of intersection of l with the line AC, and let F be the point of intersection of Γ with l that lies between D and E. Prove that the circumcircles of triangles BFE and CFD are tangent at F.

Problem 2 Prove that

$$\sum_{\text{cyc}} (x + y)\sqrt{(z + x)(z + y)} \geq 4(xy + yz + zx),$$

for all positive real numbers x, y and z. *The notation above means that the*

left-hand side is

$$(x+y)\sqrt{(z+x)(z+y)} + (y+z)\sqrt{(x+y)(x+z)} + (z+x)\sqrt{(y+z)(y+x)}.$$

Problem 3 Let n be a positive integer. Let $P_n = \{2^n, 2^{n-1} \cdot 3, 2^{n-2} \cdot 3^2, \ldots, 3^n\}$. For each subset X of P_n, we write S_X for the sum of all elements of X, with the convention that $S_\emptyset = 0$ where $\emptyset$ is the empty set. Suppose that y is a real number with $0 \leq y \leq 3^{n+1} - 2^{n+1}$.
    Prove that there is a subset Y of P_n such that $0 \leq y - S_Y < 2^n$.

Problem 4 Let $\mathbb{N}$ be the set of positive integers. Find all functions $f : \mathbb{N} \longrightarrow \mathbb{N}$ such that the following conditions both hold:

   (i) $f(n!) = f(n)!$ for every positive integer n,

   (ii) $m - n$ divides $f(m) - f(n)$ whenever m and n are positive integers.

   Each problem was worth 10 marks, and the exam was sat over 4 hours 30 minutes. The results of the British students were as follows.

Name	P1	P2	P3	P4	Total	Medal
Robin Elliott	8	10	8	1	27	Bronze Medal
Gabriel Gendler	1	9	10	0	20	Bronze Medal
Daniel Hu	10	0	10	10	30	Silver Medal
Matthew Jasper	4	0	10	1	15	Honourable Mention
Matei Mandache	10	10	9	4	33	Silver Medal
Harry Metrebian	10	4	0	0	14	Honourable Mention

   This was a very creditable performance, especially since the UK has a self-imposed rule that no student can attend the Balkan MO more than once. The current strongest players at maths competitions were therefore left at home (including two IMO gold medallists). This Balkan side was very much a team of the future. Although Robin will be going to the University of Cambridge this year, the other team members all have at least one more year of eligibility for international mathematics competitions, and it seems very likely that several of them will attend IMO 2013 in Colombia.

Thanks to his strong showing in this Balkan MO, Matei Mandache has been added to the small group of students in contention for the six places in the UK team at the 2012 IMO in Mar del Plata in July.

The competition was brilliantly organised by the Scientific and Technological Research Council of Turkey (TÜBİTAK). I thank our Turkish hosts for many kindnesses, and the UK deputy Gerry Leversha, who shows little sign of slowing down in retirement. Of course one must acknowledge the help and support of the staff and volunteers of UKMT, and the families of the young people involved. Finally, I would like to thank the team itself. They behaved in an exemplary manner throughout, and were a credit to themselves, their families and their schools.

On a personal note, it was an exceptional pleasure that my former Ph.D student, Prof Dr Huseyin Aydin, Dean of the Faculty of Science of Erzurum Technical University (http://www.erzurum.edu.tr/etu_/huseyin-aydin.aspx) was able to drop in to the competition to renew our friendship. We will be trying to set up academic links between Bath and Erzurum, a magnificent city on the "silk road" in Eastern Turkey.

Diary

As usual, in a maths competition where almost everything goes well, it is natural to draw attention to occasional imperfections for comic effect. This does not detract from the greater truth, that the competition was a tremendous success, and was enjoyed by all participants.

The diary is just one person's perspective, and the students will have experienced a completely different event.

Thursday April 26

I begin by letting my dongle get me into trouble. While clearing my email on the train, I omit to notice that we are pulling into Reading station until the train actually stops. Since I must change trains here, this is the time for a serious panic. I unplug everything and thrust it into my bag, and scamper for the door making pathetic whimpering noises, scattering pens, change and spare spectacles from various pockets. A train official takes pity on me, and holds the door and therefore the train, while passengers kindly pass me my

possessions. The conductor even waits while I check that I still have my wallet and passport before allowing the train to move off.

The second train journey is a sequence of happy surprises. I assume that I must have lost various items in my dramatic departure from the express train. However, as time goes by, I keep finding that more and more items are in my possession in odd pockets and the inner recesses of the bag.

I arrive at Gatwick airport in good time, at about the same time as Gerry, and we go downstairs from the departure lounge for an all-day breakfast. We then return to meet the students and their families, and I hand out union flags, team shirts and lapel pins.

The journey to Turkey was more arduous than expected. The easyJet flight from Gatwick to Antalya turned out to be monopolised by a golf club. This was not a golf club for the middle-classes at play. Rather, it would be the golf club of choice for the characters from the TV soap-opera *Eastenders*. When they arrived on board, there was a little crooning and some mildly boisterous behaviour. In fact they must have pre-fuelled before take-off, and were merely pretending to be sober. As soon as the plane took-off, the booze orders started flying, and two hours into the flight, they had drunk the plane dry. By this time, the volume of their senseless and ceaseless gibbering had become horribly oppressive. Even placing your fingers in your ears gave little respite.

As we landed, one of the other humans on the flight tried to point out to a golfer how unreasonable it was to hold a party on a scheduled flight. He was admonished for his elitist attitudes, and told to fly first-class in future.

All credit to the easyJet crew. They continued to act in a completely professional manner, despite the unfolding scene. It has been over a week now, and I still have a buzzing in my ears. The males shouted and the females shrieked, in a squalid parody of conversation. As you can imagine, Gerry took it all in his stride, exuding the relaxed bonhomie with which his friends are so familiar.

I rather hoped that a golf suppression unit of the Turkish police would be waiting for them in Antalya, equipped with dogs, nightsticks and knuckledusters. Unfortunately this was not the case. It turns out that Belek, the holiday resort suburb of Antalya, has 21 golf courses. Actually building golf courses to attract them seems foolhardy.

We all have to pay the 10 pound entry fee for Turkey, gathering little visa stickers in our passports. Finally we make it through customs and immigration at about 10:30pm Turkish time. Disturbingly, there is no-one

to meet us. I feign calm. When arriving at IMOs as a solitary leader, there have been occasions when I was not met, and some initiative was required to make contact with the organizers. However, when turning up with students, you do expect to be met.

After a while, I phoned the UKMT executive director in Leeds, largely to mitigate the loneliness. I began to formulate a plan: wait for an hour, and if no-one shows up, then check in to a hotel. At length, a young man called Sal turns up. He is the UK guide. There is much apologizing, but I am pretty grumpy by now, the barbarians on the plane having exhausted my goodwill toward the human race (and other primates). It turns out that while the UK easyJet plane was on time, lots of other countries are arriving late, and this has disrupted the host country's meet and greet schedule.

I am put on a bus with other leaders, and we plunge into the night. We do some more picking up at another terminal, and at last head off down the motorway. We pass a bus in the same livery as our own, so I look inside, and sure enough Gerry and the students are all slumped in their seats, looking the worse for wear. They will not know that they were under observation until they read this report.

When I arrive at the *Bel Conti Resort Hotel* it is close to midnight. I scan the lobby for Neanderthals, but it seems clear. We are told that food will be available until 1:30am. Having a distrusting nature, I dump my bags, and head straight to the restaurant. It is closing up, but I manage to grab a plate of food just before it shuts. Replete, I saunter upstairs to the bar, to discover less cynical but more hungry leaders who are now casting around for nourishment, frustrated that the restaurant is shut. There is a promise that the hotel will open for breakfast at 1am, but in fact it won't, and by then I am tucked up in bed.

Our hosts have provided a lavish collection of presents for participants. This is not only the year of the London Olympics, it is also the year of the rucksack. I was given one in the Gulf Mathematical Olympiad, one more at the European Girls' Mathematical Olympiad, and now a third at the Balkan Mathematical Olympiad. These are excellent and useful gifts, many thanks. However, future organizers please note: go easy on the rucksacks for a few years. The olive oil and scented soap are wonderful.

Friday April 27

I find the first night in a new bed rather difficult, so I don't get to sleep until about 5am. I sleep through my alarm, and wake up at about 8:40am Turkish time. I rush downstairs, have a quick breakfast, then dash back to my room for a shower. I can dress down for the opening ceremony, since I am not required on stage. A short bus journey past countless hotels takes us to the venue.

The speeches at the opening ceremony are pleasantly short. There is a tentative announcement that Turkey might be able to host EGMO, the European Girls' Mathematical Olympiad, in 2014. This is excellent news. We have a magnificent display of folkloric dancing, and connoisseurs of the form settle in for a real treat. Unusually, there is no parade of the nations.

The organizers and leaders return to their own hotel, to engage in problem selection. I look around and am delighted to see so many old friends leading and observing various teams. A Balkan MO jury is a very male environment. There is some manly hugging. We establish a schedule, and agree to meet again after lunch. This gives us a few more precious hours to study the problems.

In the afternoon we set the paper easily enough. For problem 1 we select G1 from Romania. Problems 2 and 4 are Saudi submissions, and Problem 3 was from the UK. The UK submission was made in Bath. It was a joint effort, and was constructed from an issue which arose during Carolyn Ashurst's Ph.D research. I am her supervisor, and as we resolved the difficulty, it became clear that the mathematics involved would make a suitable problem for an international competition. I therefore submitted it to the UK International Problems Committee, and forgot all about it. I was pleasantly surprised to see it again when opening the Balkan MO shortlist for 2012. The combinatorics shortlist was unusually short, and consisted of just this one question! There were lots of submissions for Algebra and Geometry, and a few for Number Theory.

As the only native speaker of English on the jury, I convene an English Language Committee based on people's willingness to sit next to me and drink coffee on the verandah. We construct the official version of the paper. This is subsequently modified and approved by the jury, and an immaculate typeset version is rapidly produced by the Italian leader, Massimo Gobbino.

I am now ahead of the game, and can head to the bar for a complimentary gin and tonic, but other leaders now have to work hard on producing different

language versions. Once those are approved, we hear mark scheme proposals from the four problem captains.

Saturday April 28

Leaders have to pack early in the morning, and we transfer to the students' site, arriving just as the exam begins. It turns out that we have moved to the *Letoonia Hotel*. Is this the Turkish word for Lithuania? This is one of a city of hotels that have been constructed to separate Ivan from his petroroubles, hence the name (or not, depending on what the name means).

We have a jury room very close to the exam hall, and I am delighted to find Gerry again. The organizers have helpfully provided two seats to a table, so he can join me in the jury room. We have some coffee, and await the first student query.

A UK student breaks the ice to ask if 0 is a natural number. After a minor grammatical correction from Gerry, I propose the reply *"Zéro n'est pas positif en Angleterre"*, but wiser heads prevail, and instead we send back the answer "no". There are three pillars of British identity: we drive on the left, the Queen is on the banknotes, and zero is not positive. There are mercifully few questions, so there is time to go back to my room and unpack.

Comparison of my room and that of deputy Gerry Leversha does not improve my humour. I have a fine bedroom, and the bed satisfies my minimum dimensional requirement that it should be wider than it is long. However, Gerry's bedroom is altogether more splendid. It has a rural annexe, a sort of conservatory which spreads out into his grounds, and is filled with soft furnishings. This will not do at all.

As the exam finishes, the story of an apparent scandal breaks. The French leader, Claude Deschamps, is spreading the news. Our Balkan Problem 4 had been used three days previously in the American national competition, the USAMO.

Given the time difference, it seems that the problem had become available on *Mathlinks* late in the evening of April 25th in Europe. On April 26th everyone was on the move, so there was only a limited opportunity for students to view the solution. Investigation revealed that, as far as we could tell, only a very small number of students had read the problem on the internet in advance of the Balkan Mathematical Olympiad, and of those, an even smaller number had read the solution.

The jury chair, Azer Kerimov, has to make a quick decision. The choices

are to proceed with the competition, or to hold an emergency jury meeting and to discuss possible extraordinary action (the discounting of all solutions to Problem 4). I counsel him to proceed without a meeting, as if nothing has happened. I am sure that many other leaders also gave advice. If we had convened a jury meeting at that stage, then there would be almost limitless scope for accusation and vituperation. It would take time to find out how this double submission had happened, and I took the view that people had important work to do (marking and co-ordinating the scripts), and that there would be an ample opportunity for an inquest at the final jury meeting.

The students' scripts arrived in the afternoon, and Gerry and I settled in for close textual analysis. Gerry is a geometry specialist, and it made sense to put him in charge of Problem 1. I would lead the co-ordination on the other three problems. It quickly became clear that we had a rather good set of scripts, given that we had an inexperienced team.

Sunday April 29

This was a day for serious tourism. In the morning we visited Perga (or Perge), where the geometer Apollonius used to hang his hat in about 250 BC. Perga has excellent rubble, and I find the connection with the foundations of conic sections enchanting. An ice-cream seems in order for obvious reasons.

We proceed to a park which contains in impressive waterfall, and finally, in a sun-toasted state, we are dropped in central Antalya for shopping. Gentlemen of a certain age repair to a shaded coffee house and start to work their way through the menu.

Monday April 30

It is co-ordination day, the day when the marking police will establish that we have marked our scripts correctly. Since the UK has submitted problem 3, we must be involved in the co-ordination of that problem for both Turkey and the guest team Turkey B, since a country cannot co-ordinate itself unsupervised. This makes the day very busy indeed.

Our student Gabriel Gendler has managed to solve Problem 2, except that he has used an invalid symmetry argument. All statements which he claims to be true are true, though the co-ordinators are suspicious of one result at first. I supply a valid proof of that statement, and we agree to meet later. In the interim, I manage to come up with a valid argument which is actually a

simplified version of Gendler's false reasoning, obtained by cunningly leaving out the bits that are wrong. This reduces the penalty for the error, and he gets 9/10 on that problem.

On problem 3, the same UK student, Gabriel Gendler is in trouble again. Is it just chance that the same student is involved? Well, no, not really. This time he has used a controversial form of induction, and to the innocent eye there seems to be a small gap in the induction step. In fact there is no gap at all, but the co-ordinators do not realise this. It turns out that the co-ordinators are reading the script as if it were written in English. In fact, it is written in Gendlerese, a modified version of the language in which quantifiers and parenthetic remarks interact to introduce multiple levels of ambiguity. If you read on, Gabriel does make his original intentions perfectly clear, but he does not do that until about page 3. When I show this to the co-ordinators, they immediately understand that the student's solution is completely correct and that he deserves full marks (and that the same cannot be said for his use of English).

The co-ordinations of the other scripts were relatively uneventful. Matei lost a mark for overlooking a triviality on Problem 2, but we were expecting that.

The final jury meeting was convened after dinner. It opened with an unreserved apology from Fawzi Al-Thukair, leader of Saudi Arabia, for the double submission of Problem 4 on number theory. The four problems of the contest had been submitted by, in sequence, Romania, Saudi Arabia, the United Kingdom and Saudi Arabia again.

I had been in contact with the US IMO leader Zuming Feng by email. From what he told me, and from Fawzi's statement, I have constructed the following simplified but, I hope, accurate summary of events.

There were a number of internationally known trainers who had worked on the maths enrichment programme for the Kingdom of Saudi Arabia. The problem in question had been constructed by a member or members of this team some time ago, and in fact the problem had been submitted to IMO 2010 in Kazakhstan, but had not been put on the shortlist. The problem was therefore returned to the submitting nation. Some, but not all, of this team of trainers ceased to work with the KSA in the (northern) autumn of 2011, and the two groups were no longer in regular communication with one another. There was at least one person in each group who felt that they had "ownership" of the problem, and the problem was submitted independently to both the Balkan MO (February 2012) and the USAMO (December 2011),

with neither party being aware of what the other had done.

Without being privy to the (doubtless complex) history of the invention and development of the problem, it is hard to know which of the two (or more) rival claimants to ownership of its intellectual property rights has the better case. Collectively, the people involved caused a foul-up, but clearly there are a multiplicity of views as to who carries the most responsibility.

It is not the case that the problem was somehow recycled from the US-AMO to the Balkan MO. That is completely ridiculous. The accident happened, and it was a 50-50 shot as to whether the Balkan MO or the USAMO would happen first. Had the competitions been separated by one fewer day in time, then it would been most unlikely that any of the students would have viewed the solution on the internet, since they would have been having an early night before the Balkan competition. On the other hand, had the competitions been separated by one more day, then large numbers of leaders at the Balkan MO would have seen the problem on Mathlinks, and the jury would not have selected it for the Balkan contest.

Thus we were very unlucky to experience this accident. However, that is how accidents happen.

Tuesday May 1

Bitter experience has it that if students are left in charge of their passports, only bad things can happen. Therefore we confiscate travel documents whenever possible, and lock them in hotel safes. This morning Gerry decides to test his safe, just to make sure that he will be able to extract the passports in good time for the return journey tomorrow. After battling with the safe for some time and getting nowhere, he eventually resorts to calling hotel reception. There is a little spare time this morning, so this is the ideal moment to explore the resources of the hotel's engineering section.

A sequence of experts arrive, all thoroughly familiar with the problem, but to little effect until the big man arrives. He tries all the usual technical fixes, before putting the safe on the floor and hitting it. This works.

Gerry is to be commended both for testing his defunct safe with 24 hours to spare, and for finding someone with the appropriate sophisticated skills.

Later in the morning, the competition set out for the Çelebi Marina, a harbour you can see easily enough on Google Maps, a little to the West of Antalya. We boarded a very large boat from the rear.

There were parrots to photograph, and loud music, but it was possible to find quieter parts of the boat in which to hide. There was serious drama as we brought the boat close to Antalya city centre. Almost everyone seemed oblivious to the situation, but there was a swimmer in the water next to a buoy, waving frantically in a "come hither" fashion. The boat picked him up. He was wearing a wet-suit and flippers, and he climbed aboard the back of the boat. He was given a mobile phone, and then things happened in short order. I spoke with a crew member, who explained that he was phoning the police because another swimmer was missing. The boat then took him close to the shore, where he was met by a police launch. Our pleasure palace then moved out to sea, and I have no idea if this story had a happy ending.

Further along the coast, we came close to the shore to view a waterfall plunging from a cliff top, straight into the sea.

We returned to the hotel to learn that the medal ceremony would begin in half an hour. I ran to my room, took a shower, and put on a suit and tie.

I sit in the front row because I am supposed to make a speech. When my name is called, the UK students make a deal of noise, for I had not told them that this would happen. I manage to get off stage before the rotten fruit starts to fly.

Then there was much congratulating, medal and certificate collecting, and at last the competition was over.

In the evening there is a farewell banquet, with waiters and a band. Initially the band is a touch loud, and jury chair Azer finds me outside, muttering about "the threshold of pain", "white noise", "the Geneva convention" and so on. He sensibly decides to make the band turn the volume down. After they do that, the musicians even get applause between songs, rather than the resentful stunned silences which had greeted their earlier intermissions.

Actually, the UK and the Saudis had the best experience, because they put the two Kingdoms at the far end of the hall from the band. It was all those Republics who formed the buffer that had to soak up the worst of it.

The deputy leader of France, Igor Kortchemski, engaged in some spirited dancing, and was no doubt preparing to do excellent work to improve Franco-Turkish relations.

Wednesday May 2

We meet in the lobby in time to catch the bus which will transfer us to Terminal 2 of the airport. Gerry reports seeing a guest and a technician

hard at work hitting a safe as he made his way to reception.

The transport supremo informs us that we are being left in the hands of the driver, and we are told that when he comes to the correct Terminal 2, he will stick up two fingers, thereby signalling that it is time for us to depart.

There is one airport, but three terminals in Antalya, in an oblique theological reference (St Paul and St Barnabas visited Perga together). The terminals are widely separated. Therefore it is crucial to get off at the correct one. We roll along the Turkish roads, through the relentless landscape of *soi-disant* 5 star resort hotels, thrown up in the past 20 years to serve the needs of the Russian tourist industry.

At length we draw up in front of a terminal, and the driver raises the traditional farewell salute. Trusting folk start to exit the bus. At this point I shout from the back that the terminal building is not numbered, but has the chilling word 'domestic' written across its front. Some people leave the bus, to interrogate the terminal. After a few minutes, they return with no definite news, but we decide, on balance, that it is not Terminal 2. We set off again, and after 10 minutes we arrive at a building which is labelled T2 on the traffic signs, and so we alight.

Luggage in tow, we make for the Air Berlin check-in. Back in Leeds, Bev had spent the previous afternoon trying, in vain, to get the Air Berlin computer to spit out boarding passes. It turns out that everyone else has the same problem, so there are queues. We cunningly select the queue run by a new and diffident member of staff, so it all takes an age.

Our journey home involves changing planes in Nuremberg. We celebrate our visit to Bavaria with ice-creams. The second leg of the journey involves an underpopulated flight to Gatwick airport. As we settle into our seats, our names were read out on the tannoy. We had to climb half-way down the steps, and identify our luggage so that it could be loaded aboard. Evidently they have sufficiently few passengers travelling from Turkey to London that their procedures are not finely honed.

The final flight is uneventful, and we enjoy several happy family reunions at Gatwick.