Romanian Masters of Mathematics 2012

UK leader's report

Bucharest, Romania; 29th February – 4th March 2012

Introduction

The UK sent a team to the Fifth Edition of the Romanian Master of Mathematics, which took place between 29th February and 4th March 2012. This has become a regular part of the mathematical competition calendar: it is a hard competition for countries that perform strongly at the International Mathematical Olympiad. The UK has been honoured to accept invitations every year since it began in 2008.

This year, the team consisted of:

Andrew Carlotti	Sir Roger Manwood's School, Kent	(Year 12)
Sam Cappleman-Lynes	Shebbear College, Devon	(Year 13)
James Aaronson	St. Paul's School, London	(Year 13)
Sahl Khan	St. Paul's School, London	(Year 12)
Joshua Lam	The Leys School, Cambridge	(Year 13)
Vishal Patil	King Edward's School, Birmingham	(Year 13)

I (Dr James Cranch, University of Sheffield) led the team; James Gazet of Eton College reprised his role from last year as deputy leader.

Questions

Paper 1

1. Given a finite number of boys and girls, a *sociable set of boys* is a set of boys such that every girl knows at least one boy in that set; and a *sociable set of girls* is a set of girls such that every boy knows at least one girl in that set. Prove that the number of sociable sets of boys and the number of sociable sets of girls have the same parity. (Acquaintance is assumed to be mutual.)

(Poland) Marek Cygan

2. Given a non-isosceles triangle ABC, let D, E, and F denote the midpoints of the sides BC, CA, and AB respectively. The circle BCF and the line BE meet again at P, and the circle ABE and the line AD meet again at Q. Finally, the lines DP and FQ meet at R. Prove that the centroid G of the triangle ABC lies on the circle PQR.

(United Kingdom) David Monk

3. Each positive integer is coloured red or blue. A function f from the set of positive integers to itself has the following two properties:

   (a) if $x \le y$, then $f(x) \le f(y)$; and

   (b) if x, y and z are (not necessarily distinct) positive integers of the same colour and $x + y = z$, then $f(x) + f(y) = f(z)$.

   Prove that there exists a positive number a such that $f(x) \le ax$ for all positive integers x.

(United Kingdom) Ben Elliott

Paper 2

4. Prove that there are infinitely many positive integers n such that $2^{2^n+1} + 1$ is divisible by n but $2^n + 1$ is not.

(Russia) Valery Senderov

5. Given a positive integer $n \ge 3$, colour each cell of an $n \times n$ square array with one of $\lfloor (n+2)^2/3 \rfloor$ colours, each colour being used at least once. Prove that there is some 1×3 or 3×1 rectangular subarray whose three cells are coloured with three different colours.

(Russia) Ilya Bogdanov, Grigory Chelnokov, Dmitry Khramtsov

6. Let ABC be a triangle and let I and O denote its incentre and circumcentre respectively. Let ω_A be the circle through B and C which is tangent to the incircle of the triangle ABC; the circles ω_B and ω_C are defined similarly. The circles ω_B and ω_C meet at a point A' distinct from A; the points B' and C' are defined similarly. Prove that the lines AA', BB' and CC' are concurrent at a point on the line IO.

(Russia) Fedor Ivlev

Results

Here are the scores of the UK team:

name	1	2	3	4	5	6	total	
Andrew Carlotti	7	0	0	6	7	0	20	Bronze medal
Sam Cappleman-Lynes	7	0	0	0	0	1	8	Honourable mention
James Aaronson	7	0	6	7	7	0	27	Silver medal
Sahl Khan	0	1	0	7	0	0	8	Honourable mention
Joshua Lam	0	0	0	7	1	0	8	Honourable mention
Vishal Patil	0	0	0	5	0	0	5	
total, all six	21	1	6	32	15	1	76	
total, top three							55	

Here are the scores of all the teams:

rank	country	score
1=	China	83
1=	Romania	83
3	Russia	78
4	USA	70
5	Poland	63
6	Italy	61
7	Ukraine	60
8	United Kingdom	55
9	Brazil	53
10	Romania B	41
11	Bulgaria	40
12	Romania F	31
13	Hungary	27
14	Serbia	24
15	Tudor Vianu	1.

Romania F, an all-girl team, deserve respect for their powerful performance, which bodes extremely well for their entry into the European Girls' Mathematical Olympiad this April.

It is interesting to consider our position among the fifteen teams on a question-by-question basis. We ranked as follows on each question:

Q1	Q2	Q3	Q4	Q5	Q6
1=	13=	3	8	2=	4=.

It is clear that our performance on those questions with a combinatorial flavour was very strong indeed.

Leader's Diary

Wednesday, 29th February

My alarm rings at 04:30, and I begin my journey with a walk to Sheffield Station. While on the bus from Luton Airport Parkway to Luton Airport itself, I pass team member Andrew Carlotti who is strolling along the grass verge: he is apparently experimenting with alternative methods of traversing South Luton.

Sam Cappleman-Lynes is waiting for us at the airport, having had an early start from Aldershot. We are joined soon enough by the roving Carlotti and the other team members James Aaronson, Sahl Khan, Joshua Lam, Vishal Patil, and trusty deputy James Gazet.

Luton is replaced in good time by the crowded arrival hall of Bucharest Baneasa airport, with about three times as many people as can fit around the luggage carousel. Andrew displays a party trick: he is able to read the name labels on baggage as they slide past. This facilitates bag retrieval considerably.

We are met at the door of the airport by three people with a Romanian Masters sign, who introduce themselves as teachers at Tudor Vianu; this is a specialist state school in Bucharest for very able students, and the origin of many Romanian mathematicians. A history teacher drives us into town; the journey is not long. The domestic arrangements are the same as last year: the students are staying at Mihail Moxa, the campus of the Bucharest equivalent of the LSE, and the adults are in a hotel just down the road.

We are rapidly reunited with old friends. We also meet the four very pleasant gentlemen from China: the last three years the team has been from Shanghai province, but this year they are sending a team from Beijing. Later we will meet two Polish and two Brazilian leaders whom we have not met before.

Dinner consists of polenta with white cheese and sour cream (a Romanian speciality), duck leg and rice, with a carrot-and-cabbage side salad. I receive a memory stick containing the proposed problems. The students settle into their rooms, with Vishal providing jokes from a Tim Vine book; soon after this James Gazet and I retire to pay the problems some attention. They are fantastic.

Thursday, 1st March

We get up bright and early, and head over to Tudor Vianu. We meet our old friends Sever Moldoveanu, the headmaster and logistical organiser, and Dan Schwarz, who this year is leading an all-female Romanian team to prepare them for the first European Girls' Mathematical Olympiad in April.

This year the problems have been selected by a committee of four: Ilya Bogdanov, Calin Popescu, Marian Andronache, and Géza Kós.

The paper is a done deal, except that two variants are proposed. The first variant is the version preferred by the selection committee, but has a question which involves concepts with which some students may be unfamiliar. The second replaces this with a safer question. The agreement is made that, if any leader feels that their team are unprepared for that question, we should use the tamer version of the paper. Brazil and the USA both feel this way, so we make the change.

The result of this is that Paper 1 contains two British problems: problem 2 is by the superveteran David Monk, while problem 3 is by Ben Elliott, who was himself a participant in the Romanian Masters only last year. Balanced against this achievement, of course, is the impressive full day of Russian problems in the form of Paper 2.

Having agreed on the paper, we begin polishing the English. While the British leaders are the only native English speakers present, several other people are uncannily close. Unless the reader is interested in our decisions on whether various clauses should be in the indicative or the subjunctive mood, there is only one noteworthy point. The original Problem 1 referred to the sets as *covering sets*. However, it is pointed out that, in English, "covering" has a connotation which might be considered unfortunate in context. I consult with some other leaders and find that this infelicity would not be lessened in translation. *Sociable sets* seems like a decorous alternative.

Meanwhile, the students are receiving a lecture on modern knot theory from Ciprian Manolescu, a Romanian mathematician at UCLA who, extraordinarily, achieved three perfect scores as a student at the IMO.

After lunch, other leaders translate the paper into their own languages, and we spend some more time working on the problems.

Friday, 2nd March

Today is the day of Paper 1. This begins smoothly. While this is happening there is a quick museum trip for the leaders. Unfortunately, it is not possible to visit Romanian museums anonymously, and James Gazet and I have both left our passports in our shared hotel room. So we stay back and relax.

The students are let out after their four-and-a-half hour paper. We meet

them briefly: Vishal has little to report, but Josh and Sahl have made various attempts, and the other three all have confident claims of Problem 1. James Aaronson also claims Problem 3.

After this, the students go off to lunch and the leaders are taken out to Otopeni, in the extreme north of the city, for a lengthy lunch to distract them while the photocopying takes place. The meal starts with heated ţuica (plum brandy) and some folk music; after a mixed meze, there is a main course of assorted multiply-nested sausages, followed by strudel and pancakes.

We return to be given the scripts by Sever. The UK students have submitted scripts on Problems 1, 2 and 3 which are respectively good, bad and ugly. Andrew, Sam and James's claims are all backed up by safe solutions to Problem 1. Sahl has made the only attempt at Problem 2: a great deal of calculation, but no strategy for tying it up. For Problem 3, James has turned in a script which is clearly packed with all the relevant ideas, but it is afflicted with horrendous handwriting, notation, and structure. I spend the rest of the day struggling with it, taking time out only to visit Ilya and Alexeey, the Russians, for a brief social call in the evening.

Ilya tells a story of Valery Senderov, the author of Problem 4. In the 70s he was a dissident and prolific problem setter. For a while he was detained, and the flow of problems had to stop. Ilya was unable to confirm that problems were smuggled out of prison, but he did mention that problems attributed to one Sender Valerianov started to appear around that time.

Saturday, 3rd March

Today is a busy day. The students are to sit Paper 2 in the morning, while we coordinate Paper 1. Then after some rapid photocopying, we are to coordinate Paper 2. This is two normal days' business; the effort required to pull this off is nothing short of phenomenal on the part of the coordinators.

The morning's coordination goes smoothly, except for Problem 3. We have still not been able to decipher some of the minutiae of James's script. We go to coordination, and are pleased to see Marian Andronache and Calin Popescu the other side of the table. They are also somewhat confused. We talk about it a bit, and then postpone.

About an hour later, Calin comes to find me. We have a quick chat. He has made significant progress on James's script, and says he is satisfied it is almost complete and that it is reasonable to award six marks. I agree, and go to sign forms and shake hands.

After a rapid buffet lunch, both coordinators and leaders alike are plunged into Paper 2. This has been quite a good day for us; we have nothing much on Problem 6, but considerable work on Problem 4 and Problem 5. We are punished mildly for a numerical slip by Carlotti on Problem 4, but otherwise

get what we would like. Géza and Ilya expertly deal with the issues raised by Problem 5, and despite their shrewdness we end up with a tidy score. Mihai Baluna politely offers a mark for Problem 6 for Sam's attempts at supplying useful lemmata.

We kick our heels for a while, while loose ends are tied. The students start to file in, and are lifted by our news of their scores. At last all is done, and the last coordinator staggers out.

The leaders gather together, expecting a rapid jury meeting to approve the scores and medal boundaries: but it never happens. On the one hand, this is a good thing, because it is very late and the students are becoming tired. On the other hand, the medal ratio appears quite low (only a little more than one-third), and based on the scores lists, the jury might well have chosen to impose different cutoffs. We will have to take more care in future, or else there may be a scandal one year.

Instead we go straight into the closing ceremony: short speeches are made and medals are awarded. James gets the first silver, and Andrew is a safe bronze. The team's shortage of metalwork does not seem to damage their credibility in the eyes of the Tudor Vianu students, and several young ladies enterprisingly manage to have their photos taken with them.

The final meal is a buffet, again separately from the students. If I had one wish (besides a return to the more relaxed four-day format) it would be for more opportunities to see the students, perhaps even for some mealtimes together. Despite my repeated strong denials to their faces, I am glad of their company occasionally. On the other hand, I would fully understand if other leaders feel differently.

Since we and the Americans alike must get up early in the morning, we excuse ourselves at a reasonable hour. It is suggested that the underground will still be running, so Razvan, Zuming, James Gazet and I descend to Eroilor station and get the last train to Piaţa Victoriei. When we reach our destination, not all the doors open. Zuming and I are adroit enough to act upon this, but Razvan and James do not do so well: we watch them slowly accelerate into the darkness. We ascertain that they will have missed the last train in the opposite direction. I do not know how I am trusted with school students when I cannot even look after our deputy leader; Zuming states the matter even more bluntly.

Sunday, 4th March

We get up at 04:30 (or "02:30", as it is known in the UK); James goes to knock upon the students, and I check out of our room. A bus appears and takes us to Baneasa airport, a building containing nothing but one very large crowd. We percolate through it. Along the way, the security people become

confused and distressed by the large number of Rubik's Cubes about Sam's person. We leave him there; he is off directly to Disneyland Paris, by way of Beauvais-Tillé airport, for some manner of excursion organised by his school.

The rest of us end up back at Luton, where almost everyone is picked up. I am surprised to meet James Gazet's wife Rachel in person: this damages my mental model of him as being a bit like Columbo. I go with the remnant, Andrew, as far as the railway station, and then make it home in time to watch the rugby.

Closing remarks

Thanks are due to many people for our successful participation in this competition. These include:

- The problem selectors and coordinators for their excellent work on these beautiful problems;

- The students' guides, Silvia, Andrei, Razvan and Sensy, for their friendliness and caring towards our students;

- All the other Tudor Vianu people who worked so hard to make this competition a success: headteacher, staff and students alike;

- Bev Detoeuf in the UKMT office for her hard work on the logistical side;

- Innumerable teachers and volunteers for getting our students into a state where we can hold our own in such competitions;

- James Gazet for his frequent selfless help and comic relief;

- Andrew, Sam, James, Sahl, Joshua and Vishal, for being good students and good people.

James Cranch (`jdc41@cam.ac.uk`), 7th March 2012.