Balkan Mathematical Olympiad 2013

UK leader's report: GCS

August 2013

The Balkan Mathematical Olympiad was held in Cyprus from 28 June until 3 July. The UK Team Leader was Dr Geoff Smith of the University of Bath and the Deputy Leader was Dr Gerry Leversha, formerly of St Paul's School. The United Kingdom participates as a guest nation in this competition, and we have a self-imposed rule that we will not send a student to it more than once. This creates a lot of churn, and gives many students the experience of international competition.

In normal circumstances, the Balkan MO is held in May, but this year that proved impossible. The Cyprus Mathematical Society stepped in to organize an emergency edition of the competition a couple of months later than usual. This had the effect that some regular guest countries, such as France and Saudi Arabia for example, were not in a position to participate. Unsurprisingly, the political dispute concerning the division of the island of Cyprus led to further complications.

There were unusual and complicated bureaucratic problems associated with taking this team to Cyprus. Thanks are due to Dr Don Collins, the BMOS treasurer, and Rachel Greenhalgh, Director of UKMT, who worked tirelessly and successfully to overcome the difficulties. Further thanks are due to the families, teachers and guardians of the students concerned for working together so effectively at speed.

The team was as follows:

Oliver Feng	Eton College, Berkshire
William Gao	Merchiston Castle School, Colinton, Edinburgh
Frank Han	Dulwich College, London
Maria Holdcroft	Willink School, Berkshire
Freddie Illingworth	Magdalen College School, Oxford
Warren Li	Fulford School, York

The team labels were subject to a permutation beyond UK control. The results were as follows

http://www.bmo2013.eu/index.php/en/contest/olympiad-results

and the performance of the British team is shown in the following table:

		P1	P2	P3	P4	Σ	Medal
UNK1	Oliver Feng	10	10	0	0	20	Silver
UNK2	William Gao	2	3	1	0	6	
UNK3	Frank Han	10	8	10	1	29	Silver
UNK4	Freddie Illingworth	10	10	10	4	34	Gold
UNK5	Maria Holdcroft	10	10	10	0	30	Silver
UNK6	Warren Li	0	3	10	1	14	Bronze

Here are the problems of the competition.

Problem 1 In a triangle ABC, the excircle ω_a opposite A touches AB at P and AC at Q, and the excircle ω_b opposite B touches BA at M and BC at N. Let K be the projection of C onto MN, and let L be the projection of C onto PQ. Show that the quadrilateral $MKLP$ is cyclic.

Problem 2 Determine all positive integers x, y and z such that $x^5 + 4^y = 2013^z$.

Problem 3 Let S be the set of positive real numbers. Find all functions $f : S^3 \longrightarrow S$ such that, for all positive real numbers x, y, z and k, the following three conditions are satisfied:

(a) $xf(x, y, z) = zf(z, y, x)$

(b) $f(x, yk, k^2z) = kf(x, y, z)$,

(c) $f(1, k, k + 1) = k + 1$.

Problem 4 In a mathematical competition some competitors are friends; friendship is always mutual, that is to say that when A is a friend of B, then also B is a friend of A. We say that $n \geq 3$ different competitors $A_1, A_2, \ldots, A_n$ form a *weakly-friendly cycle* if A_i is not a friend of A_{i+1}, for $1 \leq i \leq n$ ($A_{n+1} = A_1$), and there are no other pairs of non-friends among the components of this cycle. The following property is satisfied:

for every competitor C, and every weakly-friendly cycle $\mathcal{S}$ of competitors not including C, the set of competitors D in $\mathcal{S}$ which are not friends of C has at most one element.

Prove that all competitors of this mathematical competition can be arranged into three rooms, such that every two competitors that are in the same room are friends.

Diary

Friday June 28th I arrive first at Heathrow Terminal 1, and just as I am settling in, receive a telephone call from a legal person associated with Morgan Matthews's fictional reworking of *Beautiful Young Minds*, a romantic comedy in which, in a novel twist, love triumphs. The film-makers want the freedom to defame (or portray accurately, I don't know which would be worse) a fictional UK IMO leader, and in consequence I must sign a piece of paper to licence calumny, accuracy and similar dangers. In fact I have already signed a piece of paper to this effect, but it turns out that was the wrong piece of paper, and I must sign the right one. This is hardly a convenient moment, but I agree to sign and send a scan of the new piece of paper to the legal people from my forthcoming Cypriot HQ in Nicosia.

The team arrive, William's flight from Scotland being a little late. There is no time to take lunch, but people with sufficiently empty lives that they read the terms and conditions of their plane tickets (and we have some) assure the rest of us that there will be a meal on the plane.

We take off, and a eventually mini-meal arrives in the form of a small tray of pasta, with a little sauce, a sliced mushroom and a chicken fragment. Fortunately I do not have a large appetite, but I worry about the others. The modest nature of this meal will cause problems later.

We arrive about 9pm local time, and by the time we clear the formalities, it is dark. The Macedonians are waiting for us, to share our bus. The plan is to drive north to Nicosia, drop the leaders (Vesna of FYROM and me), and then take both sets of students to the mountains. Vesna and I scuttle into the Hilton at 10:20pm, check-in, dump our bags and just manage to get to the restaurant before it closes.

The rest of group had a more difficult time. They drove west up mountain roads until, very late, they arrived at the Rodon Hotel resort in Agros. No food was waiting for them. Fortunately I have not written *cannibal* on the

dietary requirements section of the relevant form, so Gerry is well placed to go into serious howling mode. Somehow he managed to get through to Greg Makrides, the head of most things (including the Balkan Mathematical Olympiad) in Cyprus, and very quickly a cold collation appeared. Well done Gerry, Greg and the Rodon Hotel.

Meanwhile, back at the Nicosia Hilton, I have received the shortlist of problems. All five UK submissions are on the shortlist, and two of them are my own personal efforts. In a characteristic display of good taste, my questions will not be selected by the jury.

Saturday June 29th I have breakfast outside by the pool, and run into my old friend Massimo, the Italian leader. The leaders then gather for a brief jury meeting to eliminate a problem or two for being known, and then depart for Agros to go to the 30th Balkan MO opening ceremony. Having seen the shortlist, I must not be intimate with my deputy or students, but we wave and smile across the room. There is a gratifying amount of folkloric dancing, and a play about Pythagoras is put on by local schoolchildren. We are formally welcomed, and then the leaders are sent back to Nicosia.

In the afternoon the leaders see the solutions to the problems on the shortlist, and then we choose the problems for the exam paper. A pretty geometry problem from Bulgaria is selected in position 1, and a Serbian elementary number theory problem in position 2. Jack Smith's UK submission, a tricky and unusual functional equation problem was selected in position 3. Finally a Serbian question was chosen as Problem 4.

Massimo and I form the English Language Committee, and polish the language and notation of the problems. We then present our work to the rest of the jury, who suggest further improvements.

I then scurry off for a few hours rest while the rest of the jury go to work to set the paper in eleven other languages. At the end of the process we are invited to pack our own students' envelopes with exam papers in appropriate languages, and associated instruction sheets. I am not sure that this is a very good idea because it presents a clear opportunity to breach security. A dishonest leader (with dishonest students) could slip in a note containing summary solutions. Perhaps this practice should not be encouraged?

I am beginning to be a little mystified by the Nicosia Hilton. It is a classy hotel, and is very comfortable. However, apart from the Balkan Maths Olympiad people, there are very few guests. It is an expensive place, and of course the Balkan MO is not picking up bar bills. I notice that a fairly normal bottle of wine is on sale in the bar for €44. This all looks very

strange, with the bar so expensive that no-one is using it, the high-rollers having disappeared since the economic crisis. Why do they not drop their prices to more modest levels and make some money?

Sunday June 30th We have breakfast early, check-out from the Hilton, and pile into a bus to transfer to the mountains. We close the curtains and doze. On arrival at the Rodon Hotel, I pass deputy Gerry Leversha en route to the jury room, and shake his hand.

The jury is summoned to the exam room to witness the exam papers being distributed. I try to imagine why this is a good idea, but cannot work it out. There appears to be only one clock in the ell-shaped jury room, and it is situated in a place that makes it maximally difficult to see. The organizers improvise brilliantly, and an itinerant invigilator wears a clock on his chest for the duration of the exam.

The jury retires to its rather small room. and at 10am the exam begins. The jury does not have to wait long for the first question. It is from a British student, who wants to know the definition of a *projection* in the context of the geometry question. I propose a formal abstract definition, and it is approved by the jury and sent back. There are only a few questions sent in during the 30 minute questions window, so it seems that the wording on the papers is quite clear.

There is a break for coffee, and I go to find Gerry. I find him in the *al fresco* coffee lounge, on a large balcony with a fantastic view south over Cyprus. In the far distance there is a patch of water just visible, and consulting a map we realise that we are looking at the eastern coast of the Akrotiri peninsula (Cheltenham-super-Mare).

We have a meeting with the co-ordinators who collect alternative solutions and propose marking schemes. I hand in my solution to the geometry problem which, somewhat predictably, uses areal co-ordinates and reduces the problem to linear algebra. It turns out that areal methods are not often used in this part of the world, and I later give a brief impromptu tutorial on areal techniques to those who are interested.

At the end of the exam, the British students pile out, and they have had very different experiences. William Gao has had an off-day, but has some part marks, whereas Maria Holdcroft, Freddie Illingworth and Frank Han claim to have done about three questions with Freddie having perhaps a little more, Oliver Feng has done a couple, and Warren Li about one and a half. Rumour has it that no-one has solved Problem 4 completely. Therefore the Gold Medal boundary looks certain to be in the low 30s, so Freddie might

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get a gold, and both Maria and Frank can be sure of getting at least silver medals. Warren and Oliver look likely to get bronze medals. I don't tell William, but the bronze medal cut-off at the Balkan MO is usually quite low, so there is an outside chance that he will get a bronze medal if his fragments can earn enough marks. Unfortunately, they don't.

An official marking scheme document is issued when we receive the scripts in the late afternoon. Gerry is very confident at Euclidean Geometry, so he takes Problem 1, and I look at the other scripts. Now is the time for the students to relax, and for the leaders to work really hard, and engage in fine textual analysis of the scripts.

Monday July 1st Today is co-ordination day. We have a happy time, since the co-ordination is expert and fair. Our students, save poor William, have done interesting work, and Freddie Illingworth had excelled himself.

There is a curious incident during co-ordination. While the UK is settling in to discuss a (non-geometry) problem, an Italian geometry script is thrust into my hands by a co-ordinator. The student has solved the problem by areal co-ordinates, and I am asked to verify that the method and calculation are correct. It looks and feels right, but I do not have the time to give it the full going over that I would wish. I give it my provisional blessing, and when I check later, it is in fact correct.

The jury meets at the end of the day and approves the medal cut-offs based on the results of the "official" nations. The cut-offs were 31 for gold, 20 for silver and 8 for bronze. Unfortunately William has not quite managed to get a bronze medal. Life is sometimes very unfair, for William is a very talented young mathematician. Our other team members have done better, and Freddie has secured a very clear gold medal. This underscores the strength of the UK IMO team of 2013, because Freddie is not in it! Oliver, Frank and Maria all have silver medals (Oliver having no margin of safety), and Warren has a solid bronze.

Tuesday July 2nd Today we are going out to play. In the morning we all take buses to Nicosia to visit the Archaeological Museum. This contains an astonishing collection of artifacts spanning a huge period. These objects are not properly separated and classified by era and civilization, so you have to keep your wits about you to follow what is going on. However, this is one of the great museums of the world, and every visitor to Cyprus with any interest in civilizations ought to pay a visit.

For lunch we visit the University of Cyprus where we are given a warm welcome, and attractive scholarships are offered. We listen to a passion-

ate speech about human tragedies associated with the division of the island delivered by the Rector.

As we had driven through Nicosia, we had, for a while, been travelling parallel to the *Green Line*, the UN controlled neutral zone which separates the widely recognised Republic of Cyprus, and the territory to the north which is recognized as a legitimate political entity only by Turkey. There is a giant flag of the "Turkish Republic of Northern Cyprus" somehow painted on, or carved into, the mountains to the north.

At the moment there are no active hostilities, and I read that it is possible to cross the border without too many formalities. At least one enterprising team leader actually did this, but in case this causes offence, my memory has become hazy as to who this was.

We return to the Rodon Hotel for the closing ceremony. The Minister for Education is there to make a speech and give out medals. We then step outside and have a charming farewell dinner in the gardens.

Wednesday July 3rd The return journey to London was relaxing and uneventful, except for one curious event. After we took off, the announcement from the cockpit deviated from the usual patter concerning cruising height, the likely time of arrival and impending turbulence. An officer of Cyprus Airways made a speech which made it clear that he had trenchant views concerning the division of Cyprus, and which nation was responsible.

After we arrive back at Heathrow, I go to the central bus station to pick up a rail-air bus to Reading. This proves difficult. The central bus station is so over-stretched that they cannot assign buses to stops in advance. My bus is running late, and the number of its stop is announced one minute before its scheduled departure time. Those of us passengers who scamper from the waiting room towards the stop are rewarded with the sight of the bus driving off. Sometimes I am glad that the UK has robust gun controls.

I go back to the waiting room to express my views somewhat forcefully. The central bus station is a disgrace, and I advise readers not to use it. It turns out that there are dedicated rail-air stops outside most of the terminals, and it is much safer to use those.

Summary The Cyprus Mathematical Society did a wonderful job of putting on this emergency competition at very short notice, and I thank all concerned for this fantastic effort, and for the warmth of the welcome that we received in both Nicosia and Agros.