

Report on the 2nd European Girls' Mathematical Olympiad

8th – 14th April 2013

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Introduction

After a very successful first year in Cambridge, Luxembourg took on the baton for holding the 2013 European Girls' Mathematical Olympiad. EGMO was founded to increase the participation of girls in international maths competitions. The competition had grown since last year; 22 countries participated and further countries had also hoped to join, it looks like EGMO 2014 in Turkey will be even bigger.

The selection for the EGMO team was made from performances at BMO1 and BMO2, the two rounds of the British Mathematical Olympiad. It has been encouraging to see an increase in the achievement and contribution of girls to the UK olympiad effort this year; several girls attended training camps at Oxford and Hungary and selection for the EGMO team was a close-run affair. Congratulations to all girls competing to get into the team. We hope that we will be able to build on the past year's efforts and continue to encourage girls to enjoy the beauty of mathematics.

The team consisted of:

UNK1	Maria Holdcroft	Willink School
UNK2	Elizabeth Lee	Loughborough High School
UNK3	Katya Richards	School of St Helen and St Katharine
UNK4	Kasia Warburton	Reigate Grammar School

The reserve was Ellie Holderness of Latymer Upper School. The Team Leader was Hannah Roberts of Pembroke College, Oxford and the Deputy Leader was Jo Harbour of Mayfield Primary School, Cambridge.

The Competition

The format of EGMO was amended for this year so that it now closely resembles that of the International Mathematical Olympiad (IMO). Two 4 1/2 hour papers each containing three questions (ordered by approximate difficulty) are sat over two days. Each question carries 7 marks, with part marks given only for significant progress, such that it is usually rare to be awarded 3 or 4. The scores for the UK team were as follows:

	Q1	Q2	Q3	Q4	Q5	Q6	Total	Medal
Maria Holdcroft	7	5	0	7	0	1	20	Bronze Medal
Elizabeth Lee	7	6	3	6	0	1	23	Silver Medal
Katya Richards	7	5	0	1	0	1	14	Honourable Mention
Kasia Warburton	1	7	0	5	0	0	13	Honourable Mention

The medal boundaries are 27 for gold, 21 for silver and 16 for bronze. The UK team came 11th out of 22 participating countries, and 2nd in Western Europe (behind a strong performance from Italy). This was a great result, with a particularly solid effort by all the girls on day 1 and though it may not look like it on Question 6 also, which was a hard question to score any marks on. The competition was won convincingly by Danielle Wang from the USA (a guest country) who scored an incredible 38 points. The team competition plate was presented jointly to the USA, Serbia and Belarus who all scored a combined total of 99 points.

We're pleased that three of the girls will still be eligible for the team next year, whilst Maria has an offer from The Queen's College, Oxford for October. We wish her the best of luck for her future studies, and also congratulate her on getting into the IMO9, all of whom will be training hard over the next month before the final IMO selection camp in Oundle.

The Problems

Day 1

Problem 1. The side BC of the triangle ABC is extended beyond C to D so that $CD = BC$. The side CA is extended beyond A to E so that $AE = 2CA$.

Prove that, if $AD = BE$, then the triangle ABC is right-angled.

Problem 2. Determine all integers m for which the $m \times m$ square can be dissected into five rectangles, the side lengths of which are the integers $1, 2, 3, \dots, 10$ in some order.

Problem 3. Let n be a positive integer.

- Prove that there exists a set S of $6n$ pairwise different positive integers, such that the least common multiple of any two elements of S is no larger than $32n^2$.
- Prove that every set T of $6n$ pairwise different positive integers contains two elements the least common multiple of which is larger than $9n^2$.

Day 2

Problem 4. Find all positive integers a and b for which there are three consecutive integers at which the polynomial

$$P(n) = \frac{n^5 + a}{b}$$

takes integer values.

Problem 5. Let Ω be the circumcircle of the triangle ABC . The circle ω is tangent to the sides AC and BC , and it is internally tangent to the circle Ω at the point P . A line parallel to AB and intersecting the interior of triangle ABC is tangent to ω at Q .

Prove that $\angle ACP = \angle QCB$.

Problem 6. Snow White and the Seven Dwarves are living in their house in the forest. On each of 16 consecutive days, some of the dwarves worked in the diamond mine while the remaining dwarves collected berries in the forest. No dwarf performed both types of work on the same day. On any two different (not necessarily consecutive) days, at least three dwarves each performed both types of work. Further, on the first day, all seven dwarves worked in the diamond mine.

Prove that, on one of these 16 days, all seven dwarves were collecting berries.

These were a challenging and rewarding set of questions. Q1 and Q2 ensured the paper was accessible enough that no participants received zero marks, but equally Q3 and Q6 were very difficult and very few students scored full marks on these.

Leader's Diary

The following narrative is an account of the events of EGMO from my perspective. This was my first time attending an international maths competition in the capacity of Leader and so I was a little unsure what to expect. I had however been able to pick up a few words of wisdom from more experienced leaders before I left and was very happy to be working alongside Jo Harbour, who was also Deputy last year.

Day 1 – Arrival

Today is the final day of Trinity Training Camp at which lots of great maths has been enjoyed (particular thanks to Jack Shotton, Richard Freeland and Ben Elliott for their work in ensuring the smooth running and fun atmosphere of the camp). After lunch we make our way to the porters' lodge to meet the minibus that will take us to London City Airport. Geoff Smith and Joseph Myers are also travelling with us, as they will be helping out at EGMO 2013 in the roles of coordinator (both), EGMO advisory board (Geoff) and ICT (Joseph). The minibus driver, having been told we may have lots of luggage, has brought a whole extra trailer for our suitcases. None of the girls seem to have packed an excessive amount of stuff so this ends up largely empty. The minibus driver is very chatty. Upon finding out that we're travelling to a maths competition the conversation goes something like this:

Driver: 'Did you see that documentary on a while ago about those kids that spend 24/7 thinking about maths?'

Geoff: 'Would you be referring to the documentary called 'Beautiful Young Minds'?'

Driver: ‘Yes, I think so. Are you in the same “league”?’

Geoff: ‘We’re in the same documentary.’

Driver: ‘Oh, right.’ [seems worried that he may have caused offence]

Geoff: ‘You’ll be pleased to hear that the director of that documentary is in the process of turning it into a romantic comedy.’¹ [Driver seems overjoyed at the prospect]

On arrival at the airport we discover we have an hour to spare before we can even drop our bags. The girls are sent to try to find a Union Flag, since their leaders have let them down and forgotten to bring one. They return soon after having searched all two shops to no avail. Geoff suggests they might like to buy some dinner at this point. We look on amazed as four girls manage to spend a total of nine minutes deciding which sandwich they’re each going to buy.

The flight to Luxembourg is short and sweet; it’s only an hour long and we’re offered complimentary drinks and snacks – such a novelty when you haven’t flown with anyone but Ryanair for many years. As a responsible adult, Jo collects in everyone’s passports as we stand waiting for our bags at the other end. They appear within a few minutes of us landing and everyone except me walks off to find out if there is anyone waiting to greet us. I am left standing in the middle of a nearly empty baggage reclaim hall in a slightly pathetic fit of giggles; Jo has confidently strolled off with the passports but without her suitcase and my resulting uncontrollable laughter has rendered me unable to call out her name to alert her to her predicament. I finally manage to compose myself and drag mine and Jo’s bags into the next room. The girls are relieved that they clearly have two such trustworthy and sensible adults looking out for them this week.

All contestants and leaders/deputies are being accommodated in the youth hostel AJ Luxembourg. The team have been split into pairs and each will be sharing a six-bed room with two other pairs of students. Jo and I are in four-bed rooms with other deputies/leaders respectively. We’re handed some bedding and shown to our rooms. The girls are also sharing two showers and three toilets between 36 of them. It’s 10pm Luxembourg time when we arrive, so we head straight to bed. I discover that I’m sharing a room with the French and USA team leaders both of whom were also in Cambridge last year, doing Part III maths (though we’d never met). I had expected most of the other leaders to be much older than me so this makes me feel more at ease.

Day 2 – Opening Ceremony

The next morning I get up early and am joined at 7.30 by the team; having got up earlier than the other students they have managed to avoid the queues for the showers and for breakfast. There are several exciting things about breakfast: tea, gluten-free bread for Lizzy and huge bowl of Nutella to be dolloped onto bread with a serving spoon. We have donned our purple UKMT EGMO polo shirts for the opening ceremony and after breakfast we apply matching purple nail varnish. The girls reckon this tradition should

¹For more exciting information along these lines see: <http://blog.endernews.com/2013/03/asa-butterfield-attached-to-x-y-drama.html>

be extended to the IMO. Joseph agrees.¹

Before the Opening Ceremony we have the first jury meeting (the jury consists of the team leaders from all the participating countries). We are shown the paper that has been compiled by the problem selection committee and are asked to check that none of the questions are likely to be known by the students. The Bulgarian leader is concerned that one of the problems is similar to one used for Australian training. He is reassured that it is sufficiently different, and the paper is voted in. All much simpler than I had expected. My first impression of the questions is that the girls will get on well with Paper 1, but I'm less sure how they'll find Paper 2.

Since the leaders have now seen the paper the students and deputies walk to the opening ceremony first and are seated at the front, followed by the leaders a while later. In general EGMO is much more relaxed about communication between contestants and leaders than the IMO (where leaders are often accommodated on a separate site many miles away), and we still eat all our meals together – it is taken on trust that we won't talk about maths. So this extra precaution seems a little unnecessary. It does have the advantage that the most important people, namely the students, get the best seats for the ceremony though. This is our first view of Luxembourg City in the day time. The city has an interesting natural geography; it sits at the confluence of two rivers which have carved deep meandering valleys into the sandstone. The youth hostel is situated at valley floor level, whereas most places you might want to go are at plateau level. I can see that we're going to be getting a good amount of exercise this trip (which will hopefully offset the breakfast Nutella). The opening ceremony starts with some brief speeches and then each of the teams are introduced in turn. The Ukrainians win the (hypothetical) prize for best team outfit with their matching white puffy shirts with pink embroidery (there's probably a technical name for these but I'm no fashion expert).

After lunch we have free time and the team discover to their delight that the Irish deputy, Jonathan, has brought some Set cards with him – a game the UK students have spent a lot of the training camp playing. It turns out that the version the Irish know is a fair bit more violent than the one our girls are used to, so some sort of compromise has to be agreed on first. Later, whilst the girls are shown the way to the school where their exams will take place, there's another fairly straightforward jury meeting to approve the English language version of the paper. Most of the leaders spend the rest of the afternoon translating the paper into their respective native languages. I walk up into town with the USA leader, Sherry, and decide this would be a good time to purchase some supplies to get the girls through their 4 1/2 hour exams. Halfway round the supermarket I bump into Jo and the team who've had the same idea. After much deliberation we purchase eight bars of Milka of varying flavours and hope that this will be enough fuel. Our lovely guide, Carole, offers to take us back to the youth hostel via the scenic route which is very pretty. On returning we order much needed cups of tea, and discuss tea while drinking tea, except Maria who has an unfortunate dislike of tea. Katya admits that she has brought milk, tea bags and a kettle with her in her luggage. She is representing her country well.

¹This sentence may not be entirely true.

Day 3 – Paper 1

The girls have to be up for breakfast at 6.30am on exam days and are consequently looking a bit weary this morning. Kasia is worried she might end up taking an unintentional nap during the exam and so tries to make herself nervous to induce some adrenaline. Jo and I point out that they can be grateful that the half hour walk to the exam hall should wake them up.

During the first half hour of the exam the contestants are allowed to ask questions about the paper, to clarify the meaning of the problem descriptions. It would seem the papers have been well worded to avoid confusion this year though as few queries come through to the jury.

After question time we make our way back to the youth hostel. In general the youth hostel is turning out to be a great place to be based; it's clean and warm (really very warm), has a lovely large dining area and excellent food for mass catering standards, there's a cafe/bar and lounge to relax in and free wifi everywhere. The downside is that unsurprisingly there aren't really desk facilities for when 44 leaders and deputies want to sit down and solve maths problems. Anyway, I find a bit of floor space in the lounge and have a go at the paper. It's good fun, and seems pretty accessible – I'm feeling quite out of practice at olympiad maths and figure that whatever I can do there's a good chance the girls will also be able to do, even under exam conditions, so this bodes well.

The Irish team successfully navigate their way back from the exam via the fastest route to arrive at lunch first, followed soon after by the UK team. The girls all seem in good spirits except Maria who has had a frustrating experience with Question 2, wasting quite a bit of time writing up what she thought was an elegant solution before realising there was a hole in it and going back to the standard case-by-case method. We have a short discussion of the questions over lunch and from what Jo and I can tell it seems that the girls have indeed solved the problems they're claiming to have solved, which gives us cautious hope that no-one will later be disappointed by finding out that they've made some sort of oversight.

There's now a long wait for all the scripts to be photocopied before we're allowed a look at them. During this time disquiet starts to rise amongst the jury; before lunch we had met to look over the mark schemes and approve them, however some leaders now feel that this decision was rushed and are concerned that the mark scheme for Q2 won't differentiate between those students who do have some understanding that they need to prove an upper bound for m and have some ideas for doing so, and those who don't. Jo and I feel that since leaders have now had a chance to talk to their students about what they have done, retracting the earlier decision would be particularly inappropriate. A second meeting is called in which three motions are voted on:

1. The possibility of making a modification to the mark scheme.
2. Whether to modify the mark scheme.
3. Whether the jury should make the modification or leave it to the coordinators to modify it along the lines of the suggestions of the leader of Norway.

With the result that it looks like the coordinators might have a long night ahead of them.

Day 4 – Paper 2

Day 4 starts much the same as the previous day. We begin to worry when the girls are late returning from the exam, but speculate that they may have stopped to buy cake in celebration of the fact that today is EGMO's 1st birthday. Alas no, it turns out they'd chosen to go home via a route that involves taking a lift and got out on the wrong level of the lift (they swear this was on the advice of a local man). Maria is feeling happier today and thinks she has solved Q4 and 6. On the other hand Kasia had thought she'd solved Q4 in the exam but has since found out she got a different answer to everyone else, so is feeling less upbeat. Despite their tiredness the girls spend their afternoon trying to do Q5 together, which none of them managed during the exam. Jo and I take the opportunity to go for a run before the scripts arrive. We manage to find a relatively flat route along the river that nevertheless involves climbing an old tower to cross a wall at one point. We return from our mini adventure to find the scripts have arrived much earlier today and start marking soon after. Q6 causes us the most trouble. The girls have good ideas but haven't managed to assemble them into a watertight argument within the time allowed. Maria's seems the closest to a solution but unfortunately she's changed her mind about how she's going to go about proving it halfway through so it's a little difficult to follow. Eventually, after re-writing it with the notation used in the mark scheme, I decide that I think it is complete but with one small mistake.

The girls spend the evening having fun at a blues dancing class run by the USA deputy (see the unofficial report for more details, I'm sure).

Day 5 – Coordination

At EGMO (and other international competitions) scores for each script are decided on by means of a process called coordination; team leaders and deputies sit down with panels of impartial experts to discuss each student's solution or workings and agree on a mark. We've been given a timetable with our 20 minute slots for each question scattered throughout the day but when we arrive it becomes clear that in some cases there's no need to stick to the timetable. The coordinators for Q5 are sitting twiddling their thumbs so Jo and I seize the opportunity to go and collect 4 zeros – a slightly less than exciting entrance onto the live scoreboard for the UK team. Next up is Q2 so we look over the revised mark scheme to calculate what we think the scripts are worth. All the girls have a solution to this question, though with a few mistakes and differences in how they've arrived at upper and lower bounds for m ; Maria has proved the lower bound correctly but via a method that involves a two page calculation of fourth roots and large factorials – I admire her courage. Katya on the other hand has the idea for a neat argument but states that the inequality she needs to prove is a 'well known fact'. Unfortunately for her the jury felt differently though she will certainly not be the only contestant to lose

a mark for this. When the coordination table becomes free we offer our suggestions for marks and are brusquely asked to justify them. This causes us a little concern but we do so and are relieved and surprised to discover that the coordinators agree exactly with what we've said!

This puts us into second place on the leader board! The girls are sitting downstairs in the lounge with the Latvians crowded round one of their laptops and repeatedly clicking refresh. We manage to fit in Qs 1, 3 & 4 before lunch as well. During coordination for Q4 we attempt to argue a script down by a mark but eventually accept the higher mark as it's consistent with what other candidates have received.

Our slot for Q6 is directly after lunch and I'm feeling quite nervous about the prospect. So far only a couple students have been awarded more than one mark for this question, despite many long discussions, which makes it seem unlikely that our girls will do as well as they hope. It's difficult to know since none of their quite wordy solutions fit into the current mark scheme. Co-ordination resumes and despite having a British co-ordinator on Q6, nobody has provided us with an impartial observer. This is not really necessary because Joseph is not going to be generous. I ask if we can start with Maria's script and one of the coordinators gives me a bemused look back; I imagine that for non-native English speakers scripts like this would have taken even longer to understand. I explain what I think she's done and where I think she's slipped up but it's then pointed out to me that she's made an assumption about the symmetry of the problem at the beginning that becomes false and problematic later on. This feels like a fatal blow and I'm kicking myself for not noticing it earlier. I even remember thinking it was something I should check for and then clearly forgot all about it. I ask for some thinking time and start trying to work out whether she'd have needed any new ideas to fill the gap in her case check. It seems not, but then the coordinators point us to the mark scheme they've drawn up for solutions like this one and explain that to get more than 1 mark, you have to have done something significantly more than proving the specific list of results they've written out. I look at the list and note that this is pretty much a summary of Maria's script. Whilst I feel really disappointed that this script is only going to get 1 mark, arguing is not going to help and the coordinators have far more experience and wisdom in this area than I do anyway. Neither Katya or Lizzy have explicitly considered the cases Maria missed out either so they will also get 1 mark. We shake hands on this and leave to find the girls, who had chosen not to go on the guide-led trip to the centre so that they could wait anxiously for the final updates to the scoreboard.

This was not a particularly good note to end on and so the girls are understandably feeling a bit down. Jo and I reckon that standing around talking about regrets and the 'what ifs' is probably not the most helpful or productive way to spend an afternoon so we encourage some sort of activity. Table tennis is decided on and Jo and I join in. Maria thinks she is unlikely to make the British Ping Pong team and so concentrates on mathematics instead. After a few games of round-the-world-while-doing-star-jumps we set out to the centre and partake of hot chocolate in a cafe overlooking the Ducal Palace. Kasia finds her hot chilli chocolate rather hot and chilli-ish.

In the evening there is a final jury meeting which is largely dedicated to deciding the medal boundaries. As with the IMO, the jury aim to award approximately half of the competitors medals in a G:S:B ratio of 1:2:3. Honourable mentions are awarded to those participants scoring below the Bronze boundary but with full marks on at least one question. The meeting is finished almost as soon as it has begun and the jury have generously voted to award 10 Golds (the only other sensible option being 5 Golds), with a consequent knock on effect on lower boundaries too. As a result Maria has ended up only one mark off the Silver boundary.

After dinner activities include a stroll by the river, sheltering from a hail/thunderstorm, pooh sticks and finally Karaoke.

Day 6 – Excursion and Closing Ceremony

Today all contestants, leaders and various staff take the train up to a small village in the north of Luxembourg called Clervaux. We were asked to indicate a couple of days ago what we'd like to do when we get there so the girls have decided to go on the short guided tour whilst Jo and I, having been assured there's no need to stay with them, have opted for the hike. When we arrive it's clear that moving groups is perfectly possible and Joseph decides he too would prefer a hike, though he's a little inappropriately kitted out with his laptop and pile of papers in a shoulder bag that he'd brought just in case he had a chance to do some work.

The hike turns out to be a proper hike; a few hours of lovely trails up and down hills and through varied scenery and even with a steep downhill off-trail section through woodland which Joseph is not so enthusiastic about given his extra baggage, though he manages to remain upright. On the way back we stop off briefly at the Abbey. Despite its prominent position on top of a hill and striking clock tower the inside is rather humble and there's a nice little exhibition in the basement. Jo kindly translates for me as my French is not up to scratch.

The girls have texted to inform us that Lizzy has had a terrible accident and turned into a zombie and Maria has gone missing so, deciding they may be a little bored, we head down to town without waiting for the rest of the hiking group. It turns out that whilst Clervaux is a very pretty little town there's not much to do once you've done the tour of all the main sites, but we manage to amuse ourselves until it's time to catch the train back.

As we arrive back in Luxembourg City Jo comes up with the excellent idea of getting ready for the closing ceremony before everyone else so that we have time for a cup of tea before heading out again. The girls take this quite seriously and we end up running the two kilometres back across the city, though the Hungarians still mysteriously beat us to the youth hostel. It's two bus rides to get from the youth hostel to the campus where the ceremony is being held so Jo, in another stroke of genius, decides to wear her trainers for the journey and pack her posh shoes in her bag. Unfortunately she forgets the packing-the-shoes element of this cunning plan and ends up spending the evening in a posh dress and trainers. This becomes even funnier when we hear that the Luxembourgish Prime

Minister and his wife will be attending the buffet dinner. After dinner there is the medal and certificate presentation ceremony followed by some brief speeches and thank yous. It's a very enjoyable evening and the girls top it off by encouraging all the contestants up on stage to dance the macarena. Geoff's not as impressed by their initiative-taking as I am and expresses concern over where UK mathematics may be heading.

Day 7 – Departure

Alas, it's time to head home. Our flight isn't until after lunch so we have some time to spare at the youth hostel before leaving. Though the weather has been drizzly all week this morning it's typically fairly pleasant so we sit outside chatting. The girls remember that no one has formally thanked Joseph for all the hard work he put into running the IT systems, getting all the papers printed, etc. and so decide to make him a certificate. We hold a little ceremony in which I am given the honour of presenting him with his award. This is indeed an honour as Joseph has done an excellent job behind the scenes throughout the week.

The journey goes smoothly apart from a minor delay due to some problems with the autopilot system and we arrive back in the UK safely and head our separate ways.

Thanks

EGMO 2013 has been a wonderful experience and I am very thankful for the hard work lots of people have put in to bring it all together and make it such a great week for all involved.

Thanks firstly to the UK team who were a pleasure to take away for the week, kept in good spirits even after 6am starts, have been an encouragement to one another and have worked hard over the past few months. Thank you also to Jo who has been great company and a huge help to me, especially with my inexperience, and to Geoff and Joseph whose wisdom I have been grateful to be able to draw on during the competition.

Thank you to Bev Detoef and the UKMT office for making sure all the organisation ran smoothly and particularly for quickly answering all my last minute questions and requests before we left!

Lastly, thank you and congratulations to Mike Dostert and the team of local organisers in Luxembourg. Although this competition was organised on a tight budget it was a huge success. We hope that his example might encourage other countries who are thinking of hosting a future EGMO to do so. Thanks should also extend to those who worked hard over the previous couple of years to bring to fruition the vision of EGMO. Long may its legacy continue!

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