Romanian Masters of Mathematics 2013

UK Leader's Report

Bucharest, Romania; 27th February – 3rd March 2013

Introduction

The UK was delighted to be invited to send a team to the Sixth edition of the Romanian Master of Mathematics contest, held in the Tudor Vianu National High School for Computer Science between the 27th February and 3rd March. The contest has become a regular part of the mathematical competition calendar, and is designed to be a hard competition for countries that perform strongly at the International Mathematical Olympiad.

This year, the team consisted of:

Andrew Carlotti	Sir Roger Manwood's School
Gabriel Gendler	Queen Elizabeth's School
Daniel Hu	City of London School
Sahl Khan	St Paul's School
Warren Li	Fulford School
Matei Mandache	Loughborough Grammar School

I (Jonathan Lee, University of Cambridge) was the team leader; Beverley Detoeuf of UKMT was the deputy leader, and Dan Schwarz (Romania) was an observer for the UK.

Questions

Paper 1

1. For a positive integer a, define a sequence of integers $x_1, x_2, \ldots$ by letting $x_1 = a$ and $x_{n+1} = 2x_n + 1$ for $n \geq 1$. Let $y_n = 2^{x_n} - 1$. Determine the largest possible k such that, for some positive integer a, the numbers $y_1, \ldots, y_k$ are all prime.

(Russia) Valery Senderov

2. Does there exist a pair (g, h) of functions $g, h: \mathbb{R} \to \mathbb{R}$ such that the only function $f: \mathbb{R} \to \mathbb{R}$ satisfying $f(g(x)) = g(f(x))$ and $f(h(x)) = h(f(x))$ for all $x \in \mathbb{R}$ is the identity function $f(x) \equiv x$?

(United Kingdom) Alexander Betts

3. Let $ABCD$ be a quadrilateral inscribed in a circle ω. The lines AB and CD meet at P, the lines AD and BC meet at Q, and the diagonals AC and BD meet at R. Let M be the midpoint of the segment PQ, and let K be the common point of the segment MR and the circle ω. Prove that the circumcircle of the triangle KPQ and ω are tangent to one another.

(Russia) Medeubek Kungozhin

Paper 2

4. Let P and P' be two convex quadrilateral regions in the plane (regions contain their boundary). Let them intersect, with O a point in the intersection. Suppose that for every line ℓ through O the segment $\ell \cap P$ is strictly longer than the segment $\ell \cap P'$. Is it possible that the ratio of the area of P' to the area of P is greater than 1.9?

(Bulgaria)

5. Given an integer $k \geq 2$, set $a_1 = 1$ and, for every integer $n \geq 2$, let a_n be the smallest $x > a_{n-1}$ such that:

$$x = 1 + \sum_{i=1}^{n-1} \left\lfloor \sqrt[k]{\frac{x}{a_i}} \right\rfloor.$$

Prove that every prime occurs in the sequence $a_1, a_2, \ldots$.

(Bulgaria)

6. $2n$ distinct tokens are placed at the vertices of a regular $2n$-gon, with one token placed at each vertex. A *move* consists of choosing an edge of the $2n$-gon and interchanging the two tokens at the endpoints of that edge. Suppose that after a finite number of moves, every pair of tokens have been interchanged exactly once. Prove that some edge has never been chosen.

(Russia) Alexander Gribalko

Results

The scores of the UK team were as follows:

	Q1	Q2	Q3	Q4	Q5	Q6	Total	
Andrew Carlotti	0	7	0	7	7	7	28	Silver Medal
Gabriel Gendler	0	0	0	7	0	7	14	Honourable Mention
Daniel Hu	2	7	7	7	7	4	34	Gold Medal
Sahl Khan	7	0	0	1	7	0	15	Honourable Mention
Warren Li	5	0	0	6	7	0	18	Bronze Medal
Matei Mandache	7	7	0	7	7	7	35	Gold Medal

The team scores were as follows:

Rank	Country	Score
1	USA	105
2	Russia	101
3	United Kingdom	97
4	Romania	84
5	Italy	83
6	Ukraine	80
7	Hungary	74
8	Brazil	63
9	Poland	61
10	Romania B	58
11	Bulgaria	55
12	Serbia	54
13	China	53
14=	Tudor Vianu	23
14=	Romania F	23

Leader's Diary

Wednesday 27th February

My alarm wakes me at 4:30, for the bus from Cambridge to Luton. Once there, I meet deputy Bev Detoeuf and the most of the team (Gabriel Gendler is coming out a day later). I'm looking forward to returning to Bucharest, having last been there as a competitor in the 2008 RMM. We leave Luton without incident and arrive in Bucharest Baneasa, where we are met by Matei's grandparents. They insist upon giving us *covrigi* (a traditional pretzel-bread) and seeing us on our way.

We are found by a representative of the Romanian Masters, who informs us that we're being transported to the contest with the American team, who are coming in just after us. The accommodation has now been stable for several years, with the students at Camin Moxa and the leaders

at a nearby hotel.

I drop off my things at the hotel and proceed to dinner with the other leaders. We receive memory sticks containing the papers and directions to meet the next morning. As a group we then retire to examine the proposed papers and shortlisted alternative questions. I think that the paper is really solid, and note that once again Lex Betts has stepped up and provided another excellent question.

Thursday 28th February

I get up early and break fast; Bev meets me there and we compare notes. Between us we piece together a rough schedule for the next couple of days. The UK's penchant for keeping the deputy with the students for pastoral purposes seems to have caused a little confusion about whether she is accommodated with the leaders or the students, but this has been resolved without incident.

The paper itself is essentially a fait accompli; the questions are nice and are to be changed only if they require mathematics outside of the secondary syllabi. We are solemnly reminded that the questions are intended to be challenging, and that the contest is intended to prove the top IMO teams. It's suggested that next year we may be welcoming Germany, Japan and South Korea to the contest. Ilya Bogdanov of Russia ensures that everything proceeds at a fair pace, and prevents the Jury from straying too far from the task at hand.

It is pointed out that Question 1 follows from a 1750 conjecture of Euler, which was shown by Lagrange in 1770. After some consideration, the feeling is that since we are not setting the paper to either Euler or Lagrange, the prior art is permissible. We agree that all of the questions should be taken forward, and after further discussion resolve that the mathematical details of the questions should not be altered.

The English Language Subcommittee then forms, consisting of those who offer substantive opinions on the paper as written. Lots of issues are raised with the language of Questions 2, 4 and 6, as they have terms that must be unambiguous and easy to translate. The Brazilians offer many helpful suggestions to ease translations, whilst the Hungarians and Americans help to hammer out phrasing that will precisely convey all the details. Final approval of the papers is delayed until after the opening ceremony.

The opening ceremony itself is unusually short on traditional music, instead providing a varied talent show. Speeches are given by members of the educational establishment, the city and the school, and overall the ceremony did not run overlong. In a surprise development, the Romanian

Mathematical Society introduces a new award, the "Diploma of Excellency", which is awarded for services to olympiad mathematics. The first three are awarded to Zuming Feng, József Pelikán and Geoff Smith. Sadly Geoff is not present, and so after the ceremony I go to collect it. Needless to say, this causes some confusion, as I am as plausibly a student as a leader. I am sure that Geoff's long series of awards, from two IMO Golden Microphones to an MBE for services to education, have now been sublimely completed.

The leaders then retired to endorse the English copy and translate it for individual teams.

Friday 1ˢᵗ March

We rise early and congregate in Tudor Vianu for the first day of the contest. Plenty of questions come in the first half hour, but sadly only one from the UNK's, which appears to be an excuse for Gabriel to say "Hi". Once the time for questions is over, we agree on the mark scheme. Again this is primarily a construction of the Problem Selection Committee, and it is a testament to the care with which they worked that there are so few modifications proposed. With this settled, some of the leaders go on an excursion to a local museum of folklore. I sadly have to catch up on email and work.

We are given the list of coordination times. Since the UK has proposed Question 2, I understand it to be necessary that I coordinate all four Romanian teams for the question. Fortunately Dan Schwarz is in reserve as a secret Romanian-reading weapon if needed. Unsurprisingly, it also transpires that we'll need to be in multiple places at once, as the assignment of teams to coordination times has failed to produce an injection for the union of UNK and Romanian Question 2's.

I meet Dan, and we speak briefly to the team to get a sense of how they did. Dan and I sneak out to lunch at a local Lebanese restaurant, before retiring to the hotel and beginning to mark the scripts. We have one solution to the geometry question, which is a comparatively nice trigbash from Daniel Hu. For my previous crimes against geometry I get to go over it later with more coffee; it seems correct. We have three complete solutions to Question 2, and three near complete solutions to Question 1. Helpfully, the students reported that they solved precisely the questions which they have.

As a team our question preferences don't seem to have changed much from my time; there seems to be a distinct avoidance of geometry and a preference for nominally harder questions with a combinatorial or algebraic flavour over number theory. Clearly we have been consorting

with the Hungarians too much. Amongst the leaders, discussion turns to Question 2; several students have found beautiful analytic functions solving the question, whereas the solutions we knew of were constructed less directly.

Saturday 2nd March

Today we rise an hour earlier to ease the schedule at the end of the day, so we congregate at Tudor Vianu with coffee and laptops to await questions. It becomes apparent from early questions that the Chinese team do not have the same Question 6 as everyone else, and a new translation is emitted post haste. We then settle down to coordinate the first day's problems.

There's a short back and forth over two marks for Daniel Hu's work on Question 1, though it is resolved by comparison with the mark scheme. Dan is invaluable in speeding this up, as it means that anything complicated can be discussed in English or Romanian to ensure that our view is conveyed correctly. At this stage, I've not seen the Romanian Question 2 scripts, and when I do get them I doubt my ability to mark 24 scripts in Romanian in 40 minutes. Dan very helpfully points out that I need only advise and have input on the coordination, rather than attempt to sort them out from scratch. This makes me much happier. Romanian coordination is efficient and scrupulously fair, as I had seen at EGMO; I strongly approve.

We see the students again to discuss the second day; Andrew Carlotti and Gabriel Gendler are both looking much happier, which is good. There's enthusiasm about both Questions 4 and 5, and several claimed solutions to 6. Sadly Dan has to retire early, so I go through the scripts in detail. By and large they are straightforward, but Daniel Hu's work on Question 6 takes us well off the mark scheme, which is a portent of trouble. We've also done well on Questions 4 and 5, though some other leaders are starting to keep their speculation about team performances close to their chest, so comparative numbers are hard to come by.

We are then taken to the farewell banquet, where the Leaders by and large let loose about the comparative woes of the education systems of their assorted countries. Needless to say, we also had to walk uphill to school both ways. Once we return, I settle down with coffee to resolve the matter of Hu's script, and determine that it is probably worth 4.

Sunday 3rd March

The leaders come in to coordinate the second day's questions. Again, the coordination is excellent. There's a really careful approach to finding

and penalising errors, so the discussion is entirely about the mathematics and whether the students have actually established the truth of the propositions. We pick up one mark from the coordinators being less punitive that I had been, and then I settle down with Ilya to coordinate Question 6.

Daniel's partial solution is the only really contentious issue, as it is incomplete and inductive, with no inductive solutions known to the coordinators at that stage. We break off the coordination so that I can more formally justify the claim that the script can be made into a complete solution without new ideas in a few lines. It eventually gets 4, by analogy to the mark scheme.

It becomes apparent that Matei Mandache and Daniel Hu are both in the running for Gold medals. Dan and I begin trying to count the number of scores ≥ 35 that are plausible. We establish that 34 should be a gold based on a 1:2:3 split of medals with 45 medals awarded between 90 contestants. It also seems that we're probably third overall, though we can't be certain; the team rankings are based on the total score of the top three candidates from each team, and there aren't many near-golds around.

Some time later, we get the list of awards, and trivial inspection reveals that there are rather more golds that we had expected. It transpires that the medals have been made in ratio 2:3:4, and are being awarded on that basis; thankfully these numbers match with actual mark boundaries. As is the nature of RMM this is presented as more of a fait accompli than as a matter for the jury. None of our medals vary under this change. Some rapid addition confirms that we are third overall, which is an excellent performance and a credit to the team.

We are then treated to the closing ceremony, which is even shorter on music than the opening ceremony. We disgorge from Tudor Vianu to find Matei's grandparents, who congratulate the team and take many photographs. We then proceed with some of the guides to see a little of Bucharest, which ultimately seems to cash out as finding coffee, drinking it and talking mathematics. C'est la vie.

Monday 4th March

We rise at 0300 GMT, or as it is affectionately known, "too early". We are again paired with the Americans for travel, so I find their leader and head to the bus that will take us to the students and thence the airport. The UK students appear through the gloom, but the Americans are nowhere to be found. It transpires that they cannot find their keys to return. After a short search, this is remedied and we head to the airport.

We are also the recipients of further charity by Matei's grandparents, who have insisted on providing food for us for the return trip. We reach Baneasa, and migrate to the flight without incident.

Once back in the UK, there are issues with the UK Border Agency, whose ontology apparently does not contain "international mathematics contest", and so they are confused by a group of apparently under 18 year olds without an obvious school affiliation coming through the border. Once we have established that we are not, in fact, attempting to smuggle mathematicians into the country, we are permitted to reenter. We then go our separate ways, and so far as I am aware there are no further incidents.

Conclusion

Thanks and congratulations are due to many people:

- Sever Moldoveanu and others at Tudor Vianu who worked so hard to make this contest a success;

- The Problem Selection Committee, for producing an excellent paper and mark scheme;

- Bev Detoeuf, both for her work at UKMT to enable us to go to the contest, and furthermore for coming out and ensuring that the students stayed out of any trouble;

- Dan Schwarz, for going out of his way to make things easier and helping so much;

- The guides, especially Andrei, Silvia, Ioana and Dan, for making Bucharest a warm and welcoming place to be for us all;

- The students' schools, for letting them out for a few days to compete;

- The many teachers and volunteers for UKMT, who allow us to train and prepare for these contests;

- Andrew, Gabriel, Daniel, Sahl, Warren and Matei, for working hard and being excellent company.

Jonathan Lee, (jdl43@cam.ac.uk), 1st April 2013.