

# Report on the 3rd European Girls' Mathematical Olympiad

## 10th – 16th April 2014

### Antalya, Turkey

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## Introduction

The European Girls' Mathematical Olympiad is now in its third year. The competition aims to give a large number of female students the experience of competing in an international event and hence encourage the participation of girls in olympiad mathematics. Having been based in previous years in Murray Edwards College, Cambridge, and a Youth Hostel in Luxembourg City this third edition was located in a 5\* all-inclusive hotel on the Turkish Riviera! The larger budget than in previous years allowed the inclusion of 29 teams (an increase on last year's 22), with 7 of those being entries from guest countries. The selection for the UK EGMO team was made from performances at BMO2, the second round of the British Mathematical Olympiad.

The team consisted of:

|      |                 |                                     |
|------|-----------------|-------------------------------------|
| UNK1 | Olivia Aaronson | St Paul's Girls' School             |
| UNK2 | Katya Richards  | School of St Helen and St Katharine |
| UNK3 | Eloise Thuey    | Caistor Grammar School              |
| UNK4 | Kasia Warburton | Reigate Grammar School              |

The reserve was Alyssa Dayan of Westminster School. The Team Leader was Hannah Roberts of Pembroke College, Oxford and the Deputy Leader was Jo Harbour of Mayfield Primary School, Cambridge.

## The Competition

The format of the EGMO is much like that of the International Mathematical Olympiad (IMO). Two  $4\frac{1}{2}$  hour papers each containing three questions (ordered by approximate difficulty) are sat over two days. Each question carries 7 marks, with part marks given only for significant progress, such that it is usually rare to be awarded 3 or 4. The scores for the UK team were as follows:

|                 | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Total | Medal        |
|-----------------|----|----|----|----|----|----|-------|--------------|
| Olivia Aaronson | 0  | 0  | 0  | 2  | 2  | 0  | 4     |              |
| Katya Richards  | 6  | 0  | 0  | 7  | 7  | 0  | 20    | Silver Medal |
| Eloise Thuey    | 2  | 2  | 0  | 1  | 0  | 1  | 6     |              |
| Kasia Warburton | 6  | 0  | 2  | 7  | 3  | 0  | 18    | Silver Medal |

The medal boundaries were 24 for gold, 16 for silver and 7 for bronze. The UK came 8th out of 29 participating countries in the unofficial team competition, our best result yet. Paper 1 was particularly hard this year and many students scored no marks on the first day. Hence the medal boundaries reflected this. Even so, the papers contained a great set of questions, included below.

## The Problems

### Day 1

**Problem 1.** Determine all real constants  $t$  such that whenever  $a, b, c$  are the lengths of the sides of a triangle, then so are  $a^2 + bct$ ,  $b^2 + cat$ ,  $c^2 + abt$ .

**Problem 2.** Let  $D$  and  $E$  be points in the interiors of sides  $AB$  and  $AC$ , respectively, of a triangle  $ABC$ , such that  $DB = BC = CE$ . Let the lines  $CD$  and  $BE$  meet at  $F$ . Prove that the incentre  $I$  of triangle  $ABC$ , the orthocentre  $H$  of triangle  $DEF$  and the midpoint  $M$  of the arc  $BAC$  of the circumcircle of triangle  $ABC$  are collinear.

**Problem 3.** We denote the number of positive divisors of a positive integer  $m$  by  $d(m)$  and the number of distinct prime divisors of  $m$  by  $\omega(m)$ . Let  $k$  be a positive integer. Prove that there exist infinitely many positive integers  $n$  such that  $\omega(n) = k$  and  $d(n)$  does not divide  $d(a^2 + b^2)$  for any positive integers  $a, b$  satisfying  $a + b = n$ .

### Day 2

**Problem 4.** Determine all integers  $n \geq 2$  for which there exist integers  $x_1, x_2, \dots, x_{n-1}$  satisfying the condition that if  $0 < i < n$ ,  $0 < j < n$ ,  $i \neq j$  and  $n$  divides  $2i + j$ , then  $x_i < x_j$ .

**Problem 5.** Let  $n$  be a positive integer. We have  $n$  boxes where each box contains a non-negative number of pebbles. In each move we are allowed to take two pebbles from a box we choose, throw away one of the pebbles and put the other pebble in another box we choose. An initial configuration of pebbles is called *solvable* if it is possible to reach a configuration with no empty box, in a finite (possibly zero) number of moves. Determine all initial configurations of pebbles which are not solvable, but become solvable when an additional pebble is added to a box, no matter which box is chosen.

**Problem 6.** Determine all functions  $f: \mathbb{R} \rightarrow \mathbb{R}$  satisfying the condition

$$f(y^2 + 2xf(y) + f(x)^2) = (y + f(x))(x + f(y))$$

for all real numbers  $x$  and  $y$ .

## Leader's Diary<sup>1</sup>

### Day 1: Travelling

Having met the team briefly at Trinity Training camp a few days previously we meet again for only the second time at Gatwick South Terminal. Eloise is the only one of us who's had to get up early, having made the intrepid journey down from The North. Thanks to regular prompts from the two experienced members of the team (Kasia and Katya), I have this year remembered to pack a UK flag. The girls have brought along turquoise nail varnish to match our team t-shirts so, no last minute airport shopping is required and we're all set to travel! The UK contingent also includes Geoff Smith (EGMO Advisory Board) and family, and Joseph Myers (IT man).

The plane journey is not without event – about an hour before landing Jo very suddenly starts feeling faint and nauseous. Thankfully a lovely flight attendant called Charlotte is close at hand and she deals with the situation superbly; after a few minutes Jo is supplied with Oxygen and this seems to do the trick. Charlotte tells us that the Oxygen is required almost every day, though usually by people who haven't eaten breakfast.

It's a short bus ride from the airport to the hotel and when we arrive our guide, Serpil, is waiting to greet us. The Turkish hosts have put together huge goody bags for us including EGMO-branded mugs, USB sticks, and Jo's favourite, Turkish delight. It's quite late, Jo still isn't feeling great and we haven't yet managed to have dinner, so this combination of factors makes the lengthy hotel check-in process (which we can't really understand due to not speaking Turkish) a little stressful. But we do get to the hotel buffet before closing time and then head off to bed.

### Day 2: Paper preparation day

On day 2 the Jury (which is made up of all the team leaders from the various countries) must approve the papers and then translate them into all the required languages. When it comes to the approval of the paper we are shown the questions but given very little extra information about them and only a couple of minutes to read them, so the purpose is mainly to check that none of them are immediately recognisable as known problems. Afterwards a small English Language Committee is set up to work on finalising the English Language version of the paper. Here, as well as correcting all the American spellings, our job is to leave as little room as possible for misunderstanding the questions.

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<sup>1</sup>For a more entertaining account of events from a different perspective, read the team's unofficial report.

We're also trying to make it easy to translate and avoid lots of foreseeable clarification questions being asked by the students. We struggle a little with the very wordy pebble question (Q5) but otherwise there are no issues.

Mid morning it's time for the Opening Ceremony. This is a combination of welcoming speeches from the Organisers and other officials, the introduction and photographing of all the teams and, wonderfully, a performance of traditional Turkish dancing (no opening ceremony is really complete without traditional dancing)! It is clear that the Turkish organisers have taken a no expense spared approach to hosting EGMO: the entire length of the stage is decorated with a garland of (real) flowers and there are printed EGMO banners, complete with mathematical-symbol-designs, hanging either side of the room and around the hotel. We were also supplied with colour-coded EGMO polo shirts in our goody bags; pink for contestants, bright blue for deputies, dark blue for leaders etc. Unfortunately it turns out that the UK team t-shirts that we brought with us are exactly the same bright blue colour as the t-shirts for deputies. This is a recipe for much confusion. Since leaders (who have now seen the paper) are separated from their teams for the opening ceremony, it's visually obvious that I'm either in the wrong place or wearing the wrong t-shirt. A group of yellow-clad guides approach me and try to work out what's gone wrong. I try to explain that I really haven't switched my T-shirt with anyone and keep pointing to the UKMT logo, but my gesturing skills clearly need work and it seems easiest to just agree to go and change after the opening ceremony. It wasn't until much later in the week that our poor guide finally understood why all her team had been wearing the 'wrong' t-shirts.

In the afternoon we all have some free time whilst the paper is being translated. Activities have been put on for the girls so they go off to try out rifle shooting whilst Jo and I enjoy the indoor swimming pool (the weather is a little miserable). Today is also the only time leaders have to try out the paper before being given solutions and mark schemes in the morning, so I disappear off to attempt the problems for a bit before dinner. I don't get very far with any of them but during the evening, whilst looking over the different language versions with other leaders, discover that I'm not the only one who's been finding it hard.

### **Day 3: Paper 1, the sun comes out!**

The girls have been told that they need to meet 45 minutes before the exam time. Our guide has learnt that they're not always the most punctual group, but the process of getting everyone seated in the exam hall does actually end up taking half an hour. Each team is briefed in turn before being shown individually to their seats. Across the hall in the Jury room we have a lengthy discussion over the mark scheme for Problem 1. Part of the issue seems to be that whilst the general consensus among the jury has settled on a couple of minor changes being made to the original mark scheme that was proposed by the coordinators, there are differences of opinion on how we should vote to approve these changes. It is possible that tensions are running a little higher than usual since many leaders are aware that their students will be finding this paper much harder than they were expecting (I know that this is concerning me).

Indeed, the girls haven't found the exam very easy, three of them think they may have done question 1 and Olivia is full of adrenaline having also tried to scribble down a solution to question 3 in the last minutes, but Eloise has not enjoyed it at all. Thankfully there is the prospect of a sunny afternoon spent in the pool to lift everyone's spirits. Whilst the girls play frisbee and try out the water slides Jo and I go for a long walk along the beach. Neither of us have ever been anywhere quite like this before – it's a seemingly never ending stretch of beach-front hotel complexes. The view of the Mediterranean Sea and the backdrop of snowcapped mountains still makes for a lovely walk though.

Dinner is great as usual, in fact perhaps now would be good time to mention the food at this place. It's amazing – there's so much choice and it's all delicious, if sometimes a little mysterious. The dessert selection is particularly good, mainly consisting of baklava and wobbly creme-caramel type puddings along with plenty of fruit. Jo and I decide to implement a '3 desserts per meal' rule (mainly to aid our own self-control), though Jo has managed to get round this today by eating 2 lunches. Sneaky.

## **Day 4: Paper 2**

The girls are understandably nervous before exam day 2. Jo and I have been trying to remind them that we want them to enjoy the experience of EGMO rather than seeing it as a competition where they need to prove themselves, but I'm not sure how much that has helped.

The jury meeting is much faster today and the organisers have planned for all leaders, deputies and other academic staff to go on an outing for the afternoon. After a lovely outdoor lunch (with even tastier baklava than that at the hotel buffet) and a short trip to a waterfall our outing culminates in a visit to the old town in Antalya. We have an hour or so to wander round so Jo, Joseph and I pick up a map (of the more illustrative kind) from the tourist office and decide on a route which will roughly take us round the outside of the old town towards Hadrian's gate. The Polish leader and deputy join us too. After a short period of pleasant exploration we get a little stuck on the navigation front and eventually approach a couple of men sitting on the edge of a public square. They are very friendly but don't seem keen to engage with our map reading problem, instead they simply tell us that we've reached 'the best ceramics shop in town'. This is good to know, but not quite what we were looking for. After some more joking around they point us in the right direction. Hadrian's gate turns out to be a little disappointing – a couple of thousand years old and intricately carved but also covered in graffiti. We make sure to stop by the best ceramics shop in town on our way back to the coach though; it is indeed a very lovely shop.

The girls seem to have had a good time while we were out. Kasia's speedy swimming and Jungle Speed training the previous night have resulted in her being victorious in a game involving fetching spoons from the bottom of a swimming pool.

One small disadvantage of being out during the day is that we now have a lot of marking to do before coordination in the morning, including Question 1 scripts from

Turkey A and B.<sup>2</sup> Jo has a cunning method for speedily getting an idea of how much these are worth, and it only takes her a few minutes to scan over them for the presence of a 2 or 2/3. Some of the girls have not had time in the exam to write out neat versions of their solutions but despite this their workings are generally clear and easy to allocate marks to, so none of the scripts cause us a headache, in fact there are some really nice solutions in there. Unfortunately Olivia has had a particularly bad first day, having misunderstood problems 1 and 3, and Eloise, having been thrown by the difficulty of day 1 and panicked by the lack of geometry on day 2 has not had a good second day. Both have made substantial progress on other questions though, for which they should get well-deserved marks.

We head to bed shortly before midnight. The Ukrainian leaders and their team, who have been sitting opposite us all evening, are still up and hard at work.

## Day 5: Coordination

It's coordination day! We have 20 minute appointments with pairs of coordinators throughout the day, one for each question, plus observing the Turkish team's coordinations to guard against foul play. Since two of the problems on the papers were submissions from the Netherlands, the Dutch leader and deputy have a very busy day ahead, and had to miss out on yesterday's excursion to get their marking done. The girls are off on an excursion today, which is definitely a good thing since it means they will be less tempted or able to obsessively check the live scoreboard!

Once things get underway it's clear that problems 4 and 5 are causing the most issues, with coordinations regularly taking at least an hour. Most of our coordinations are fairly straightforward, but coordinating with the Turkish teams means we get to witness others' coordination styles. Not everyone adheres to the 'what the markscheme says goes' attitude as much as Jo and I do, leading to long arguments ending in exactly the same result. Perhaps we're just not pushy enough though. We are last on the schedule for coordinating question 4. We have heard rumours that the coordinators for this problem are quite strict and unwilling to give away part marks, leaving many teams coming away with lower scores than they had expected. Thankfully we agree immediately on the scores for the first three scripts. When it comes to Kasia's script I suggest a 6 since her solution is essentially complete but there are a couple of areas that could do with some more explanation. The lead coordinator expresses his disagreement so I start panicking a little and going through where I think she's dealt with each bit of the proof. He interrupts though to tell us that actually they think it is worth 7, and suggest we take the 7 without any further discussion. Of course we oblige and all head to dinner.

After dinner there's a Jury meeting to decide the medals. There are no major issues here, though there is a lot of discussion around deciding the Bronze boundary given that the possible values are 6–8 and one mark is quite a significant difference in that range.

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<sup>2</sup>The convention is that scripts from the host country are also looked at by the leaders of the country that proposed the problem. Problem 1 is a UK submission.

There is also no agreed method for how we should vote on the boundaries when there is more than one possible option, so it seems many people leave having not had the opportunity to vote for their favourite set.

Olivia is the one member of the team who will still be eligible to compete next year, so she asks if we can go over her scripts together and we spend some time doing that before bed. For the 5th night in a row the girls decline the invitation to join the disco in the basement, much to the disappointment of our guide.

## **Day 6: Closing celebrations**

Jo and I get up a bit early to go for a run along the beach. We did invite the girls but none of them seemed even the least bit tempted. Despite running in the same direction for half an hour we still can't see the end of all-inclusive-hotel-land. Post-run we eat our last breakfast in the restaurant. No more pancakes or freshly-baked pastries for us sadly, except the ones the girls smuggled away with them for the trip tomorrow. The morning is spent chilling in and by the pool, though Joseph and I also spend a fair amount of time battling with the online check-in system. At 12.30pm we eat our last lunch in the restaurant. Many tears are shed as we enjoy eating the freshly-baked flatbread for the last time. The whole team seem in good spirits for the closing ceremony, which is lovely, and we cheer on Kasia and Katya as they receive their silver medals with the rather dramatic Pirates of the Caribbean theme tune blaring in the background.

For our evening celebration a boat trip has been organised. In summary, this turns out to be a full on boat party. We leave the marina on the far side of Antalya in the early evening and over dinner travel across the bay to a point where a waterfall cascades over the cliffs into the sea. On the return journey the second floor of the boat turns into a disco and we pause in the middle of the bay for a long time to allow the party to continue. The top deck, as well as being slightly quieter, also has a great view of the twinkling coastline. Despite all the fun of being on a boat, bed seems like a very attractive but unattainable option right now, particularly since we know we have to get up at 6 to go to the airport. We are allowed a few hours sleep though and the return journey back to the UK is surprisingly smooth – no one requires any medical attention. A success!

## **Concluding remarks**

It is wonderful to have the opportunity to take a group of girls to compete at EGMO. The experience clearly has some resoundingly positive aspects – the girls actively throw themselves in to meeting and socialising with as many different teams as possible. They also really benefit from and are an invaluable presence at UK training camps. There are some harder aspects too though; I think girls have more of a tendency than boys to compare their performance with others, worry about what others think, and question whether they deserve to be on the team. This can add a character building aspect to the experience as well, but I guess that's not necessarily a bad thing, and I hope that the net effect on the girls' enjoyment of mathematics is still a positive one.

Many thanks are due: Thanks to the girls for being great company and such a pleasure to take away; to Jo for her companionship, for looking after us all and not even coming close to losing our passports; to the organisers for hosting an excellent EGMO with great attention to detail; to Geoff and Joseph for all of their hard work and sensible comments in Jury meetings ensuring the smooth running of the competition; to our guide, who was very diligent in her care of the girls; to Beverley Detoef and all those in the UKMT office for arranging our travel, visas, t-shirts and much more; and, lastly but importantly, to all those girls who took part in the UKMOG and BMO this year – we hope these competitions will continue to widen the participation of girls in Olympiad Mathematics.