Balkan Mathematical Olympiad 2015, Leader's report

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Summary, observations and thanks

This is the UK/ROI team leader's report on the 32nd Balkan Mathematical Olympiad, held in May 2015 in Athens. Our team comprised five British students (Alex Harris, Lawrence Hollom, Sam Kittle, Kirsten Land and Philip Peters) and one Irish student (Luke Gardiner). The team was accompanied by three adult volunteers: myself (leader); Gerry Leversha (deputy leader); and Jill Parker (pastoral). The competition consisted of a single four-and-a-half hour exam paper comprising four questions, one from each of the standard olympiad areas of algebra, geometry, combinatorics and number theory. The questions were as follows.

1. Let a, b and c be positive real numbers. Prove that

$$a^3b^6 + b^3c^6 + c^3a^6 + 3a^3b^3c^3 \geq abc(a^3b^3 + b^3c^3 + c^3a^3) + a^2b^2c^2(a^3 + b^3 + c^3)$$

2. Let ABC be a scalene triangle with incentre I and circumcircle (ω). The lines AI, BI, CI intersect (ω) for the second time at the points D, E, F, respectively. The lines through I parallel to the sides BC, AC, AB intersect the lines EF, DF, DE at the points K, L, M, respectively. Prove that K, L, M are collinear.

3. A jury of 3366 film critics are judging the Oscars. Each critic makes a single vote for his favourite actor, and a single vote for his favourite actress. It turns out that for every integer $n \in \{1, 2, \ldots, 100\}$ there is an actor or actress who has been voted for exactly n times. Show that there are two critics who voted for the same actor and for the same actress.

4. Prove that among any 20 consecutive positive integers there exists an integer d such that for each positive integer n we have the inequality

$$n\sqrt{d}\{n\sqrt{d}\} > \frac{5}{2}$$

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where $\{x\}$ denotes the fractional part of the real number x. The fractional part of a real number x is x minus the greatest integer less than or equal to x.

The students performed well, especially given that for half of the students this was their first international competition. All in all, we obtained four bronze medals, mainly due to a solid performance on the first two questions. The precise breakdown follows.

	Q1	Q2	Q3	Q4	Total	Result
Luke Gardiner	3	10	–	–	13	Bronze
Alex Harris	10	10	0	0	20	Bronze
Lawrence Hollom	10	1	0	1	12	Bronze
Sam Kittle	0	–	2	0	2	–
Kirsten Land	0	1	0	–	1	–
Philip Peters	10	10	0	0	20	Bronze

We are greatly indebted to everyone who made the Balkan Olympiad a great success this year. Our particular thanks go to the chair of the jury, Prof. Theodoros Bolis, and the chair of the problem selection committee, Prof. Anargiros Fellouris, along with all of the other organisers and guides who contributed to the smooth running of the event. Of course, we are also grateful to everyone on the British and Irish side who helped train the students and foster their enthusiasm for mathematics. These volunteers and family members are too numerous to mention by name, but we would like to single out Bev Detoeuf of the UKMT with all her help with the logistics for the competition. Many thanks to you all.

Leader's diary

Disclaimer: The leader's diary has two main purposes: to give everyone an insight into what goes on behind the scenes in international maths competitions; and in some small part to amuse. In the cause of this, undue attention may have been placed on the various mishaps that occurred – it should be understood that I found the competition in general very pleasant, and don't mean to imply otherwise.

May 3rd

As planned, we meet at Heathrow Terminal 2 ("The Queen's Terminal") at 10:30, and the students say goodbye to their parents before receiving their ever-important UKMT shirts. Having checked in online, we are directed to the self check-in desks, where we obtain duplicate boarding passes, and thence proceed to security.

We have left plenty of time to resolve any problems that come up before our flight, and are not disappointed. Luke has not been allowed to take a pair of compasses on his ultra-high security flight from Ireland, and the airport staff aren't impressed with "Gerald" Leversha's attempts to impersonate the "Gerry" Leversha that has been booked in. Additionally, there has been a mix-up with the registration fees, and Gerry has to step in temporarily, withdrawing enough cash to prop up the Greek economy during our trip. Despite the delays, we still clear security in good time and board our flight to Athens.

After an uneventful flight, we arrive in Athens airport and I say farewell to the students as I am whisked off to the leaders' hotel for the business of selecting the papers. I am still a little early for dinner, so I hang around watching the sea and chatting to the chair of the jury, Theodoros Bolis, who is a very amiable chap. After dinner, we proceed to the jury room, complete with disco ball, and are presented with copies of the problems. These are generally of a very high standard, and I stay awake late into the morning trying to solve a particularly beautiful combinatorics problem.

Deputy leader and observer[1]

Once I've been separated from the students, they are taken with Gerry and Jill to the seaside hotel where they will be staying for the duration of the competition. There, the students are introduced to their guide, who seems quite taken aback that the team would rather read and study than swim in the sea the other side of the road.

While the students are in their guide's hands, Gerry and Jill are informed that there is no room in the hotel for them, and they have been put in a separate hotel, which according to Gerry is located roughly at the edge of the observable universe. This is particularly unfortunate for Jill, who is here in a specifically pastoral capacity. Still, there is little they can do in the immediate future, so they take a taxi to their accommodation.

Star Wars Day

Regretting my late night shortlist party, I wake up begrudgingly and stand under the shower. This has only a single control, which seems to affect flow rate and temperature simultaneously – woe betide the man who likes his showers cold and pummelling. Fortunately I am not he, so I head to breakfast refreshed, where I meet Massimo and Vesna, the Italian and Macedonian leaders.

The first task for the day is to receive the official solutions and continue working on the problems. However, before long we are herded into a coach to join the students for the opening ceremony, which takes place at their hotel. The podium is set up like a mini UN, with a long table for the leaders covered with small national flags. I find my seat next to the Irish flag, and my other seat next

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[1]For the first few days I was not allowed to contact the students, deputies, observers or the outside world in order to protect the integrity of the shortlist. Thus my account of their first few days is largely patched together from what they would tell me later in the competition.

to the union flag, so I perform a quick and politically unlikely amalgamation and sit down.

The ceremony is very pleasant. One of the organisers clearly understands the virtue of brevity, and the whole thing runs like tightly-wound clockwork. After a few short speeches, we proceed to the parade of teams, which this time just consists of the teams standing up when their name is called, like some sort of international Whack-a-Mole. All the students seem very pleased to be here (as they should be), and the Turkmen and Azeri teams are showing off some snazzy uniforms. Not that the UK/ROI team notice, the peculiar L-shape of the room has rendered half the students invisible to the other half.

Still, it was an enjoyable ceremony, and I give a wave and smile to the team as I return to the leaders' hotel for the main event: the deciding of the paper. We begin by weeding out the known problems, and categorise the remainder in terms of difficulty. Suddenly problem 1 is selected: an easy-ish inequality that sadly rules out many beautiful abstract algebra problems. Massimo and I exchange looks. In a second snap poll, the jury takes up problem C1, at the expense of a very challenging combinatorics problem I had had my eye on. Gruntled I am not.

However, at this point the jury realises it's about to select the paper too quickly, so we entertain a long debate on how to decide the remaining questions. Eventually, N6 (a Serbian proposal, and for me *the* question on the shortlist) becomes our hard problem. The Romanian leader invokes the spirits of Thales and Euclid in support of the purest geometry questions on the shortlist, and accordingly the jury takes G4 to complete the paper.

Then Fawzi, the Saudi leader, and I get to work producing the official English paper. We completely rephrase the combinatorics question, and I take the opportunity to expunge the non-words "favorite" and "center" from the paper. The jury accept our wordings, although while I'm out of the room, the 3366 film critics in problem 3 mysteriously all become male. It seems that we will be finished with the work well in advance of bedtime.

However, it is not to be so. After dinner, I detect a flaw in problem G4, and decide to hit the jury with the nuclear option: problem 2 on our paper is just a degenerate case of Desargues' theorem. Normally, this would rule it out as a suitable problem, but this time my nuke falls on deaf ears. After this, it doesn't take long to resolve the matter of the remaining mark schemes, and I head to bed. Security forbids me from wishing the students luck, but I have my fingers crossed for them.

Deputy leader and observer

Gerry and Jill spend their day looking for hotels closer to the students, but to no avail. Nonetheless, the beneficent hand of Fate rewards their efforts when the organisers suddenly find a place for Jill in the students' hotel, and she is promptly carted off to her new lodgings. No corresponding room appears for Gerry, however, and for the remainder of the competition he remains based in his hotel somewhere in the vicinity of Betelgeuse. Having to visit the leaders'

and the students' hotels regularly, he quickly becomes a regular patron of the Athenian taxi service, from which he learns much about the Premiership efforts of Manchester United.

May 5th

Today is contest day. After some collective dawdling, the leaders board their bus to meet the students at the contest site, the mathematics department of the University of Athens. This boasts stunning views over the surrounding hillside, although we don't have much time to appreciate this before we head into the building. We are sitting in a room nearby the contest hall, and the students have the opportunity in the first half hour to ask written questions of clarification, which we are there to answer. One of the Serbian students puts forward the question "Does problem 1 have to be an inequality again?", but the remaining questions are disappointingly sensible.

After answering the questions, we still have four hours to kill before the students finish the exam, so we head to the Acropolis museum for a guided tour. With the aid of several models, our guide explains the history of the Parthenon as a place of worship variously Hellenic, Christian and Islamic. We then continue into the main body of the museum, where I find the remaining paintwork on the ancient statues particularly interesting.

After lunch, I head to the students' hotel to chat with them after the exam. However, the exam started an hour late and their bus takes an age to return, so we only rendez-vous shortly before four. I say a brief hello before diving out of the stampede for the buffet, worried that in the seven hours since breakfast, the students may have turned feral. Over lunch, we have a more civilised chat. It seems a good proportion of our students are claiming solutions to either problem 1 or 2, but progress on problems 3 and 4 is thin on the ground.

Later, I retire to the leaders' hotel to mark the scripts. The scripts are due to arrive at 6pm, but they aren't anywhere to be seen. I take the opportunity to send a lot of emails, but I have run out of things to send long before the scripts finally limp through the door at 10 o'clock. Despite the very late hour, I summon Gerry from his hotel in the Oort cloud and we have a first look through the scripts.

The solutions claimed by the students seem pretty solid generally, though Lawrence's partial trigonometric solution to question 2 seems unsavable. As they predicted, Sam and Kirsten don't have much, but Sam's work on problem 3 looks tantalisingly close to a new solution. I stay up trying to find a quick fix to Sam's crucial lemma, and by 3am I succeed and collapse into bed.

May 6th

Today, the students and Jill are out visiting the Acropolis museum and exploring, giving them some well-earned relaxation. Gerry and I are up for coordination, which entails meeting with some local organisers and agreeing the marks with them. The coordinators have been provided with photocopies of

the students' scripts, but not all of the scrap paper has been photocopied, so occasionally we need to provide this.

Our first coordination is question 3, with a pair of Greek coordinators playing the classic Good Cop/Bad Cop routine. We agree to five non-contentious zeroes, and then begin examining Sam's script. I explain to the coordinators how to fix Sam's crucial claim, and run them through how he concludes. The coordinators seem surprised that this approach works, but happy that this is worth non-trivial marks. I suggest three, but Bad Cop points out that Sam has made a minor error in his introduction, so in the end we agree that 2 is the fair mark. The whole thing is very civil, and I am in a good mood to face the other coordinations.

Problem 4 is up next. I don't think we have anything worth any marks, except perhaps a little comment in Lawrence's rough work, which identifies the crucial bounding that needs to be done to connect to the number theory. The coordinators feel this is worth a mark, and we are inclined to agree.

Surprised and happy, we head next to the problem 1 coordination, with two very cheerful coordinators. Three tens and two zeroes are quickly agreed, and then we turn to Luke's script. The complicating factor here is that, due to running out of time in the exam, Luke has concludes his proof appealing to Schur's inequality, which his instincts have rightly told him is the final crucial step. However, he told us that he didn't have time to work out the details of the Schur argument, which are quite subtle, so we argue that his script is an especially meritorious attempt. Pleased with our honesty, the coordinators agree, and after a short discussion we settle on three marks.

The final coordination occurs quite late in the day, and the problem 2 coordinators, no doubt keen to get home, seem in a generous mood. The tens and the zeroes are quickly settled, and Kirsten picks up a single point for a useful remark in her rough work. In Lawrence's incomplete trig bash, he has also made a useful non-trigonometric observation, and he too is rewarded with a point.

Once the coordinations are finished, we have the final jury meeting, where the chair of the jury passes on to the organisers of the next Balkan Olympiad. If all goes to plan, the next hosts should be Albania, but the Macedonians are ready to step in should they have difficulties. In any case, the competition should be in safe hands for the next year, and I wish them the best of luck in organising a competition at least as enjoyable as this year's.

The main event of the meeting, though, is the setting of the medal boundaries. As expected, 20 marks isn't enough for a silver medal, but the bronze boundary is unusually low at 12 marks, meaning that four of our students get bronze medals. This is great news, and I find the team to let them know. They are suitably pleased, and the rest of the day passes in an enjoyable and exhausted blur before I head back to my hotel to crash out.

May 7th

Today is the day of the closing ceremony and presentation of medals. We expect that the UK media are full of reports on the competition, but we hope that more

minor news stories are not totally eclipsed.

Before the ceremony, we all board buses for the group excursion to the site of the battle of Marathon, taking a scenic route over a mountain pass. The site itself isn't particularly impressive. Aside from some informative signs near the entrance and a central tumulus, there is not much to see except fields. I content myself with photographing interesting flowers and blue butterflies.

After our short visit, we head to the beach for a break. Gerry, Jill, Kirsten, Sam and Luke head off to get a coffee, while Alex, Philip and I paddle in the shallows. The water is beautifully cool and clear, and we can see tiny fish scurrying off into the seaweed. We spot some tiny crabs, and have to dissuade Philip from trapping them in his water bottle. Somehow I can't see us getting that through customs.

After an iced chocolate, we head back to Athens for the closing ceremony and dinner. The festivities begin with a series of addresses from various mathematical dignitaries, beginning in English, but soon switching into Greek. One speaker talks (in Greek) about how maths is a universally understandable language, while another (also in Greek, but with translations provided) talks about maths being a pylon of our civilisation. After the initial addresses, we are treated to a few piano pieces from the next Vangelis, and a fascinating display of Greek folk dancing which gets one of the local organisers leaping up and down the aisles.

After this is the presentation of the medals. There is one perfect score from a Turkish student, and only six gold medals in total, all of which are very well-deserved in the face of this tough paper. There is much cheering for the home team, but Gerry, Jill and I put in a good effort as our four medallists drape themselves in their respective flags and are given their medals.

After the presentation is over, we walk back to the hotel for a final dinner together, and the students gradually disappear to get some sleep for the flight back the following day.

May 8th

Despite an obscenely early start, the return journey to the UK passes without incident, and the British students are reunited with their parents at the terminal. We say our goodbyes, and to Luke as he boards his flight to Ireland, and the Balkan Olympiad is over for another year. We very much enjoyed the competition, and am sure will all have fond memories.