

RMM 2015 – UK Report

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Introduction

The Seventh Edition of the Romanian Master of Mathematics was held in Bucharest between 25th February and 1st March 2015. This is a hard competition for school-aged mathematicians from invited countries. The UK has been honoured to accept invitations to attend each edition since the first in 2008.

This year, the UK team was selected based on tests at the winter camp in Dombóvár, Hungary, and consisted of:

Joe Benton	St. Paul's School	(16)
Liam Hughes	Robert Smyth Academy	(17)
Sam Kittle	Simon Langton Grammar School for Boys	(18)
Warren Li	Eton College	(17)
Harry Metrebian	Winchester College	(18)
Harvey Yau	Ysgol Dyffryn Taf	(15)

James Gazet of Eton College was deputy leader.

The results of the UK team were:

	P1	P2	P3	P4	P5	P6	Σ	
Joe Benton	7	5	0	7	7	7	33	Gold Medal
Liam Hughes	7	0	0	7	6	0	20	Honourable Mention
Sam Kittle	7	6	0	0	0	3	16	Honourable Mention
Warren Li	7	6	2	7	7	2	31	Silver Medal
Harry Metrebian	7	1	0	0	7	0	15	Honourable Mention
Harvey Yau	7	5	2	7	7	1	29	Silver Medal

The cutoffs for medals were 22 for bronze, 27 for silver and 32 for gold. This represents a very strong team performance by the UK, and all our students and their trainers deserve congratulations for the hard work they have put in during and between our camps which has led to this success. This is the second-strongest medal return by a UK team at this competition, and Joe Benton's fifth place in the individual rankings is the highest for a UK

student since my own teammate Jonathan Lee came third in the inaugural RMM 2008. Congratulations to Chenjie Yu from China who placed first with a remarkable 39 points.

At this competition, the team scores were given by the sum of the top three contestants from each team and were: Russia (105), USA (100), China (96), United Kingdom (93), Hungary (84), Ukraine (80), France (74), Italy (73), Peru, Poland, Romania (all 72), Serbia (71), Bulgaria and Brazil (69), Romania B (58), Mexico (56), Tudor Vianu (45).

Congratulations are due to Russia, USA and China for the excellent solutions from all their students on the very challenging papers. This year a team from the Shanghai area represented China. Russia and USA will similarly choose their teams for the IMO in Thailand later this year from a broader pool of students than for RMM, and based on these results they surely seem set for another impressive performance.

For the UK to come ‘best of the rest’ is an excellent result. It is also very pleasing to see us coming neck-and-neck with Hungary. Our joint winter camp held in Hungary over every New Year is now a firmly established tradition, and to see our students improving together, while also forming strong friendships, is hugely rewarding.

Problems of the Contest

This contest follows the same structure as the IMO. On consecutive days the students sit exams lasting 4.5 hours. Each exam has three questions.

Day One

Problem 1. Does there exist an infinite sequence of positive integers a_1, a_2, a_3, \dots such that a_m and a_n are coprime if and only if $|m - n| = 1$?

(PERU)

Problem 2. For an integer $n \geq 5$, two players play the following game on a regular n -gon. Initially, three consecutive vertices are chosen, and one counter is placed on each. A move consists of one player sliding one counter along any number of edges to another vertex of the n -gon without jumping over another counter. A move is *legal* if the area of the triangle formed by the counters is strictly greater after the move than before. The players take turns to make legal moves, and if a player cannot make a legal move, that player loses. For which values of n does the player making the first move have a winning strategy?

(UNITED KINGDOM) JEREMY KING

Problem 3. A finite list of rational numbers is written on a blackboard. In an *operation*, we choose any two numbers a, b , erase them, and write down one of the numbers

$$a + b, \ a - b, \ b - a, \ a \times b, \ a/b \ (\text{if } b \neq 0), \ b/a \ (\text{if } a \neq 0).$$

Prove that, for every integer $n > 100$, there are only finitely many integers $k \geq 0$, such that, starting from the list

$$k + 1, \ k + 2, \ \dots, \ k + n,$$

it is possible to obtain, after $n - 1$ operations, the value $n!$.

(UNITED KINGDOM) ALEXANDER BETTS

Day Two

Problem 4. Let ABC be a triangle, and let D be the point where the incircle meets side BC . Let J_b and J_c be the incentres of the triangles ABD and ACD , respectively. Prove that the circumcentre of the triangle AJ_bJ_c lies on the angle bisector of $\angle BAC$.

(RUSSIA) FEDOR IVLEV

Problem 5. Let $p \geq 5$ be a prime number. For a positive integer k , let $R(k)$ be the remainder when k is divided by p , with $0 \leq R(k) \leq p-1$. Determine all positive integers $a < p$ such that, for every $m = 1, 2, \dots, p-1$,

$$m + R(ma) > a.$$

(BULGARIA) ALEXANDER IVANOV

Problem 6. Given a positive integer n , determine the largest real number μ satisfying the following condition: for every set C of $4n$ points in the interior of the unit square U , there exists a rectangle T contained in U such that

- the sides of T are parallel to the sides of U ;
- the interior of T contains exactly one point of C ;
- the area of T is at least μ .

(BULGARIA) NIKOLAI BELUHOV

UK Team Diary

One excellent aspect of this year's competition was the extent to which leaders and students were encouraged to mingle, rather than being housed, fed and entertained separately, as is sometimes the case elsewhere. Given this, it felt sensible to present this brief account of the UK delegation's trip jointly, rather than creating a separate student report. Joe reports on behalf of his teammates on those moments for which the students had their own perspective.

Wednesday 25th February

We have spent the night at a hotel close to Luton Airport, so we can proceed to our flight on foot. Walking in front of a bright red sunrise to a bright orange terminal to depart on a bright pink plane leaves me with a sense of colour overload not experienced since I last watched *South Pacific*. The three hour flight to Bucharest is unremarkable. Sam has fallen asleep with pencil poised halfway through a long expression where every other term is 2012^{2012} , and Harvey makes rapid progress on a dodecaheagonal Rubik's Cube.

Soon afterwards we arrive in Romania and get lifts to the Moxa accommodation complex of the University of Economics where the students will stay. There are clearly mild cultural differences concerning what levels of privacy middle-aged adults might expect to enjoy, but the organisers have done a good job, and all is resolved satisfactorily. Of the students, Harry and Sam will share with two Brazilian boys who are due to arrive in the middle of the night, and the remaining four have a dorm to themselves, complete with precarious looking upper bunks.

Joe: Slightly surprisingly, we have been given seven guides, an entire Year 11 class at the Vianu school. Four of them take us on a walking tour round the central area of Bucharest, including the imposing Victory Square, and Herăstrău Park. A sign informs us we should not toboggan down the brief slope between the path and the lake. We heed this advice.

Later in the evening, James and I diverge for the first leaders' meeting. Old friendships are renewed, and the proceedings are informal and brief, allowing as much time as possible to get to grips with the questions. A proposed pair of papers is circulated by chief problem selector Ilya Bogdanov, and we get to work in James' room. Our immediate impression is that we like them a lot, and this is reaffirmed over the coming hours as we explore them further.

Thursday 26th February

The leaders get to work finalising the papers. My confidence in the quality of the questions has grown even stronger overnight, and so I am not surprised when these are approved fairly rapidly. I propose swapping questions 3 and 5 based entirely on my own prejudice regarding their relative difficulty, and it turns out that others feel similarly, and this is approved.

Next, we must finalise a definitive wording of the questions before they are translated into languages for 14 other countries. Various people have strong views on commas, how many times one should use the word ‘let’ in a given sentence, and whether ‘open’ or ‘interior’ are more likely to be found ambiguous by students. Perhaps unexpectedly, a question submitted by the UK, courtesy of Lex Betts, causes the most problems for wording. In the end, it seems easiest to avoid ambiguity by completely rephrasing it in terms of the blackboard setup that will now appear as Q3 on the paper, accompanying Q2, the work of our own Jeremy King.

Joe: Meanwhile we enjoy a more comprehensive tour of Bucharest, past the old city and the Palace of the Parliament, then on to an excellent lecture by Călin Popescu. He tells us about topological dimension, and we learn that triangles are two-dimensional, though unsurprisingly the real challenge is deciding precisely what ‘triangle’ and ‘two-dimensional’ actually mean.

The opening ceremony is a well-organised affair in the grand university hall with several generous speeches from the mayor and other local dignitaries, and representatives of Tudor Vianu school. On the way home, the students examine their goodie bags, featuring various stationery and an RMM polo shirt. The leaders have not been missed out, though I wonder whether their guesses at sizes may have been informed by my predecessors? Certainly I will have to eat a lot of the omnipresent potato salad to run any danger of fitting into this item before the end of the competition...

After our winter camp in Hungary, the students are now connoisseurs of Eastern European cuisine, and remain unfazed by even the most remarkable display of gherkins. While James and I catch up on work, they relax before tomorrow’s festivities by starting another round of the card game which I am apparently not allowed to name. Suffice it to say, it has a similar quality to *The Archers*, offering a nonsensical background murmur which proves surprisingly supportive to research productivity.

Friday 27th February

Harry reports over breakfast that he spent some of the night helping dismantle a hyperactive burglar alarm, but it seems everyone is feeling well-prepared for the first day of the contest.

James and I have carefully assembled a selection of fruit for the UK team's refreshment, but, after Snow White themed questions regarding our intentions, the apples prove substantially more popular with the Hungarian students.

After approving answers to a handful of questions, mostly about the nature of the 'first turn' in Q2, we are free, so I return for a walk around the serpentine Herăstrău Lake. The boundary of the lake seems to have Hausdorff dimension slightly greater than 1, but in any case, it is pleasant to stop halfway round its seemingly infinite perimeter to work on some problems about multitype branching processes. I also stop at the orthodox cathedral, from which my own college chapel could learn plenty about how a solemn space can be gold without being gauche.

Our students seem unsure whether to be upbeat or not, but we have a complete set of solutions claimed for Q1, and some cautious reports of progress on Q2. To avoid wasting time worrying about the recent past, some of the guides scoop up the Russian, American and UK teams for a walking tour of Bucharest old town and the stylish Cișmigiu park. As in 2008, I observe that Bucharest enjoys a surfeit of excellently-equipped playgrounds almost everywhere, but a total absence of children using them. On this occasion, the younger members of our team are reluctant to rectify this.

I get started on the Q2s after dinner, and in a pleasing reversal of what often happens at some competitions, our two students claiming partial solutions have actually done rather better than they suggested. Sam in particular has been very clear about what he can and cannot do, and might even end up scoring seven. James and I convene at his hotel to discuss the challenging Q3 which seems to be equally clear-cut, so it is a hard-working evening, but a lot less drawn-out than it might have been. It is good to see that our recent active efforts to encourage the students to improve their write-up style are paying dividends.

Saturday 28th February

There is much to squeeze into the programme, and so the second paper starts an hour earlier. James and I spend the day based at Tudor Vianu, the specialist maths and computing school that has hosted this competition since its first incarnation in 2008. Even coming from schools which regularly send students to such international competitions, we find it remarkable to see how much explicit emphasis they place on academic excellence here. Where British observers might expect lists of prefects and photos of glorious football teams, here instead we see students posing with medals and Romanian flags from contests around the world.

Our first task is to coordinate yesterday's problems. Qs 1 and 3 are agreed extremely rapidly, with the problem captains very complimentary of the UK boys' number theory

solutions. Q2 has all the ingredients to suggest a long slog, but coordinators Lucian Turea and Radu Gologan have clearly thought very carefully about the UK scripts. Everything they say is sensible and easy to make consistent, so we are finished and happy comfortably within our 20 minute slot.

James and I also have to supervise the Romanian scripts for Qs 2 and 3 as these are British submissions. My schoolboy Latin helps a little bit, and we eventually agree on how to pair up comments in the solutions with points on the markscheme just in time to meet our students as they finish. Joe is full of excitement, having completed all three in the closing moments, while others are disappointed, having been slightly thrown by the geometry, but overall spirits are high.

The team are squirrelled off by the guides, and I have the afternoon to engage with the Q5 scripts. We have five solutions, and all are fine, but might not appear fine to a casual observer. Liam has opted for the Jackson Pollock approach to truth, where statements of various levels of interest and veracity are independently sprayed freely across three pages, though after a while I am convinced that every line does follow from something somewhere else on the page.

While working, I realise that having a ground floor room in an Eastern European hostel has its drawbacks. That said, opening the window gives the chance for an experiment to determine exactly which genre of music is found least appealing by lingering smokers. Enescu's *sonate dans le caractère populaire roumain* proves successful, despite its local heritage.

I have a better idea what's going on in all our students' arguments in time to venture down to the slightly baffling Bucharest metro towards the farewell dinner, which retains its name despite not falling on the final evening. The students are deliberately separated from the leaders, but no attempt is made to enforce this and everyone mingles freely. This year the Chinese team comes from the Shanghai area, and their leader teaches at their high school. He has recently spent a term in Reading and, together with Warren and Harvey, we have a highly enthusiastic conversation about differences in education systems between our countries.

The UK students' table seems to have been chosen for especially ponderous service, but the 30 seconds they are given between their desserts arriving and the bus arriving proves sufficient. I feel judged when I arrive back at my room and find the Hungarian deputy leader still working on his problematic geometry, so make sure to have at least a nominal further glance at our Q5s before setting an early alarm.

Sunday 1st March

The only people in Victory Square at 6.30am are stray dogs, and stray leaders heading to the school to prepare for their coordinations. I'm apprehensive about being asked to go through Harry's solution to Q5 line-by-line, but though the effort to understand everything felt purposeful, it wasn't necessary, as we get what we request almost immediately. Exactly the same thing happens on the other second day questions, so James and I are kicking our heels by 9.30, and the possibility of a return to bed feels very inviting while we wait for the other countries' scores to clear.

*Joe: Meanwhile, we are at the mysterious 'Hostel X', in order to visit an 'escape room'. This was not actually as dubious as it first sounds. As Dominic explained, we were to be locked in a room for an hour during which we would have to solve a number of puzzles in order to escape. [DY: think of *The Crystal Maze* but with less leopard-print.] This turns out to be extremely enjoyable, as we gradually discover the collective significance of some masks, a chessboard and a couple of UV torches, but also quite difficult. Sam, Harvey, Andrei and I manage to escape with barely five minutes to spare but the others do not quite finish, although they assure us that their room was by far the more difficult of the two...*

We meet the students, who are discussing Morse decoding and similar things with great enthusiasm. En route home, James thinks that we have been too hasty to accept a flaw in Joe's solution to Q6, since it suddenly dawns that it could be fixed with the addition of a single \geq sign. Our original coordinators have gone home, but chief coordinator Mihai Bălună graciously takes a second look, and agrees with our re-assessment, so James' defibrillator can go back in the box.

The bronze and silver medal boundaries have settled naturally, and after brief discussion the jury decides to round up the number of golds to ten, which is surely the right decision. This leaves the UK with three honourable mentions, two silvers, and a gold. Though some of our students might be disappointed to lie just below a boundary, they all recognise that this contest features challenging problems and experienced contestants, many from countries with far more strenuous training programmes than ours. By any measure, this is a fantastic team performance, and James and I are very proud of them.

The closing ceremony is held in the atrium of Vianu school, and after an encouraging speech from the headteacher, the medals are awarded fairly swiftly. Joe reports that the hardest aspect of winning a gold at RMM is the necessity to smile on stage continuously for three minutes. Russia is announced as the winner of the team competition, with a very impressive set of performances, closely followed by the USA.

With the entire evening clear, the UK and USA teams head to Piața Romana to celebrate each other's successes. The Romanian guides and the UK leadership have slightly different views about what constitutes an appropriate venue for this, but in the end everyone is

entirely happy to gather in the common area at James' hotel. This has been an excellent competition, and it is wonderful to see students, guides and leaders from all teams finding so much in common and much to learn from one another.

Monday 2nd March

My roommate departs for a train to Budapest at 4am, and the accommodation staff are enthusiastically dismantling the bunkbeds in the adjacent room at 6am, so it is fair to say I might have slept better. Two cars take us out to the airport, precisely one of which thinks we are having a race through the rush hour traffic. Suffice it to say, I would probably like to cycle round the *Arcul de Triumf* even less than its Parisian counterpart.

The students' recently acquired metalwork doesn't quite take us over the baggage weight limit. The Wizzair boarding procedure leaves a little to be desired, but the party looks keen for little except sleep, so it makes no difference to do this sparsely. As ever, the arrivals barriers at Luton only just manage to hold back the legions of adoring fans. Goodbyes are exchanged before we head our separate ways, though we will meet again for more worthwhile mathematics in just over three weeks at our next training camp in Cambridge.

Conclusion

RMM 2015 was a very well-run competition, and the mathematical aspects more than justified its reputation as the most challenging and worthwhile contest currently available to school-age mathematicians. Everyone involved with the UK setup hopes it will continue to flourish for many years to come!

Thanks are due to many people, especially:

- Everyone at UKMT who arranged all aspects of our participation at this competition.
- Radu Gologan and the RMM organisers, including Tudor Vianu school, who again ran a fantastic competition. Particular thanks are due to the problem authors and the problem selectors producing a beautiful pair of papers, and local organisers, especially Iulia Manicea for accommodating all our other needs.
- The UK team's guides, Irina, Irina, Adina, Cristi and Andrei, who worked tirelessly to keep the team informed and entertained.
- James, who was as always an excellent person with whom to travel, mark, and discuss combinatorics, the universe and everything.
- The students, who showed continual enthusiasm for challenging mathematics, and were perfect representatives of the UK and our mathematical enrichment activities.