IMO 2016 UK Team Leader's report

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The International Mathematical Olympiad is the world championship of secondary school mathematics. It is a nomadic festival, and is held every July. Each country or territory may send up to six students, and these contestants face two exams sat on consecutive days. Each exam lasts 4 hours 30 minutes, and each paper consists of three problems. In terms of problems which students may face as part of a normal secondary education, the three problems on each paper would be successively rated: *very hard indeed*, *completely impossible* and *surely you are joking*. However, the students who participate at the IMO are among the strongest young mathematicians of their generation, so for them we adopt the inaccurate category names *easy*, *medium* and *hard* for the problems.

Dominic Yeo has taken the lead role in UK team training this year, and any reflected glory from the UK team's splendid performance at IMO 2016 in Hong Kong shines upon him. It makes sense to regard his report as the principal one for 2016, with mine containing a few incidental observations.

The British team was found through a selection process based on performances in test exams. They were

Joe Benton	St Paul's School, Barnes, London
Jacob Coxon	Magdalen College School, Oxford
Lawrence Hollom	Churcher's College, Petersfield, Hampshire
Warren Li	Eton College, Windsor
Neel Nanda	The Latymer School, Edmonton, London
Harvey Yau	Ysgol Dyffryn Taf, Whitland, Carmarthenshire, Wales

and the reserves were

Rosie Cates	Hills Road VI Form College, Cambridge
Michael Ng	Aylesbury Grammar School
Thomas Read	The Perse School, Cambridge
Renzhi Zhou	The Perse School, Cambridge

The IMO of 2016 took place in Hong Kong, a Special Administrative Region of China. Dominic Yeo (Oxford University) was Deputy Leader and principal organizer of team training camps. Jill Parker was Observer with students, and I was UK team leader, sitting on the IMO Jury. As well as the official UK delegation, there were another couple of UKMT volunteers in Hong Kong. James Cranch and Joseph Myers are there as experienced anglophone co-ordinators, and Joseph has an extra brief to shadow IT specialist Matjaž Željko to build some redundancy into IMO computer support.

Here is a table showing the performances of the British students:

	P1	P2	P3	P4	P5	P6	Σ	Medal
Joe Benton	7	7	2	7	2	1	26	Silver medal
Jacob Coxon	7	1	0	6	7	3	24	Silver medal
Lawrence Hollom	7	7	0	7	3	0	24	Silver medal
Warren Li	7	7	2	7	7	3	33	Gold medal
Neel Nanda	7	7	0	7	2	7	30	Gold medal
Harvey Yau	0	7	0	7	7	7	28	Silver medal
	35	36	4	41	28	21	165	

The cut offs are 16 for bronze, 22 for silver and 29 for gold. Here are the problems:

Day 1

Problem 1 Triangle BCF has a right angle at B. Let A be the point on line CF such that $FA = FB$ and F lies between A and C. Point D is chosen such that $DA = DC$ and AC is the bisector of $\angle DAB$. Point E is chosen such that $EA = ED$ and AD is the bisector of $\angle EAC$. Let M be the midpoint of CF. Let X be the point such that $AMXE$ is a parallelogram (where $AM \parallel EX$ and $AE \parallel MX$). Prove that lines BD, FX, and ME are concurrent.

Problem 2 Find all positive integers n for which each cell of an $n \times n$ table can be filled with one of the letters I, M and O in such a way that:

- in each row and each column, one third of the entries are I, one third are M and one third are O; and
- in any diagonal, if the number of entries on the diagonal is a multiple of three, then one third of the entries are I, one third are M and one third are O.

Note: The rows and columns of an $n \times n$ table are each labelled 1 to n in a natural order. Thus each cell corresponds to a pair of positive integers (i, j) with $1 \leqslant i, j \leqslant n$. For $n > 1$, the table has $4n - 2$ *diagonals* of two types. A diagonal of the first type consists of all cells (i, j) for which $i+j$ is a constant, and a diagonal of the second type consists of all cells (i, j) for which $i - j$ is a constant.

Problem 3 Let $P = A_1A_2 \ldots A_k$ be a convex polygon in the plane. The vertices A_1, A_2, $\ldots$, A_k have integral coordinates and lie on a circle. Let S be the area of P. An odd positive integer n is given such that the squares of the side lengths of P are integers divisible by n. Prove that $2S$ is an integer divisible by n.

Day 2

Problem 4 A set of positive integers is called *fragrant* if it contains at least two elements and each of its elements has a prime factor in common with at least one of the other elements. Let $P(n) = n^2 + n + 1$. What is the least possible value of the positive integer b such that there exists a non-negative integer a for which the set

$$\{P(a + 1), P(a + 2), \ldots, P(a + b)\}$$

is fragrant?

Problem 5 The equation

$$(x - 1)(x - 2) \cdots (x - 2016) = (x - 1)(x - 2) \cdots (x - 2016)$$

is written on the board, with 2016 linear factors on each side. What is the least possible value of k for which it is possible to erase exactly k of these 4032 linear factors so that at least one factor remains on each side and the resulting equation has no real solutions?

Problem 6 There are $n \geqslant 2$ line segments in the plane such that every two segments cross, and no three segments meet at a point. Geoff has to choose an endpoint of each segment and place a frog on it, facing the other endpoint. Then he will clap his hands $n-1$ times. Every time he claps, each frog will immediately jump forward to the next intersection point on its segment. Frogs never change the direction of their jumps. Geoff wishes to place the frogs in such a way that no two of them will ever occupy the same intersection point at the same time.

(a) Prove that Geoff can always fulfil his wish if n is odd.

(b) Prove that Geoff can never fulfil his wish if n is even.

Extra Information

The word "fragrant" refers to the meaning of "Hong Kong", which is "fragrant harbour". The problems were proposed by Belgium, Australia, Russia, Luxembourg, Russia and the Czech Republic respectively. The authors were Art Waeterschoot, Trevor Tao, Alexandr Gaifullin, Gerhard Woeginger, Nazar Agakhanov & Ilya Bogdanov (so two authors for Problem 5) and Josef Tkadlec. The answer to the obvious question about the second composer is "yes".

Analysis of Results

Here are the countries ranked in the top 21 at IMO 2016. There were a total of 109 countries sending contestants.

Rank	Name	Points	Medals
1	United States of America	214	GGGGGG
2	South Korea	207	GGGGSS
3	China	204	GGGGSS
4	Singapore	196	GGGGSS
5	Taiwan	175	GGGSSS
6	North Korea	168	GGSSSS
7	Russia	165	GGGSB
7	United Kingdom	165	GGSSSS
9	Hong Kong	161	GGGSSB
10	Japan	156	GSSSSB
11	Vietnam	151	GSSSSB
12	Canada	148	GGSSBH
12	Thailand	148	GGSSBH
14	Hungary	145	GSSSBB
15	Brazil	138	SSSSSB
15	Italy	138	GSSSHH
17	Philippines	133	GGSSHH
18	Bulgaria	132	SSSBBB
19	Germany	131	SSSBBB
20	Indonesia	130	SSSBBB
20	Romania	130	SSSSSB

This was the first ever top ten performance by the host territory, and the first top 20 performance by the Philippines.

The UK team collectively scored 165 points, putting them joint top of Europe with Russia, the team with which we shared 7th place. This is the best UK performance since 1996 when the UK finished 5th.

Other continental champions were Africa: South Africa, Asia: Republic of Korea, Australasia: Australia, North America: USA and South America: Brazil.

Bitter local rivalries include the annual Nordic knife fight: Sweden (1), Denmark (2), Norway (3), Iceland (4). The Baltic republic bar brawl finished: Lithuania (1), Estonia (2) and Latvia (3).

The monarchy roll of honour (taking a liberal view of Grand Dukes, Princes and elected Kings) is UK (1), Japan (2), Canada (3), Thailand (4), Australia (5), Sweden (6), Saudi Arabia (7), Netherlands (8), Spain (9), Belgium (10), New Zealand (11), Malaysia (12), Morocco (13), Denmark (14),

Norway (15), Luxembourg (16), Cambodia (17), Jamaica (18) and Liechtenstein (19).

Other European countries which performed well were Hungary [14], Italy [16], Bulgaria [18] and Germany [19]. Romania had, by their high standards, a modest year in position [20].

The results of Commonwealth and anglophone countries (taking an extremely relaxed interpretation of the word anglophone) were USA [1], Singapore [5], UK [7], Hong Kong [9], Canada [12], Philippines [17], Australia [25], India [34], Bangladesh [35], New Zealand [53], Malaysia [56], South Africa [58], Cyprus [63], Sri Lanka [66], Ireland [75], Nigeria [88], Trinidad and Tobago [88], Myanmar [96], Uganda [98], Kenya [99], Jamaica [102], Ghana [104], and Tanzania [106]. Note that many of the teams ranked lower on this list did not send a full team of six students.

Personally I received a fair amount of teasing about Brexit, and its supposed impact on team performance. This teasing mostly and mysteriously stopped after the IMO results were published. Spurred by this insolence, I can report that the medal count of the UK and that of the best six students of the rest of the European Union were equal; two golds and four silvers in each case. However, it must be admitted that the Rump European Union outscored the UK 178 to 165. The REU team consisted of Attila Gáspár of Hungary (35), Filippo Gianni Baroni (31) from Italy, and four participants chosen from a set of two Czechs, a French person, an Italian and two Swedes, all of whom scored 28 points.

Diary

July 4 As I walk across the concourse outside Heathrow T5 I hear the characteristic tones of Richard Quest, CNN's poster boy for business class travel. As you probably know, he sounds like a concrete mixer on steroids. I am just wondering why someone has parked a TV tuned to CNN outside T5 as I turn my head to the right to see Richard Quest in person, doing a piece to camera. I muse as to whether I should linger, and offer to give CNN my views on the global crisis, Brexit, the US election, the future of the Labour Party and whether or not 0 is a natural number. Perhaps the world is not yet ready. The flight from West to East is not happy, and I am not sleepy. I watch five films of progressively inferior quality.

July 5 I arrive in Hong Kong and am met by the friendly and efficient

organizers. The drive to the Kowloon Harbour Hotel is longer than I had anticipated. There is a lot of geography in Hong Kong.

My arrival at the Kowloon Harbour Hotel involves a minor disaster, as I succeed in breaking the bottle of single malt Scotch which I had somehow acquired in Heathrow duty free. I check in, and the staff attempt to explain everything at once. I am in shock from the loss of whisky combined with sleep deprivation, and grasp lots of pieces of paper in the hope that they may be relevant.

July 6 Next day we have the exciting all day meeting of the IMO Advisory Board sitting in committee. Since Joseph Myers does not have enough to do already (IT, co-ordination) the IMOAB ropes him in to take notes during their day long committee meeting. This has the advantage (or disadvantage) of allowing IMOAB secretary Gregor Dolinar to spend more time contributing to the discussions.

First item on the agenda is the American offer to host IMO 2021. After several attempts, we establish a decent Skype link to Michael Pearson of the Mathematical Association of America in Washington DC, but unfortunately the US leader Po-Shen, who should be here in Hong Kong at the IMOAB meeting, is not physically present. This is in contrast to my state of being not mentally present. It turns out that Po-Shen has also underestimated the impressive quantities of Hong Kong geography, and he arrives a few minutes late.

Next comes the presentation from Nazar Agakhanov of the offer by the Russian Federation to host IMO 2020 in St Petersburg. After that we process a huge amount of IMOAB business, over which I draw a veil.

In the evening I get hold of the IMO shortlist, and start to enjoy the problems.

July 7 Over breakfast I can now start to make social contact with large numbers of other team leaders. This is a very happy time of the year, when we meet again after a year apart (the ideal basis for a social relationship).

There is one small disadvantage for me, because I am the elected chair or president of the IMO advisory board. This means that whenever someone has a problem, then it is very tempting to share it with me. This is because I have magical powers, and can make problems go away instantly. Also some people have ideas (a dangerous side effect of airline travel), and it is well known that meal times will be improved if they share these excellent suggestions with me.

We begin with a jury meeting in the well appointed jury facility. The

meetings are chaired by Kar-Ping Shum, who often delegates matters of detail to his assistant Andy Loo. It turns out that Andy enjoys decorating the proceedings with spurious formality, swinging freely between vague allusion to Robert's *Rules of Order*, and light pastiche of Erskine May's *A Treatise upon the Law, Privileges, Proceedings and Usage of Parliament*.

There is some initial discussion concerning the schedule for the day, and eventually Kar-Ping Shum endorses the jury's wish to work on the shortlist all day. In the evening we receive the solutions.

July 8 We discuss the merits of various questions, and eventually select a paper. The problem selection protocol I suggested in 2013 continues to enjoy wide support in the jury. I am slightly surprised that only one geometry problem makes it on to the paper, since most years there are two such problems. I think that the *zeitgeist* is that we should choose problems primarily on merit, and not be so constrained by the problem area. This may have happened because the 2013 protocol guarantees a wide range of problem areas will appear in positions 1, 2, 4 and 5, so that issue can be left to the algorithm, and jurors can think more about suitability, elegance and beauty.

July 9 The English language committee (i.e. those members of the jury who have nothing better to do) meets after breakfast. I have prepared draft versions of a possible paper but I do far too much talking at the IMO, and my beloved colleagues Michael Albert of New Zealand and Angelo di Pasquale of Australia kindly agree to conduct the meeting. Problem 6 has my name in it, but we suggest that other countries use whatever name suits them best. The Dutch leader Julian Lyczak seems to have made the best use of the opportunity, and the person involved in Problem 6 for Dutch students is Amalia. This honours HRH Catharina-Amalia, Princess of Orange. Thus she becomes the princess with the frogs. However, I am pleased to discover that "Geoff" is a suitable Chinese name.

There are substantial and fruitful discussions between the jury and the co-ordinators concerning mark schemes.

We also have the annual joint meeting of the IMOAB and the jury. The jury accepts the recommendations that they take up the generous offers by the Russian Federation and the United States of America to host IMO 2020 and IMO 2021 respectively. These offers are greeted by warm applause. Changes to regulations are discussed and in some cases approved.

There is an announcement by the Ethics Committee that a historical case had been discovered where a student competed at IMOs while in breach of educational status regulations. The student's name has been removed

from the official record. It is clearly the responsibility of team leaders to ensure that contestants adhere strictly to all IMO regulations, and if in doubt, consult the secretary of the IMO Advisory Board.

Before the IMO Ethics committee was introduced, there were occasional irregular incidents at IMOs. Let everyone be aware that the Ethics Committee has teeth.

July 10 Today we have the opening ceremony so I put on a suit. When I arrive I see the celebrated algebraist Efim Zelmanov who I knew slightly a long time ago. It is a pleasure to catch up with his news. I try to catch a glimpse of the UK team, but fail until they walk across the stage in their stylish and elegant uniforms.

There are many speeches, interspersed (thank goodness) with some live contemporary music composed for the IMO by the Hong Kong composer Dr Mui Kwong Chiu. The composer has donated the music to the IMO, so future editions of the competition are free to use it. There is a splendid orchestra, and various guest musicians and drummers. I try to keep my own speech light and short, and then I administer the IMO Oath to two members of the Hong Kong team.

After the ceremony I am interviewed by Reuters TV so I pretend to be important and say positive things about IMO 2016 and Hong Kong. I try to sound dynamic and look thrusting. My alter ego Richard Quest often copies this technique.

At the welcome dinner, a Chinese Banquet, The Hong Kong elite is out in force: Chief Executive C Y Leung (i.e. the PM) and Minister of Education Eddie Ng head the cast of officials, along with many other important figures, including the well known mathematician Tony Chan of HKUST which is the students' site. The speeches emphasize the huge state support for STEM in Hong Kong.

There is an initial toast for the cameras, followed by a large number of delicious courses. I had not previously realised the extent of the influence of the temperance movement in South East Asia.

This is my natural milieu of course. Everyone is very positive and extremely friendly. Towards the end of the evening I am called outside to do a short piece to camera to encourage the contestants who will sit IMO Paper 1 tomorrow. I imagine six hundred contestants scanning *Facebook* in the hope that some foreigner in a suit will make encouraging noises about the exam. This is obviously the best form of preparation for an IMO. I have some fun by making a cryptic reference to Problem 6: "I'll be with you in a special

way". I will later learn that my mischief was successful, and many contestants wasted their time looking for the meaning of this remark. In reality, it was simply an joke about the frog problem.

July 11 The exam is happening at the students' site, and there is 30 minutes at the start of the exam for students to ask questions of clarification. There are one or two teething problems getting the Q&A session going. After the chaos, I discuss the matter with various specialists, and we modify the procedures to make things run more smoothly the next day.

After Q&A we go on a excursion. This starts with lunch. You have to board a boat to get to the Jumbo Floating Restaurant. Once again I admire the iron grip of the temperance movement. The Czech and Bosnian leaders Martin Panák and Jelena Radović are paraded on stage in traditional costumes. Perhaps this is a prelude to human sacrifice?

Next on the agenda a trip to to see the panoramic view from Victoria Peak. I wander off in search of shade, and discover to my surprise that I have a minder, Queenie Lee. She is there to make sure I do not get into trouble. This is perhaps the hardest job at IMO 2016.

The coach descends from the summit, giving us magnificent views. We soon arrive at Hong Kong Polytechnic University, handily positioned close to the leaders' hotel. We start with tours of various research projects. I visit an anechoic chamber and am disappointed to hear myself when I speak. After that we listen to a panel of local experts discuss olympiad mathematics as well as aspects of mathematics in the modern world.

When we return home in the evening, we get access to our students' scripts for the Day 1 paper. It looks quite good, except that poor Harvey Yau has melted in the face of Problem 1.

July 12 We implement the improved Q&A procedures. It is now possible to send standard pictures rather than redraw them for every student, and there are modifications to the queuing rules. Also, instead of the leaders quietly and confidently concentrating on the matter in hand, thereby causing complete chaos as they did on Day 1, we install Geoff at the front to hector, encourage and bully the jury into some kind of order. This works surprisingly well, but after a while I get a bit fed up with shouting at people, and my voice starts to deteriorate. We really do need to find a better way to do this.

Next stop HKUST, the Hong Kong University of Science and Technology. I have an excellent student room in a flat occupied by several good friends. There is vigorous air-conditioning, and I am not far from the UK team.

I have a meeting with the UK delegation. They are in good spirits, and

are quite optimistic about their performances. They all seem to enjoy one another's company, and get on well with Dominic and Jill. I am given a briefing on the pre-IMO camp in the Philippines, their adventures, and the Mathematical Ashes Competition. The Ashes have been retained (again). I am feeling quite sorry for our Australian friends.

July 13 and 14 These are the co-ordination days. Dominic and I have studied the scripts carefully, and we think we know what they deserve. There is a glitch at the start, and things start a little late. Also there are only a handful of chairs outside the co-ordination room. Extra seating is obtained in short order, and when co-ordination starts, it proves very efficient. Our scripts have been carefully read, and there are very few matters to discuss in detail. The only things which hold us up are when students have done things which are not quite covered by the marking scheme, and the co-ordinators are very sensible and natural justice prevails.

There is one instance where Dominic notices that we have been treated a shade more generously than the Australians, and we know that the same issue will arise for at least one Polish script. Dominic alerts everyone to the issue, and the co-ordinators make a minor adjustment to ensure that we are all treated fairly.

The final jury meeting is in the evening of July 14. There is an election to the IMOAB and Dávid Kunszenti-Kovács, team leader of Norway is elected. As a contestant, Dávid competed seven times for Norway. His breadth of experience includes obtaining all possible results, from gold medal through to nothing at all.

We remember the late Geoff Ball by standing in silence. Geoff was the IMO Deputy Leader of Australia nine times in the 1980s.

Some amicable disputes between leaders and co-ordinators are resolved by the jury, and we set the medal boundaries according to the procedure introduced last year. The jury sees the gold, silver, bronze pie charts of the 8 most plausible medal boundary selections. Four of them violate the "no more than 50% get a medal" rule, and some members of the jury immediately move that we should break our rule. A 2/3 vote of the jury is required to do this, and this cannot be mustered. Therefore we focus on the four remaining medal boundary selections and make our choice. Then it is revealed what we have chosen. The cut offs are 16 for bronze, 22 for silver and 29 for gold.

July 15 The closing ceremony is in a hall that we have not visited before. The event begins with high-voltage Oom Pah Pah involving robust Hong Kong youth jumping around like cheerleaders while playing large brass

instruments.

A leading Hong Kong politician disappeared as the medal presentation ceremony began. I suspect this was a precaution in case he was to see a flag to which he would have been obliged to take offence. In fact the people who might have caused the offence behaved with perfect restraint, and no such flag was shown. The politician surfaced again when there was no danger of seeing anything controversial. Good work all round.

The final banquet was in a huge hall, with (approximately) each country having its own dedicated round table. Note to other host countries: this is the perfect way to run a final banquet. Once more I sit with other people almost as important as myself including Carrie Lam. Assiduous readers of these reports will recall that Josh Lam competed for the UK at IMO 2011 and IMO 2012 while he was being educated in England. Carrie Lam is Josh's mum and, incidentally, Chief Secretary of Hong Kong, head of the civil service.

The audience is treated to my surprise speech when I am told, with no notice whatever, that I will say a few words of appreciation.

After the dinner there is finally time to relax, and to enjoy banter with both the team and old friends.

July 16 I fly home relaxed and so tired that I sleep the whole way across China. Unfortunately I am on a plane a couple of hours ahead of the main party. My only regret about IMO 2016 was that I did not spend more time with the UK team. Thank goodness that Dominic and Jill do such a good job. We lose two students to university this year, but the other four are available for selection for IMO 2017 in Brazil, and Harvey Yau will also be available for IMO 2018. He will spend the next 12 months drawing neat geometry diagrams I am sure.

I muse on the plane how sad it is that no girl won a gold medal at IMO 2016. The only two years with no female IMO gold medallist this millennium are 2013 and 2016. Work is being done all over the world, including at EGMO, to address this matter. Clearly it is very much work in progress.

A happier thought is that it has become clear during IMO 2016 that Norway will soon offer to host IMO 2022 and Japan to host 2023. These countries have well funded and well organized maths olympiad organizations, so the medium term future of the IMO looks secure. I have been continuing my campaign to get an Australian or New Zealand IMO in 2028 when the total eclipse of the sun of 22 July (π day) will pass through Wyndham, Kununurra, Tennant Creek, Birdsville, Bourke, Dubbo, Sydney and Dunedin.

Perhaps the obvious thing to do would be to put the students in Kununurra and the jury in Tennant Creek.

Thanks

This was a remarkably strong performance by a wonderful team. Sitting behind these young people is huge support, both in intellectual terms but also in terms of support from families and schools. There is also the UKMT infrastructure, and the army of volunteers and staff who create the competitions, publications, camps, mentoring and other activities which underpin the UK contribution to the IMO and other mathematics competitions.

I must specifically thank *Oxford Asset Management* for their financial support for the IMO team. Their natty logo adorns the team blazer.