

Balkan Mathematical Olympiad 2017 – UK report

Ohrid, Republic of Macedonia

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The Balkan Mathematical Olympiad is an annual competition for secondary school students. Eleven countries in Eastern Europe send official teams, and a generous handful of other nations participate as guests. The 2017 competition was hosted by the Republic of Macedonia, and took place next to lake Ohrid.

The UK has a self imposed rule that no student may attend the BaMO¹ more than once. This rule ensures that more students get a chance to experience an international contest.

The UK team for 2017 consisted of:

Hugo Aaronson	St Paul's School, London
Sam Bealing	Bridgewater High School, Warrington
Emily Beatty	King Edward the VII School, Sheffield
Thomas Pycroft	Whitchurch High School, Cardiff
Yuta Tsuchiya	Queen Elizabeth's School, Barnet
Naomi Wei	City of London School for Girls, London

Overall the event was a huge success, and would not have been without the hard work of a very large number of people. Thanks are due to the team members, their families and their teachers, and also to many people involved with the UKMT, particularly Dominic Yeo and Bev Detoeuf. The team was accompanied by Gerry Leversha, Jill Parker and myself. Gerry and Jill were consistently marvellous throughout the week, and made being the team leader great fun. I am also grateful to the organising committee for putting on such a well run competition, and to Slagjana Brsakoska in particular for being unfailingly welcoming, patient and helpful.

The results of the team were as follows:

	P1	P2	P3	P4	Σ	
Hugo Aaronson	10	0	10	3	23	Bronze
Sam Bealing	10	10	10	0	30	Bronze
Emily Beatty	10	0	10	1	21	Bronze
Thomas Pycroft	2	0	0	0	2	-
Yuta Tsuchiya	5	0	0	0	5	-
Naomi Wei	2	10	5	0	17	Bronze

The medal boundaries were Gold: $\Sigma \geq 39$, Silver: $\Sigma \geq 31$, Bronze: $\Sigma \geq 16$.

Technically the BaMO is an individual competition, but the countries with the highest aggregate scores out of a possible 240 were Bulgaria (226), Serbia (208) and Italy (187). The UKs total of 98 placed us fourteenth overall. Congratulations to Bulgaria and to the seven students with perfect scores who hailed from Serbia, Romania, Italy, Greece and Bulgaria ($\times 3$).

¹The ‘a’ is not standard, but avoids confusion with the British Mathematical Olympiad.

The problems

- Find all ordered pairs (x, y) of positive integers such that:

$$x^3 + y^3 = x^2 + 42xy + y^2.$$

- Let ABC be an acute triangle with $AB < AC$ and let Γ be its circumcircle. Let the tangents to Γ at B and C be t_B and t_C respectively and let their point of intersection be L . The line through B parallel to AC intersects t_C at D . The line through C parallel to AB intersects t_B at E . The circumcircle of triangle BDC meets the side AC at T where T lies between A and C . The circumcircle of triangle BEC meets the line AB at S where B lies between A and S .

Prove that the lines ST , BC and AL are concurrent.

- Let \mathbb{N} be the set of positive integers. Find all functions $f : \mathbb{N} \rightarrow \mathbb{N}$ such that:

$$n + f(m) \text{ divides } f(n) + nf(m)$$

for all $m, n \in \mathbb{N}$.

- There are $n > 2$ students sitting at a round table. Initially each student has exactly one candy. At each step, each student chooses one of the following operations:

- Pass one candy to the student on their left or the student on their right.
- Divide all their candies into two, possibly empty, sets and pass one set to the student on their left and the other to the student on their right.

At each step the students perform their chosen operations simultaneously.

An arrangement of candies is *legal* if it can be obtained in a finite number of steps.

Find the number of legal arrangements.

(Two arrangements are different if there is a student who has different numbers of candies in each one.)

Solutions

The outline solutions below are not full formal write ups, I have omitted some (hopefully) easy details, and also tried to indicated how the different solutions might have been arrived at naturally.

Problem 1

Possible initial thoughts about this equation might include:

- (i) I can factorise the sum of cubes on the left.
- (ii) How can I use the 42?
- (iii) The left is cubic and the right is quadratic, so if x or y is very large there will be no solutions.

The first two might lead us to rewrite the equation as $(x+y)(x^2 - xy + y^2) = x^2 - xy + y^2 + 43xy$. The number $43 = 42 + 1$ is prime which is good news since we have $(x+y-1)(x^2 - xy + y^2) = 43xy$.

Now we can employ some wishful thinking: if x and y happen to be coprime, then $(x^2 - xy + y^2)$ has no factors in common with x or y so it must divide 43. This feels like a significant step except for the fact that x and y may not be coprime.

This suggests writing $x = dX$ and $y = dY$ where d is the highest common factor of x and y .

We end up with $(dX + dY - 1)(X^2 - XY + Y^2) = 43XY$ so $X^2 - XY + Y^2$ equals 1 or 43.

The first of these readily gives $X = Y = 1$. A neat way to deal with the second is to assume $Y \leq X$ so $43 = X^2 - XY + Y^2 \geq Y^2$. This gives six cases for Y which can be checked in turn. Alternatively you can solve $X^2 - XY + Y^2 = 43$ for X and fuss about the discriminant.

In the end the only solutions turn out to be $(x, y) = (22, 22), (1, 7)$ or $(7, 1)$.

Another reasonable initial plan is to bound $x+y$ (using observation (iii) above) and then use modular arithmetic to eliminate cases until only the correct solutions remain. Working modulo 7 is particularly appealing since $7|42$ and the only cubes modulo 7 are 0, 1 and -1 .

We have $x^3 + y^3 = (x+y)^3 - 3xy(x+y)$ and also $x^2 + 42xy + y^2 = (x+y)^2 + 40xy$ so the equation becomes $(x+y)^3 = (x+y)^2 + xy(3xy + 40)$. Now using $xy \leq \left(\frac{x+y}{2}\right)^2$ leads to $x+y \leq 44$.

This leaves a mere 484 cases to check! The modulo 7 magic is not really enough to cut this down to an attractive number, and although the approach can obviously be made to work, to call it laborious is an understatement.

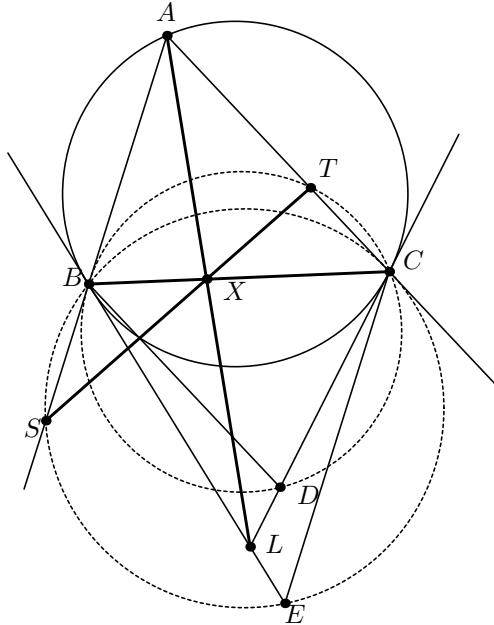
Other possible approaches, such as substituting $u = x+y$, $v = x-y$, seem to lie somewhere between the two described above in terms of the amount of fortitude needed to pull them off.

Problem 2

How we attack this problem depends on how much triangle geometry we can effortlessly recall – a good knowledge of some standard results helps a great deal.

We might instantly note that AL is a symmedian of ABC , and so divides the line BC in the ratio $c^2 : b^2$. Now the plan is to show that ST also divides BC in this ratio.

Since we are working with ratios of distances, Menelaus' theorem may prove useful.



A key step is to notice (based on a careful diagram) that AC is tangent to circle CBS . Once spotted this is easy to prove. The parallels give us $\angle B = \angle BCE$, and, since BE is tangent to circle ABC , we have $\angle EBC = \angle A$ by the alternate segment theorem. Now angles in a triangle give $\angle C = \angle CEB$, and we have the converse to the alternate segment theorem. We obtain AB is tangent to circle CBD in the same way.

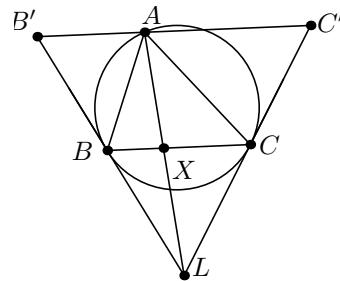
Now we have some tangencies and want some ratios.

Tangent-secant yields $AT \cdot b = c^2$ and $c \cdot AS = b^2$ or, equivalently, $b \cdot CT = b^2 - c^2$ and $c \cdot BS = b^2 - c^2$.

By Menelaus we know that ST divides BC in the ratio $AT \cdot BS : AS \cdot CT$ and it's all over bar the shouting.

If we are not lucky enough to have the stuff about the symmedian at our fingertips, we can still get essentially the same solution with a bit more work. We begin with the second half of the proof above, and establish that ST divides BC in the ratio $c^2 : b^2$. Now we need to prove that AL divides BC in the same ratio.

The next (non-obvious!) step is to draw a parallel to BC through A as shown.



Now $\triangle ABC \sim \triangle B'AB \sim \triangle CC'A$ and the ratio $B'A : AC'$ which equals $BX : XC$ drops out as $c^2 : b^2$ as required.

Clearly knowing the standard symmedian configuration and corresponding ratio is an enormous advantage.

Finally it is worth noting that, to the right sort of mind, the problem screams out for areal coordinates. These turn out to kill it fairly easily, not least because all three circles pass through at least two vertices of $\triangle ABC$.

Problem 3

The striking thing about this problem is that the relation concerns divisibility rather than equality. How can we exploit this? We are given that $n + f(m)|f(n) + nf(m)$ but we can certainly add or subtract multiples of the left hand side from the right hand side and preserve the divisibility. This leads to a key idea:

‘Eliminate one of the variables from the right hand side.’

Clearly $n + f(m)|f(n) + nf(m) - n(n + f(m))$ so for any n, m we have

$$n + f(m)|f(n) - n^2 \quad (\star)$$

This feels like a strong condition: if we fix n and let $f(m)$ go to infinity, then $f(n) - n^2$ will have arbitrarily large factors, which implies it must be zero.

We must be careful: this argument is fine, so long as the function f takes arbitrarily large values. (We also need to check that $f(n) = n^2$ satisfies the original statement which it does.)

We are left with the case where f takes only finitely many values.

In this case f must take the same value infinitely often, so it is natural to focus on an infinite set $S \subset \mathbb{N}$ such that $f(s) = k$ for all $s \in S$. If $n, m \in S$ then the original statement gives $n+k|k+nk$ where k is fixed and n can be as large as we like.

Now we recycle our key idea and eliminate n from the right.

$n+k|k+nk - k(n+k)$ so $n+k|k - k^2$ for arbitrarily large n . This means that $k - k^2 = 0$ so $k = 1$, since it must be positive.

At this point we suspect that $f(n) = 1$ for all n is the only bounded solution, so we pick some t such that $f(t) = L > 1$ and try to get a contradiction.

In the original statement we can set $m = t$ and get $n + L|f(n) + nL$. Eliminating L from the right gives us nothing new, so how can we proceed? Well, we have an infinite set S such that f is constantly 1 on S so we can take $n \in S$ to obtain $n + L|1 + nL$

Using our key idea one more time and eliminating n from the right, we get $n+L|1 - L^2$ for arbitrarily large n which is impossible if $L > 1$.

A rather different solution can be found by playing around with small values of m and n .

As before it helps to establish (\star) but now $(n, m) = (1, 1)$ gives $1 + f(1)|f(1) - 1$.

The left is bigger than the right, so the right must be zero - $f(1) = 1$.

Now try $(n, m) = (2, 1)$ and obtain $2 + f(2)|f(2) - 4$. Subtracting the left from the right gives $2 + f(2)| - 6$. Since $f(2) \in \mathbb{N}$ the left is a factor of -6 which is bigger than 2. This gives $f(2) = 1$ or $f(2) = 4$.

In the first case we can plug this back into the orginal statement to get $2 + f(m)|1 + 2f(m)$. Now taking two copies of the left away from the right we have $2 + f(m)| - 3$.

Thus $2 + f(m)$ must a factor of -3 which is bigger than 2, so $f(m) = 1$ for any m .

Before proceeding with the case $f(2) = 4$ we take another look at our strong result (\star) . Setting $n = m$ gives $n + f(n)|f(n) - n^2$ so taking $f(n) - n^2$ away from $n + f(n)$ shows that

$$n + f(n)|n + n^2 \quad (\dagger)$$

Let see if we can use (\star) and (\dagger) to pin down the value of $f(3)$, using $f(2) = 4$.

From (\star) we have $3 + 4|f(3) - 9$ and from (\dagger) we have $3 + f(3)|12$. The second of these shows $f(3)$ is 1, 3 or 9, but 1 and 3 are too small to work in the first relation.

Similarly, setting $(n, m) = (4, 3)$ in (\star) gives $4 + 9|f(4) - 16$ while $n = 4$ in (\dagger) gives $4 + f(4)|20$. The latter shows $f(4) \leq 16$ so $13|16 - f(4)$. The only possible multiples of 13 are 0 and 13, of which only the first one works. Thus $f(4) = 16$.

Now we are ready to try induction. Assume $f(n-1) = (n-1)^2$ and use (\star) and (\dagger) to obtain $n + (n-1)^2|f(n) - n^2$ and $n + f(n)|n + n^2$. The latter implies $f(n) \leq n^2$ so the former becomes $n^2 - n + 1|n^2 - f(n)$. If $f(n) \neq n^2$ then $n^2 - f(n) = 1 \times (n^2 - n + 1)$ since any other multiple would be too large. However, putting $f(n) = n-1$ into $n + f(n)|n + n^2$ implies $2n - 1|n(1+n)$. This is a contradiction since $2n - 1$ is coprime to n and clearly cannot divide $1+n$.

Problem 4

One possible initial reaction to this problem is that there is rather too much movement of caramels² at each step to keep track of easily. This leads to the question: ‘How little can I do in, say, two steps?’ If every student passes all their caramels left on one step using (b), and all their caramels right on the next step, then no caramels move. (This is rather too little movement.) Let us see what a small change to this sequence can accomplish. We choose a student with at least one caramel. At the first step, she passes one caramel to the right and any others she has to the left. Every one else passes everything left. At the next step everybody passes everything right. The effect of this is that exactly one caramel has moved exactly two places to the right. Similarly, there is a double step which moves exactly one caramel to places to the left.

If we have not already done so, now is the time to start working through some small values of n .

The case $n = 3$ yields a useful observation. Going two places (let’s call this a *double jump*) to the left on a triangle is the same as going one place to the right. Indeed if n is odd, say $2k + 1$, then k double jumps to the left moves the caramel one place to the right and vice versa.

It is now clear that, if n is odd, any arrangement of caramels is possible. We simply move them into position one at a time.

In the case $n = 4$ it seems hard to get all the caramels into one place. Indeed, if we limit ourselves to double jumps, then we can only get $(1, 1, 1, 1)$, $(1, 2, 1, 0)$, $(2, 2, 0, 0)$ and rotations of these arrangements. What can we say about these? Well it seems that students one and three always hold two candies between them. Having noticed this, it is not too hard to make a more general observation: if n is even then a double jump cannot change the total number of caramels held by the odd numbered students. However, double jumps are not the only moves available to us. For example, it is possible to go from $(2, 2, 0, 0)$ to $(3, 1, 0, 0)$. A double jump now gives $(2, 1, 1, 0)$ as well.

To squeeze maximum value out of the $n = 4$ case, it is worth looking at the arrangements we have not yet managed to get to. They are (rotations of) $(4, 0, 0, 0)$, $(3, 0, 1, 0)$ and $(2, 0, 2, 0)$. What do these have in common? They are precisely the arrangements where the even numbered students hold all the caramels. Can we prove that these are illegal? Well, what can we say about an arrangement which precedes one of these elusive ones? This question leads to the last big idea in the solution to this problem. If after some step the even numbered students have all the caramels, they cannot have had any at all before the step, else they would have passed at least one caramel to an odd numbered student.

Turning this around gives a crucial lemma for even values of n . Let’s call a caramel held by an odd numbered student an *odd caramel* and define *even caramels* similarly. Let’s call an arrangement with at least one odd caramel and at least one even caramel *balanced*. If the arrangement is balanced before some step, then it will be balanced after the step. The initial position is balanced, so every legal position is balanced.

Finally we are on the home straight. We claim that every balanced position is legal. Using double jumps we can move to $(\dots, \frac{n}{2}, \frac{n}{2}, 0, \dots)$. Now we need to tinker with the numbers of odd and even caramels. There are lots of usable sequences. For example:

$$\begin{aligned} &(\dots, a, b, 0, \dots) \\ &(\dots, a - 1, 1, b, \dots) \\ &(\dots, a - 1, b + 1, 0, \dots) \end{aligned}$$

can be used to change the number of odd caramels provided $a - 1 \geq 1$.

Once we have the correct number of odd and even caramels, they can be moved into place using double jumps.

It remains to observe that there are $\binom{2n-1}{n}$ possible arrangements of caramels, and that if n is even, then $2\binom{\frac{3n}{2}-1}{n}$ of these are not balanced.

Another sensible approach is to think about which steps are reversible. It turns out that many are, including all those where the students all use option (b).

It is possible to argue that if n is odd, then we can start with any position, move to (\dots, n, \dots) reversibly, then move to the initial position reversibly. Playing the whole tape backwards shows all positions are legal.

If n is even it is possible to start from any balanced position and reversibly move to $(\dots, n - 1, 1, \dots)$ and thence to the initial position so we are done.

²The word ‘candy’ was a little too grating for my delicate British ears. I am grateful to the Italians for suggesting the more elegant alternative.

Leader's Diary

Monday 1st May

Our flight to Skopje leaves early on Tuesday morning, so we will be spending tonight at the Luton airport Ibis hotel. My bus deposits me at the main terminal, and I spend some time failing to find the hotel in the sea of building works. When I finally arrive, most of the team are already assembled. I mention that getting here from the terminal is tricky and am politely informed that this is false.

The other team members arrive in time for dinner and a few maths problems. They seem happy enough but are fairly quiet. I suspect their calm exteriors conceal some understandable nerves: Naomi represented the UK at EGMO 2017 in Zurich, but for the others this will be their first international maths competition. Everyone opts for an early night.

Tuesday 2nd May

We leave the hotel promptly at 7am and follow the single straight path to the terminal. We get to the gate in good time and wait patiently. Shortly after our scheduled departure time we are informed that our plane has now arrived in Luton which is surely better than the alternative. Once airborne the flight goes smoothly. The toddler in front of us is mercifully quiet, but has a knack for transferring momentum through the seat and into Gerry's patellas. I offer to swap seats with Gerry who duly moves one place to the left. Three minutes later the toddler does likewise.

We arrive only slightly late and are met at the gate by some of the organisers. The small chess master class under way in the arrivals hall turns out to be the Italian team with whom we share a four hour bus ride to Ohrid. I introduce myself to the leader and deputy in LOUD – S L O W – ENGLISH. To my relief it becomes apparent that their command of the language is at least as good as mine.

We deposit Jill and the students at a large Hotel a few miles outside Ohrid and drive another thirty five minutes to the leaders hotel in Struga. This is a small no frills affair, and we are the only guests. On arrival we learn that Gerry is not on the list. In years gone by Gerry has been deputy leader at the BaMO and has therefore stayed with the students. Jill, suprema of all things pastoral, has been an 'Observer with students'. The intention this year was to reclassify Jill as deputy leader, freeing Gerry up to be 'Observer with leader' so that he and I can work on mathematical matters together. Details of this creative arrangement have not made it to Macedonia, but our hosts kindly agree that Gerry can stay, and he is given a key to room 24. None of the rooms are labelled 24, but since room 16 is directly above room 8, I suggest that the room directly above 16 is intended. The room is vacant, but the key does not work, creating a security problem for Gerry's effects. When he closes the door the handle comes off in his hand. I suggest that taking it with him would solve the security problem. A few minutes later he has a new room with a fully functioning door and key.

After dinner we are given a shortlist of twenty-six problems to consider by 10am the next morning. Fortified with coffee and chocolate, the Italian leader and I spend a few hours working on the combinatorics in the dining room. We are kicked out shortly after eleven, and by midnight I decide sleep will be a more mathematically productive use of my time.

Wednesday 3rd May

Shortly before 5am I become aware that the hotel is close to a Mosque. In my case the call to prayer also functions as a call to mathematics, and I return to the shortlist. At 10 o'clock the jury meet to discuss the problems. We weed out those that have appeared elsewhere and classify the rest as Easy, Medium, Medium/Hard and Hard³. This process is made more difficult by the fact that time constraints have forced many (all?) of us to refer to the solutions to some problems, rather than solving them from scratch. There is, therefore, a tendency to equate the difficulty or otherwise of the problem with the complexity of the official solution. I think the discrepancies between my personal classification and what we end up with are explained by this tendency, but nothing looks wildly out of place.

There are a number of constraints on constructing the paper. We need one question from each of the four main areas (algebra, number theory, geometry and combinatorics) and also need a range of difficulties. There are only two easy problems, and the jury quickly settle on the number theory option. We are also short on usable combinatorics. I have a soft spot for the Medium/Hard C5 despite its use of the word 'candy'. A likely alternative is C6 which concerns convex n -gons and is resplendent in the

³The Saudi Leader jokingly proposes the alternative categories 'Medium Nice', 'Medium Ugly' and so on. I approve wholeheartedly.

'Hard' column on the board. This particular column is sparsely populated, and it makes sense to choose something from here next. Since the UK is a guest nation, I do not vote on the jury, but I am a little hazy on whether or not I am allowed to propose motions to be voted on by the other leaders. The chairman does not object, and I argue that C5 should be considered as a candidate for problem 4. The other 'Hard' options are technical and, I maintain, not sufficiently elegant. The motion is carried, and C5 is selected shortly afterwards. Choosing problems 2 and 3 goes smoothly, and by lunchtime we have a paper.

After lunch I get to work on the official English wording of the paper, with the help of the leaders from Serbia and Romania. Both the geometry and the combinatorics prove controversial, and we go through several iterations before being bussed to the students' hotel for the opening ceremony. I brace myself for a long session of speeches, a parade of flag waving contestants and some local music and dancing. Our ever efficient hosts have other ideas: we are warmly and officially welcomed, the Macedonian national anthem is played and we are out in under five minutes.

Back in Struga the other leaders get to work translating the paper into their native languages. I field further questions on whether it would be simpler if all the students mentioned in problem 4 were male, whether the set of points between A and B includes A , and so on. At dinner I sit with Arslan, the leader from Turkmenistan. I learn that Arslan is sometimes spelt without an 'r' and means lion in Turkmen. We chat briefly about C. S. Lewis before returning to the papers. Everything is done before 11pm which feels like a victory.

Thursday 4th May

Today is competition day, and after breakfast the leaders move the students' hotel. This represents a substantial upgrade, complete with stunning views of Lake Ohrid. During the first half hour, contestants may ask questions about the wording of the problems. These questions are brought to the jury who decide how to respond. Predictably, most concern the problem 4, and I am relieved that the modal answer is: 'Read the problem carefully.'

Once the questions are dealt with, we discuss draft mark schemes. The quickest solutions to problem 2 will rely on quoting facts about the symmedian, and Gerry is keen to ensure that students who are ingenious but inexperienced are not too heavily penalised. He eloquently convinces the jury that someone who establishes that 'it suffices to show AL divides BC in the ratio $c^2 : b^2$ ' should get nearly full marks. This is sensible since adding the phrase, 'and this is well known' would probably be enough for a perfect score. I am pleased to see justice being served, but wonder whether the set of candidates who benefit from this ruling will turn out to be empty.

After the exam Gerry and I join the students at lunch and hear how it went. In general they seem to have enjoyed the paper, though questions like 'Did problem 1 need *that many* cases?' make me slightly nervous.

The scripts need to be photocopied before Gerry and I get to see them, so we have some time to relax on the terrace. To the great credit of the organisers, we receive a tidy stack of folders early in the afternoon and are able to get to work much sooner than I had feared. There are three particularly long attempts on problem 1 to wade through, and it is unclear how much Hugo's partial solution to problem 4 might be worth, but the rest is just a matter of careful checking. We retire at a fairly civilised hour.

Friday 5th May

The main business for today is finalise the scores. The coordinators have worked long into the night to get a sense of what the scripts are worth and now need to agree final marks with the team leaders. I have decided that Emily's lengthy assault on problem 1 is essentially flawless, but Yuta's attempt is harder to judge. His method is morally sound but woefully inefficient, and an error near the end means he misses quite a number of cases. The mark scheme is silent on the matter, but I think he has got more or less half a complete solution – we will see.

Coordination is well organised and fair. We get exactly what we ask for on problem 1, and problem 2 is entirely uncontroversial. The problem 3 coordinators are happy enough with a slightly odd explanation at the end of Emily's script to give it 10, and kindly award Naomi one more point than we ask for. I have prepared for our discussion of Hugo's work on problem 4 by writing up what I claim is the solution he was working towards. My argument has six steps of which Hugo has three, so I start by asking for $10 \times \frac{3}{6}$ points. In the end we agree on 3, though perhaps 3.5 would have been nice.

In the meantime most of the students have gone on a trip to a reconstructed iron age settlement called the Bay of the Bones. The prehistoric inhabitants lived on platforms above the surface of lake

Ohrid, and apparently used ropes and fetters to prevent their young children from straying too near the platform edge. I wonder whether this problem solving strategy could be adapted for my own use, but eventually reject it as unpromising.

After dinner there is a final jury meeting to agree the medal boundaries. At the Balkan Mathematical Olympiad at most two thirds of official (as opposed to guest) participants receive medals and these medals are Gold, Silver and Bronze in approximately the ratio 1:2:3. Given this fairly clear framework, I am surprised at the length of the meeting and particularly enjoy the moment when a leader calls for the jury to vote on whether or not the jury should vote on whether or not to break the rules of the competition. In the end there is no real controversy and the only disappointment from my perspective is that Sam has missed a silver medal by one point.

Saturday 6th May

Today there is a joint trip into Ohrid for leaders and contestants. Our excellent tour guide takes us round the major sites of this beautiful town and reminds us that it is cheaper to fly from Luton to Ohrid and back than to spend the weekend in London.

Between Samuel's fortress and the Church of St Clement there is plenty of time to chat about mathematics. The Italian team sat their national maths olympiad at the hotel yesterday, and copies of the paper have been fairly widely distributed, so there is plenty to talk about. Our team can be heard discussing ways of finding arbitrarily large complete subgraphs in a particular graph on the naturals, so I assume they have not been too badly traumatised by Thursday's exam. Gerry claims that one of the geometry questions is a 'very well known result about isodynamic tetrahedra' though he fails to produce a single on-line reference, or, for a short while at least, a proof of the statement.

In the evening we learn that the closing ceremony and final dinner have harmoniously coalesced, so we sit and applaud between mouthfuls. In general I think most things are improved by the addition of dinner, but the arrangement does mean we do not see entire teams on stage. Had there been a uniform parade, the UK would have performed well thanks to Dominic Yeo's good taste, though we would have struggled to compete with Saudi Arabia's traditional garb or the sharp black suits from Turkmenistan.

After dinner there is time for traditional folk dancing, and I am thrilled when all our students decide to give it a go. Aspiring BaMO contestants should note that it is fast becoming a tradition for the entire UK team to dance at this competition.

Eventually Jill, Gerry and I call it a night and leave the team playing cards with the Albanians.

Sunday 7th May

Our bus to the airport leaves at 6.30am. The students play Indian poker, and Gerry establishes that the well-known result about isodynamic tetrahedra is easy to prove if you know how. We arrive in good time and clear security with nary a hiccup.

However, we are still waiting at the gate when our scheduled time of departure comes. Ordinarily this would not be much of a problem, but we have to change in Budapest on the way home and Wizz Air do not seem to do transfers as such. When we land we will (hopefully) have two hours to clear immigration, collect our bags and then check them in to our next flight. The window is fairly tight, and I am relieved when we make up the lost minutes during the flight and land exactly on time.

Those of us who are (for the time being) EU citizens are invited to join a short, steadily moving queue at passport control. Yuta and Naomi are travelling on Japanese and Chinese passports respectively, and their queue is a little longer. I wait with them until I am confident that they will be visible from Hungary, and then cross the border myself. To describe their movement as glacial would give a misleading sense of inevitability to their progress: an hour passes.

Eventually I send the rest of the group on ahead on the grounds that n new tickets are bound to cost more than \sqrt{n} . Finally Yuta arrives at the desk and is waved through. Naomi is not so lucky. It transpires that a) Chinese citizens need visas to pass through Hungary in transit and b) these visas cannot be acquired at the airport. I am not entirely surprised by a) but rather blind-sided by b). Yuta and I have no choice but to leave Naomi in the care of Hungarian immigration, who say they will take her back through the airport to the departure gate so that she does not need to enter the country. The problem of her suitcase is elegantly solved by classifying it as outsize baggage, which can be done within sight of the immigration desk. Making these arrangements leaves Yuta and I with fourteen minutes till our gate closes, and we still have bags to check in. We run to the back of the check in queue and are reassured by a Wizz Air employee that we are just in time. When we reach the desk ten minutes later we are informed by another Wizz Air employee that the first Wizz Air employee was mistaken. Some

rushing around ensues and a lady at the ticket booking office (not a place I had planned to visit today) suggests another neat solution. Yuta and I should simply take all our luggage through security and then pay a fine at the gate for bringing oversized ‘hand luggage’. The idea that anyone could innocently confuse Yuta’s case with hand luggage is risible, but, once we have jettisoned our toiletries, the plan works beautifully. I am the last person to board the flight, and am not unaware of the fact that no one has seen Naomi since immigration. Fortunately the Hungarians have been as good as their word and she is safely aboard. A full delegation, with a full complement of luggage, lands punctually in Luton and the adventure is over.