

Balkan Mathematical Olympiad Report

Collectively Written by the Team of Six

7 to 12 May 2018

Lemma

This will be the best student report ever written.

Proof

According to Alex, this is well-known so the proof is omitted.

Monday — Day 1

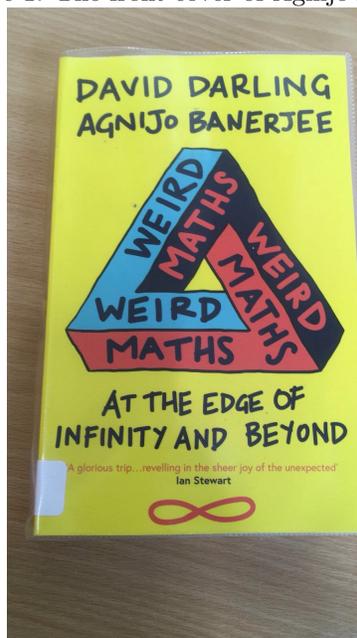
*A group of friends meet at Heathrow;
To Serbia they aim to go.
For an interesting test
Will bring out the best
In the Serbians, yes, but us? No.*

Problem Monday.0:

Find as many typos as possible in the following report.

The story begins as we all arrive at the airport in front of the WHSmiths. Jill successfully recognises Agnijo and Tom as mathematicians because they are 'quiet, well-behaved and not causing trouble'. If Alex had been the first to arrive, maybe Jill would have spent longer trying to identify us. Characteristically, Alex has decided to wear an orange jacket, orange T-shirt and orange watch, all in slightly different shades of orange. We consider looking in the shop for a copy of 'Weird Maths' (currently retailing at £12.99 on Amazon, written by and highly recommended by Agnijo) but end up forgetting to do so.

Figure 1: The front cover of Agnijo's book



Continuing with the theme of books, Tom has predictably forgotten to bring Aron's book about physics which he picked up for him at a physics awards ceremony. It is agreed that instead Tom will endeavour to ask his parents to bring it when we arrive back at Heathrow on the return journey (spoiler: they also forget). Agnijo has remembered his physics books — this is lucky as he has a physics exam on Tuesday. And Vesna has given us a book as well which is full of past Balkan questions — this ends up being passed from person to person (the best way to get rid of it seems to be to point to a geometry problem and say to Aron, "I bet you can't do this geometry problem") as nobody really wants to carry it around. We pass airport security before Aron has the bright idea of putting it in a bag instead. For some reason, Agnijo has brought his EHIC card, despite Serbia not being in the EU, which results in a puzzled expression on his face as he is searched for suspicious items. Thankfully Agnijo has remembered to put his compasses in the hold.

When we get on the plane, we discover that the British Karate Team has also boarded the same plane. We collectively decide that if they need a reserve, we can put Alex forward as he was once a Wimbledon ball boy and therefore has some association with sport.

On the plane, we all decide to do some inequalities in the hope of there being an inequality on the Balkan paper on Wednesday (spoiler: there isn't), apart from Agnijo, who does physics in the hope of there being some physics questions on his physics exam on Tuesday (spoiler: there aren't many). Many of the questions begin with the phrase 'Using your knowledge of physics' (this is important for the rest of the report).

Figure 2: Our first view of [what we're pretty sure is] Serbia from the plane window



We arrive in Serbia and the first piece of cultural information which we learn is that mobile data costs \geq £5 per MB for each person (this really is true). We quickly decide only to use WiFi instead and thankfully the hotel does indeed offer WiFi at some times during the day — these times, unfortunately for us, being completely undecided and random, depending only on the quality of the WiFi at any given time.

Vesna now unfortunately has to leave us since she is meant to be with Dominic for the next two days.

We examine the Balkan Olympiad website and discover that Jill has been promoted to the rank of 'Leader'. The general consensus is that although Dominic is technically the leader, Jill deserves the rank more so far since Dominic has abandoned us to help select problems and we haven't seen him at all.

Tuesday — Day 2

*Today's to relax and have fun,
And the prep work is now nearly done.
Therefore we, Sandra, Jill
Take a walk up a hill
And we're less stressed (according to some).*

On Tuesday we begin the day with some breakfast. Aron quickly becomes fond of a triangular prism-shaped 'cheese' and proceeds to eat not just his own two blocks of cheese, but also both of Agnijo's. We also have some tea, which Alex claims 'tastes like caffeine'.

Agnijo now goes to take his physics exam.

Sandra (our friendly and very helpful guide) suggests that we go on a walk to Avala Tower at some point during the day. This would involve walking up a hill (or, if you're Serbian, 'mountain') for roughly 40 minutes before climbing the tower using a lift.

So in the morning, most of us sit outside for a bit and puzzle over how we would approach solving a horrible, non-symmetric, not-even-cyclic, fifth-degree, three-variable inequality without resorting to calculus. Everybody except Alex gives up within ten minutes of trying it, and Alex himself gives up after around two more hours, by which time we've almost forgotten all about it anyway. This is a pointless exercise because calculus is never useful in any maths Olympiad ever. (Apart from this year's RMM... oh yes and the RMM before that...)

Problem Tuesday.1:

Prove that for any positive reals a, b, c , we have that

$$a^5 + b^5 + c^5 \geq 5abc(b^2 - ac)$$

When does equality hold? (This problem was credited to David Monk in the book 'Introduction to Inequalities'.)

Agnijo now finishes his physics exam.

Eventually we go back to our rooms for a bit and then have lunch. We present the reader with the following Scottish Advanced Higher Physics-style question:

Problem Tuesday.2:

Agnijo and Aron have sat down to eat lunch. From their frame of reference, Jill and the others have not yet materialised. Jill and the others have also sat down to eat lunch. From their frame of reference, Agnijo and Aron have not yet materialised. Using your knowledge of physics, explain this situation.

(Answer: Agnijo and Aron have sat down behind a pillar and could not see and could not be seen by Jill and the others until everyone had finished lunch. The reason for this is that light does not diffract or produce an observable interference pattern when it passes by a pillar.)

Lunch involves beef, which Agnijo does not eat. He is offered the vegan option. Agnijo says that he would indeed like the vegan option instead, and is thus presented with a plate of fish. All our dishes are served with a salad consisting of vinegar with some lettuce in.

After lunch it is revealed that we will need to pay to enter Avala Tower. Luckily, there is an exchange office not too far away. Unluckily, it does not exchange pounds — only euros. Aron sets up his own exchange office with the rest of the team as intended customers, so that he can get euros by giving his pounds away. Aron's exchange rate: £1 to 1.4 euros.

The Azerbaijan team has also decided to come with us, both to exchange money and to walk up the hill. Giles asks, "What is an Azerbaijan?"

Problem Tuesday.3:

What is an Azerbaijan?

It is famously known that when Jill last went up a hill (to fetch a pail of water), her friend and walking partner, Jack, somehow managed to break his crown. Jill had rather a tough time herself, when she came tumbling after. Luckily we do have vinegar (with the odd lettuce) and indigo paper (see tomorrow) in case there is a repeat of this event.

A black dog follows us all the way up the hill, and in typical mathmo-style imaginative fashion, we name it 'Dog'. Dog, unfortunately, will not be allowed into Avala Tower as there is a sign representing the fact that members of his/her species are explicitly forbidden from entering, as well as guns, smoking, and ice cream.

The tower itself is extremely impressive, although we comment that it looks a little like a cartoon rocket as it stands on a tripod. We ask Agnijo to use his knowledge of physics to explain why it is so stable, but Agnijo replies that he is not, unfortunately, able to use his current knowledge of physics to explain this. Knowledge of history is also required, as the tower was bombed in 1999.

Figure 3: Sandra and the team at the top of Avala Tower



After our brief explanation of British politics to Sandra, she in turn explains some of Serbia's own history and culture, which we all find very interesting. We proceed to walk down the hill, and we quickly find we are being followed by Dog who has waited for us. We hope that this is just Dog being friendly, and not a bad omen.

For some reason, Aron wants to mention the word 'goat' here as a reference to a previous Balkan student report. So here goes: "goat". Done. Now goat to the next page and read about what happened on Wednesday.

Wednesday — Day 3

(Warning: the following limerick contains spoilers for those of you who have not yet attempted the questions.)

*A complete quadrilateral again,
With two ants which can walk on a plane.
And for Alice and Bob
Powers of two do the job
But the primes p and q are a pain.*

Today is

1. The day of the actual exam paper
2. The day after the day on which the opening ceremony should really have been, and also
3. The day of the opening ceremony.

The breakfast is composed of hot dogs and some cheese, although not the kind of cheese that Aron likes, fortunately for Agnijo. We hope the hot dogs have no relation to yesterday's Dog and are not bad omens. Alex says he would prefer coffee for the high caffeine content, but we shall have to make do with tea for now.

We then spend about 5 to 10 minutes staring at a list of people and their associated rooms, to work out in which room each contestant will be taking the exam. Between now and the opening ceremony, Aron somehow manages to forget his three-digit room number so we have to go back afterwards to check. This is definitely a bad omen.

Anyway, the opening ceremony is where we briefly see Dominic — for the first time since the Trinity camp. Rather than say anything, he throws us a Union Jack flag. We search for a secret message giving a hint as to how to solve question 1 or something, but we cannot find anything. Just as Aron's English GCSE poetry anthology indicates, "it's just a piece of cloth." Sitting on a table, due to the lack of multiplication ability of the organisers, we wave the flag, hesitantly — none of us know which way up it should be.

Problem Wednesday.1:

Which way up should the Union Jack go?

After a short musical interlude, we hear the opening speech. Far from being a "traditional, boring speech which nobody remembers" (quoted from the speech), in this speech, the speaker, Teodor von Burg, refuses to wish us good luck, on the grounds that everybody would just get one extra mark and the medal boundaries would be raised accordingly. This does beg the question set out in problem Wednesday.2.

Problem Wednesday.2:

Had the speaker wished good luck, and everybody got exactly one extra mark, and the medal boundaries were raised by a mark, what would have happened to the gold boundary in the actual competition?

Just before we sit the exam, we are told about the special 'indigo paper' (which turns out to be white and not indigo) which we have to write on. This involves putting two sheets of paper together, such that when you write on top of one sheet, a copy is produced on the sheet below. Aron, to save time, cleverly stacks all his paper in one big bunch so that he doesn't have to remember to take two sheets at a time. It is only after he has written the first page of his first proof that he realises that his work has been copied not just onto the sheet immediately below, but also all the sheets below that one. Hence he immediately has to ask for more paper. Giles and Tom, who are doing the paper in the same room as each other, are told to "break your leg, as you Brits say," and we desperately hope that the possibly bad omen of Dog yesterday was not a foreshadowing of a more extreme version of Agnijo's sprained ankle at this year's RMM.

Problem Wednesday.5:

A quadrilateral $ABCD$ is inscribed in a circle k , where $AB > CD$ and AB is not parallel to CD . Point M is the intersection of the diagonals AC and BD and the perpendicular from M to AB intersects the segment AB at the point E . If EM bisects the angle CED , prove that AB is a diameter of the circle k . (Balkan 2018/1)

Problem Wednesday.4:

Let q be a positive rational number. Two ants are initially at the same point X in the plane. In the n -th minute ($n = 1, 2, \dots$) each of them chooses whether to walk due north, east, south or west and then walks the distance of q^n metres. After a whole number of minutes, they are in the same point in the plane (not necessarily X) but have not taken exactly the same route within that time. Determine all possible values of q . (Balkan 2018/2)

Problem Wednesday.3:

Alice and Bob play the following game: They start with two non-empty piles of coins. Taking turns, with Alice playing first, each player chooses a pile with an even number of coins and moves half of the coins of this pile to the other pile. The game ends if a player cannot move, in which case the other player wins. Determine all pairs (a, b) of positive integers such that if initially the two piles have a and b coins respectively, then Bob has a winning strategy. (Balkan 2018/3)

Problem Wednesday.6:

Find all primes p and q such that $3p^{q-1} + 1$ divides $11^p + 17^p$. (Balkan 2018/4)

Pedants among you will of course notice that 5, 4, 3 does not occur as an increasing sequence within the set of natural numbers. However, the problems have been labelled in the order in which they should objectively have been numbered in the paper itself. For some reason, the first question was both a first question geometry and not a simple angle chase, with the 'short and easy' solution involving harmonic ranges and Brokard's theorem. Aron finds this solution, but otherwise this doesn't seem to be such a successful question for the overall UK team.

Questions 2 and 3 are somewhat more doable, and in fact every UK student takes a rational approach to question 2 (yes that pun was intended) and everybody except Giles thinks that they have solved question 3.

Question 4 does not go well. In total, it will turn out that we will score only one mark combined for this question.

At the end of the exam, we open our envelopes, take out our stacks of indigo paper, and in dey go. (The rest of the team want to point out that this joke was Nathan's fault.)

We see Dominic after the exam and this time, we actually get to speak to him. He is highly inquisitive about how we think it went, and enquires about the problems in question. Since everybody is so exhausted after a stressful paper, nobody wants an Inquisitor!

(This is an in joke which readers are not expected to get.)

We think of going to a restaurant for lunch, but using Vesna's knowledge of Serbian geography combined with Agnijo's knowledge of physics, we deduce that we cannot walk fast enough to get there and back in a reasonable length of time.

In the afternoon, we head out to Belgrade to walk around the city before going to the Nikola Tesla museum. This excursion was kindly organised by Sandra and her friends.

Nathan manages to get himself temporarily lost in Belgrade while getting a drink of water, but finds his way back relatively quickly (as it turns out that mathematicians are indeed recognisable for their characteristic quietness, good behaviour, and lack of ability to cause trouble). We go to a church, which, according to the guides, people started to build some centuries ago. Clearly, the builders working on the church are some of the laziest ever, as although the underground part is decorated highly ornately with gold paint and elaborate depictions of Jesus and other people with halos, work is still unfinished in the main part of the church, with loads of metal scaffolding. Aron uses his knowledge of GCSE RS to deduce that, had Jesus been present, he would have dramatically flipped some tables due to the souvenir shop within the church.

Figure 4: The church



Walking from the church, we see a water fountain. Aron feels thirsty so queues at the back of a queue of approximately 20 mathmos, and just as he gets to the front, he fears that the water may not be drinkable, despite numerous Serbian guides telling him that it definitely is. Aron, still suspicious of the water, declines.

We arrive at the Nikola Tesla museum, and we watch a film about Nikola Tesla. The UK team collectively wrote the following haiku about Nikola Tesla:

*Nikola Tesla
Using his knowledge of phys-
Ics invented things*

The Nikola Tesla museum was in fact very enjoyable. We learned about how Nikola Tesla used his own knowledge of physics to invent an Tesla coil which ionised air. Some slightly worrying things were said by our guide, such as "Oh, by the way, nobody has a pacemaker here, right?" just after "This generates half a million volts." Nobody gets electrocuted, though, so all is fine.

Here's an interesting quote found by Agnijo: *'I loved that pigeon as a man loves a woman, and she loved me. As long as I had her, there was a purpose to my life.'* — Nikola Tesla

We finish the day by going to a restaurant to eat dinner. We are played some traditional Serbian music/other music/both (we aren't sure which) as we have salad, which, when compared to the salad at the hotel, has

1. More lettuce and
2. Considerably less vinegar.

The food is exceptionally tasty, anyway, and we all have a great time.

It's worth mentioning a little tip: when in Serbia, always order a taxi beforehand if you plan to get one back, since Vesna and Dominic suspect that we are slightly ripped off by the taxi drivers. The three taxis charge the same amount of money each, which proves a modern version of the old proverb: "Honour among thieving taxi drivers".

Thursday — Day 4

*On Thursday, some high hopes come down,
And so later we go into town.
Therefore when we return,
Our confirmed scores we learn,
And Aron is given the crown.*

Unlike before, the day does not begin with all of us having breakfast. Instead, the day begins with everybody except Alex having breakfast, and Alex himself being asleep.

So, since we are all very nervous about what our scores are going to be and whether or not we shall get a medal, and if we do get a medal, what colour it will be, we decide to gather in Agnijo and Aron's room to discuss the paper and make ourselves even more nervous. Agnijo, fearing that Serbian food might not actually be edible, brought both a box of chocolate biscuits and a box of chocolate truffles. The chocolate biscuits prove to be particularly popular.

Tom is hoping to get a 'pity mark' for proving the converse of question 1 — unfortunately this is way easier and has basically nothing to do with actually solving the problem. Giles is fairly confident about what the rough range of his score — he attempted question 2 and got it right. Aron mixed up the words 'odd' and 'even' on question 3, and somehow remembers that he did so. Nathan, like Agnijo, appears to have done well, having solved questions 2 and 3, with expected partial marks for question 1. We don't know about Alex yet because Alex is still asleep.

To take our minds off things, Agnijo gives us a problem involving rooks, dominoes and a chessboard.

Problem Thursday.1:

Consider an n by n chessboard. Define a rook to be a 1 by 1 square which cannot be placed in the same row or same column as another rook. Define a domino to be a 2 by 1 rectangle which cannot overlap, either with other dominoes or other rooks. For which positive integer values of n is it possible to tile the chessboard completely with n rooks and the rest dominoes?

Meanwhile, Alex finally enters. Tom is curious about Alex's synthetic solution to question 1. It takes Tom five times to say 'slowly' before Alex stops saying random letters and actually draws a diagram to refer to. There is one point in the proof which Tom thinks might be a little under-detailed, and that Alex quotes as 'well-known'. When asked about it, Alex replies, "It's definitely

true because I checked.” When Tom asks Alex to explain what he means by ‘checked’, Alex says that he ”drew a few diagrams and it looked right”.

On a more serious mathsy note, the lemma Alex needed to prove is actually an interesting problem in itself, and we leave it as an exercise to the reader to find a nice synthetic proof of Wednesday.5 using this idea.

Problem Thursday.2 (Alex’s Lemma):

Given a triangle ABC , let the internal angle bisector of angle BAC meet side BC at D . Let I be a point inside the triangle, on AD , and let BI and CI intersect sides CA and AB at E and F respectively. It is known that angles CBE and DIC sum to a right angle. Prove that I is the incentre of triangle ABC .

(Answer: according to Alex, this is a well-known fact so the proof is omitted.)

Meanwhile, Dominic enters. It’s incredible what he can do to lower stress levels in a room and reassure everybody that they’ve all done well, as the first thing he says is, ”Who would prefer it if I told you your predicted score privately, if it’s much much lower than you expected?” It turns out that this question is totally meaningless, as although everybody replies that they don’t mind either way, he still calls people out of the room so that they can discuss their scores with him.

It turns out that Alex will get nothing for question 1. This seems slightly unfair, as, although he says that the result which needed to be prove was well-known when it wasn’t, he has still reduced the problem to a simpler one and the team collectively feels he deserves at least some marks for this. As well as this, his questions 2 and 3 have many errors in them so his scores are likely to be considerably lower than 10. Agnijo, expecting a 4 for question 1, finds out that he can now receive at most a 2, and Tom, despite having the right ideas for question 2, will probably only get a 6 due to the fact that he has missed out the case $q = \frac{3}{2}$.

On the bright side, Agnijo will likely receive a mark for his observations about question 4, as well as a probable full marks on questions 2 and 3. Aron appears to have done very well, and is likely to get full marks or near full marks on each of questions 1, 2 and 3.

After lunch, there is another excursion, to Belgrade again. We arrive outside the government buildings, and there are political messages on posters stuck to the fences outside. Some of them refer to the protection of ‘Albanian terrorists’ by ‘EUleks’ (Is this us? If so, not for much longer). On a completely unrelated note, we learn from Jill that there is ongoing tension between the nations of Serbia and Albania.

We visit a castle and walk around. Sandra takes photos of us gazing thoughtfully into the middle distance on a bench at the top of a hill (again, if you’re Serbian, read ”mountain”).

Figure 5: Alex gazing thoughtfully into the middle distance



Figure 6: Tom gazing thoughtfully into the middle distance



When we arrive back at the hotel, we eat and quickly proceed outside to see what scores and medals each of us has got. The UK results are below.

Contestant	Q1	Q2	Q3	Q4	Total	Medal
Agnijo (UK1)	1	10	10	1	22	Bronze
Nathan (UK2)	3	10	8	0	21	Bronze
Alex (UK3)	0	5	3	0	8	
Tom (UK4)	0	6	10	0	16	Bronze
Giles (UK5)	0	8	0	0	8	
Aron (UK6)	10	10	9	0	29	Silver
Total	14	49	40	1	104	Two Golds and a Bronze
Min	0	5	0	0	5	
Max	10	10	10	1	31	Silver

Note that two guest contestants got 29, one of these being Aron. Since the medal boundaries are decided by how well the Balkan contestants did, the silver boundary was initially set at 31. It was only after a petition by Dominic and Vesna went around the jury that this was lowered to 29, which didn't affect any Balkan contestants.

But before we praise Dominic too much, it's also worth mentioning that Dominic also tried to stop Aron getting a silver medal by claiming that his solution to question 2 was not worth 10 marks. Luckily for Aron, the jury found this to be an unreasonable suggestion. It's unclear whether Dominic is on Aron's side or not. Dominic says that he is on the side of "completely correct mathematics."

Unfortunately, Tom got 16, which is not coprime to 4 so we cannot construct a homogenous polynomial with integer coefficients which is equal to 1 for each contestant to Q6 from IMO 2017, to describe the team's performance.

Agnijo was the only UK contestant to get marks on all four questions, and also to get two tens on questions 2 and 3. Nathan got 0 for question 4, despite solving it (the reason for this is that he solved it on the bus coming back from Belgrade and didn't have a supply of indigo paper to hand). Alex was the only UK contestant to have exactly one question with a score equal to his contestant number. Tom was the only contestant to have a score as a perfect fourth power which was also the square of his contestant number. Giles was the only UK contestant to get score numbers which all have two lines of reflectional symmetry. Aron was the only contestant in the entire competition to get a score equal to his overall position.

Strangely the Azerbaijan team (so that's what an Azerbaijan is!) seems to have performed oppositely to us, doing very well in questions 1 and 4 but somewhat less well in questions 2 and 3.

We proceed to Agnijo and Aron's room to discuss the scores, and while Tom goes and puts his rucksack in his own room, Giles mysteriously disappears. When Tom returns, it is revealed that Giles has used his knowledge of physics to climb into a small cupboard, although he fails to make a dramatic exit from the aforementioned cupboard when Tom walks in.

Dominic and Vesna encouraged us to talk to some other people, so we head downstairs and do indeed talk to some other people. We first talk to some friendly-looking people who turn out to be the Romanian team, all of whom scored more than the UK's top scorer. We play their (simpler) version of the traditional UKMT card game, Irish Snap, but according to the Romanians, the aim is to get all the cards by snapping first. This is wrong but we go along with it anyway.

We introduce them to some new rules, such as the 'sixes' rule (originally developed by Agnijo), and the 'saying snap when the card has the same number as the card below' rule, which, strangely, they seemingly did not know. If any Romanians are missing any cards, we may have accidentally stolen one of them — sorry.

We also decide to talk to the Albanian team. They turn out to be very friendly indeed. The Turkish team arrive and we talk to a couple of them as well.

Never mind — we go to another table to talk with two Serbian guides and a Serbian contestant. We decide to teach them another traditional UKMT card game, which begins with M and rhymes with 'Yau'. For those wishing to know more about the game, the only rule we are allowed to tell you is this one.

Friday — Day 5 (Before the Sleep Bit)

As it always does, the game turns to chaos. This is because Nathan does something to change the game somewhat (we can't say any more) and we nearly run out of cards. It is quite late by the time we eventually go to bed.

Friday — Day 5 (After the Sleep Bit)

*A journey which takes rather long,
And Dominic says a word wrong.
"So is this a real cave?"
And then Vesna's dad gave
The awards to some eighty-four strong.*

Today is the day of the big excursion, and requires us to leave at 9am precisely. The journey should take two hours. For those of you who are skilled at mental arithmetic, you will have no doubt worked out that we should arrive at 11am.

Unfortunately, there are some of us (or, more specifically, Alex) for whom 11am is a usual sleeping time and so getting up in the morning to be ready by 9am is a real challenge. Amazingly, possibly due to Tom's incredibly complex alarm clock system, and the fact that Tom is sharing a room with Alex, we do actually make it out before 9am, ready for a swift departure.

Our first visit is to a school in a town called Valjevo. We have already discovered that it is a legal requirement for every public building in Serbia to display a portrait of Nikola Tesla (famous for using his knowledge of physics to invent things) and it is great to see that this school has theirs proudly in their front hall.

We are shown a presentation about the school itself, almost as if we are ourselves prospective students, and we learn that the school has a rich history, and many famous alumni (provided you've heard of them). The school specialises in maths and sciences, and we all agree that it sounds very appealing. Given that we have been away from our schools in the UK for a week, it seems that, by the end of the presentation, we now know more about this school than any of our own.

We see their library, which Aron says "looks like a display by Ikea" and which Dominic says "looks like a hipster cafe". Readers may draw their own conclusions about what Aron and Dominic respectively choose to do with their free time.

After this, we get on the bus and head to a research centre where Vesna used to go to maths camps. They too have a library, filled with scarily advanced-looking research projects, mostly in Serbian and written by secondary school students. For physics experiments, the research centre has a 'deep underground laboratory', through whose windows the Sun was shining brightly.

Next is the third part of the excursion, where we visit a big cave. Since the guides are also here, this gives Aron the opportunity to ask interesting and intellectual questions, such as "Is this a real cave?" Sandra looks a little confused for a second before replying that "yes, it is a real cave." Aron nods thoughtfully.

We discover on the way back that we are again being followed by a dog, with almost exactly the same appearance as Dog from day 2. With this in mind, Tom and Agnijo decide on a systematic way of naming all dogs encountered. The method is an inductive definition, which is described below:

Let $\text{Dog}_1 = \text{Dog}$. Now for each successive dog, we consider the highest subscript of all previous dogs. Let this be k , where k is an element of the set of British natural numbers. Then name the dog in question Dog_{k+1} . This covers all possible instances of dog encounters, and so we're done.

Problem Friday.1:

Use the inductive definition of dog naming, combined with your knowledge of maths, combined with your knowledge of dogs, to name the dog which we encountered on Friday.

(Answer: Dog_2 .)

Figure 7: A picture of Dog_2



After a large part of the journey on the way back, Aron points out of the window and says, "We can't be far. I can see Avala Tower." Tom looks out of the other window. "Where?" Aron replies "No, out of this window — you really can't miss it." Dominic interrupts: "We didn't. We, being NATO, didn't."

When we get back to the hotel which has WiFi, Aron tells Dominic about some solutions to question 1 on the AoPS website — Sam Bealing (see Balkans 2017) has posted not one, but two separate bash solutions to the questions, using barycentrics/areals and complex numbers respectively. Also, as always with a lot of problems on AoPS, we find a highly arrogant comment which does nothing but describe just how easy and 'simple' the user found the problem — as a team, we would collectively like to reassure readers that we found the problem deceptively extremely hard (see scores for details).

Sam has also texted Aron to tell him how one part of the problem is trivialised by the converse of Blanchet's theorem. Dominic, after seeing said message, tells us that the theorem would not be quotable in an Olympiad, since it is by no means familiar to "anyone with a reasonable work-life balance".

We talk about question 1 a little more and Dominic mentions the 'pedal triangle,' pronouncing the 'e' short, as in a bicycle pedal. Aron becomes very happy as he remembers that, during the Trinity camp, he was mercilessly mocked by Tom and Ben for pronouncing it as such. Tom maintains that the 'e' is long. Nobody else cares.

[Note: some members of the team got the opportunity to ask Geoff at the Tonbridge camp about the pronunciation of the word 'pedal' when an adjective. Geoff said that the 'e' was absolutely meant to be long.]

At the closing ceremony, we conclude that the whole competition seems to have been set up as an elaborate family reunion for Vesna, given that we find out that her father is the head of the jury. Aron receives an extra cheer from the Italians, due to the fact that they also got a bronze medal upgraded to a silver because of the UK leaders' petition, but he later manages to break the ribbon attached to his medal somehow. Once everybody who needs to has gone up to collect their medals, Alex takes the flag and wears it as a cape for the rest of the evening.

Figure 8: Nathan and Jill at the Closing Ceremony



We find some Bosnians to talk to, and teach them the same card game as we played with the Serbians on Thursday. Once again, Nathan causes absolute chaos by doing something to change the game somewhat (again, we really can't say). After playing some 'cheat' as well, they introduce us to a card game called 'Donkeys' which has literally nothing at all to do with donkeys. Giles, unfortunately, loses, and has to shout some things as loud as he can as a forfeit.

Alex and Giles go off to play some volleyball in the dark with the Bosnians and others; those of us who find sport in general somewhat less appealing (or are simply too exhausted) decide to go inside instead to pack.

Saturday — Day 6

As before, the day begins with us waking up — but at 6am Serbian time. Miraculously, we all manage successfully exit the hotel on time and Tom manages not to forget anything.

For breakfast, Aron is very happy because, having waited for four whole days, he finally gets to have the cheese that he likes again. He becomes disappointed, though, when nobody else lets him eat their blocks of cheese as well.

We get into the (booked) taxi and head towards the airport. We get there two hours early, so we entertain ourselves in a coffee shop for a while.

Nobody wants to do any maths when we board the plane, so we take the time to read through the airline magazine (which refers to “Stalin, who was later replaced by Lenin”...) and all of us set our watches back one hour.

After a relatively uneventful arrival, we walk through Heathrow arrivals and leave.

Thanks

- Thank you to all the Serbians who helped to organise this competition — and particularly Sandra, who was an incredible guide.
- Thank you to everyone we met from other countries taking part: it was great to talk to you and we wish you the best in future maths activities.
- Thank you to the UKMT which funded the trip.
- Thank you to Dominic, Vesna and Jill for being such great and motivational team leaders — also Dominic for the flag.
- Thank you to you, dear reader, for getting through the report (we realise now that at 25 pages we should have probably made it a bit shorter.) We hope you enjoyed it.

