

Balkan Mathematical Olympiad 2022

Unofficial Report

Introduction

For the first time in two years, the Balkan Mathematical Olympiad was held entirely in person, in the scenic city of Agros, Cyprus, from the 4th to the 9th of May. Everyone on the UK Team thought this was one of the best weeks of their lives, and what follows is hopefully a faithful report on this very eventful experience.

Day 1 - Tuesday 3rd May

One by one, we arrive at the Heathrow Terminal 5 Premier Inn in a well-defined sequence¹, greeted by Jamie. Lingde informs us that it is his birthday, which is quite surprising considering Sida's is the day before, and Thomas' is in 4 days' time. (Any enthusiastic mathematicians are welcome to work out the probability of consecutive birthdays in a group of 6).

Then we play a Logic Mafia, with four different types of detectives. Lingde tries to rat Sida out, who manages to win the second round. Then we try Contact, and this somehow devolves into naming elements of the periodic table, carried by Jamie and Eleanor.

After depositing our luggage, we meet with Ina and have dinner. There we profess our hatred towards inequalities, apart from Ben and Lingde. Then we decide to have a look at Lagrange Multipliers before moving onto Projective Geometry on a Napkin, a crash course by Sida. Lingde and Hayden are visibly confused and have no clue what the others are talking about. During the dinner, we also discover that UKMT safeguarding has evolved, most likely for the better.

Robin arrives towards the end of the dinner, and distributes out our blue Balkan Mathematical Olympiad polo shirts, in an assortment of sizes. Lingde and Sida try to spot a typo after the 'Kindom' on the Winter Camp shirts, but with no success, although it appears that the original version of the shirt had spelt 'Cpyrus' wrong. Thomas accidentally ordered an XL shirt, so receives one. Finally, Ben was distraught at the fact that nobody else watches football, and we all retreat to our rooms at roughly 22:00 for an early 6 o'clock departure tomorrow.

¹The sequence goes Ina, Lingde, Jamie, Sida, Thomas, then the other 3 arrive together.

Day 2 - Wednesday 4th May

At 5:30, Thomas, Lingde and Sida wake up for breakfast; alas it was not there. We were making progress on sorting the logistics out, but unless we were planning to consume sufficient food in 3 minutes, it was in vain.

We squeeze into the shuttle bus to Terminal 5 at 6:01, and make it through the airport without much of an issue (though Thomas did leave his laptop in his bag). As we weren't being served food on the plane, we give Pret a lot of business (as Ina likes to say), and settle with our assortment of sandwiches and drinks. Dominic gave us a book of Balkan problems with a warm message, and some random lady gave Hayden a suitcase to temporarily look after.



Figure 1: Aeroplane selfie!

It seems that UKMT put everyone with an L or XL shirt on the aisle seat, which is unusually thoughtful. Everyone is equipped with a problem to solve during the 4.5 hour flight: Eleanor with IMO 2014/2, and Sida and Thomas with IMOSL 2014/G3. Sida and Thomas are successful in their geometric ventures, and then Eleanor and Sida move onto an inequality from Sida's mentor (more on him later) that was unusually tight. Here is the problem for the brave of heart:

For any non-negative real numbers a, b, c prove $a^2 + b^2 + c^2 \geq \sqrt{3(a^3b + b^3c + c^3a)}$.

As we are landing, we are informed that our plane was selected for Covid testing, which has us worried. Sida then uses some statistics to calculate it is only a 33% chance that at least one of us had Covid; this doesn't help the mood. But spoilers: we were all negative.

At the airport, we are greeted by a man who took our passports and any % speedruns us through the landing process. Twice we board airport buses in which we wait for more than 10 minutes, but the journey takes less than 1 minute.

In our coach, we plant ourselves in the middle of the Greeks, with the Bulgarians at the back. The Greeks are very nice and social, and have amazing English. Then we start telling maths jokes, including but not limited to:

All the functions are holding a party to worship the king, $f(x) = x$. Then $f(x) = e^x$ tries to join the party, but all other functions say that there is no point for him to integrate in.

Why do parallel lines always get into arguments?

Because unless they project, they never concur.

Where in a room should you go to warm up?

In the corner, because it's always 90 degrees.

Pause for laughter and applause.

We reach the Rodon Hotel, and go to our rooms. Hayden, Lingde, Sida share one (with an amazing view), Ben and Thomas another. Eleanor is delighted at the thought of getting an Azerbaijan roommate. She is even more delighted when said roommate, Fidan, informs her that they sleep with the lights on in Azerbaijan. Fidan is, however, lovely and spends a lot of time with the UK team throughout the rest of the week. Nobody has camper beds, and we all settle down without needing to fear for death by collapsing bed.² After an intense game of First to 3 Three Player Rock Paper Scissors, it was decided that Lingde would get the bedroom, while Sida and Hayden would get the single beds in the living room.



Figure 2: Balkan-y view (joke courtesy of Sida).

During dinner, we compare households, and realise there was a pretty even divide between those that only drink filtered and boiled water, and those that could leave water for a week without bother. We also discuss dreaming maths; Thomas has evolved from dream-solving problems to dream-creating problems, including a geo that actually turns out true. After being fascinated by the abundance of stray cats near the pool, we debate over the flavour of

²Possibly a reference to Michael Ng's 2016 Balkan MO Report...

a certain dessert (there are 10− arguments for custard, egg, and 0+ for coconut), and finish the day off with a traditional game of Mao. Unfortunately for Thomas, it is his first time playing, and this acts as a great source of entertainment for the rest of us.

Day 3 - Thursday 5th May

Hayden wakes up having proposed a combi problem, then forgotten it. To exercise our brains in the morning, we start the day off with a good and easy maths problem: IMOSL 2019/N8. We set off with different approaches: Eleanor tries fixing b , Lingde went straight to quadratic residues, and Sida does some algebra slog. After 2 hours, Lingde claims that one of his four cases could be solved by proving some Jacobi symbol equals -1 , but then realises his algebra was wrong a few times, and Sida claims to have a working solution, which Eleanor has a read-through but is ever-so-slightly disgusted at the bashing required. Thomas also solves Balkan 2021/4 in this time.



Figure 3: Group photo! It was difficult getting the flags the right way up.

Before lunch, we decide to take a group photo (involving Eleanor jumping out the window of her room) with a three-eyed cat watching. During lunch, Eleanor talks about her farm life in New Zealand, and the level of water quality described causes Thomas to die internally. Lingde also informs the group of his 2400-day Duolingo streak, to the dismay of Sida with merely a 1600-day streak.

Next up is an excursion to Limassol, where we get free merch from the Cyprus University of Technology, almost swaying us away from the UK, and visit Limassol castle. We find various ancient rocks and tablets, and a figure of a man with funny eyes. Then, somehow we lose our original tour guide, and follow a new one back, hence repeating every attraction twice. Ben asks Thomas if he can drink his one day-old mango juice from Pret, and Thomas agrees hesitantly in concern for Ben's health, which Ben and Eleanor find unfamiliar.



Figure 4: Another group photo, another beautiful view.

Likely due to British luck, it begins to rain, and Ben uses this opportunity to ‘wash’ the British flag. We board the coach to the opening ceremony in Laniteio Lyceum, a school.

After Thomas and Sida take the opportunity to drink as much water as possible, we enter the hall and the opening ceremony begins. The traditional music and dance was lovely, followed by speeches and the presentation of the teams.

We manage to get all flags the right way around *FORESHADOWING*. Italy brought balloons along with their flag, and the Cyprus team gets the loudest cheer.

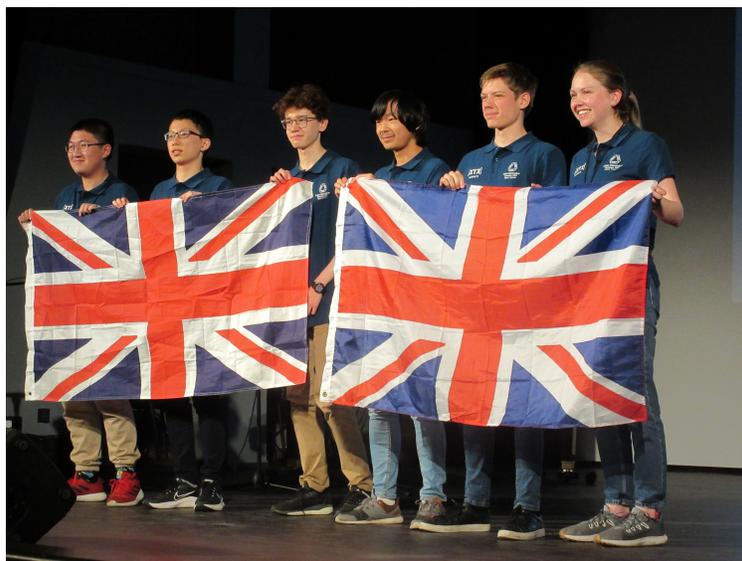


Figure 5: Team UK!

On the way back, we begin with a balloon fight with the Italians, which devolves into a

full-on war with headshots and backhands. Thomas tries his hardest to hit Hayden, who was protected by Lingde, and manages to do so. After this was karaoke, and we possibly jinx ourselves with a resounding rendition of ‘We are the Champions’. We also rick-roll the rest of the bus.

At dinner, Ina introduces us to a probability question involving people sitting on their designated seats, or not, Lingde jokes that the answer is 50% because “it either happens or it doesn’t”, to which Ina replies “correct”, not hearing the latter half of the sentence. Then we reach the natural conclusion that we will see a dinosaur on the way back because we didn’t see one on the way here.³

Tomorrow is exam day, so we all had an early night.

Day 4 - Friday 6th May

Once again the day begins with one of Hayden’s peculiar dreams; this time he and Sida miss their alarm clocks and wake up at 9:40, and then he solves Problems 1 and 2 but doesn’t write them up.

We enter the exam hall at roughly 9:00, and the exam begins some time later. Fortunately for all but Ben and Lingde, there was no inequality. Problems can be found here.

Upon discussing the exam, we discover that (SPOILERS): Lingde tried Cartesian-bashing Problem 1; Sida and Thomas realised Y was the A-dumpty point (Eleanor: the WHAT?); Hayden and Eleanor jumped straight to Problem 4 combi; Ben used analysis in Problem 3. It turns out Eleanor has quoted properties of the A-dumpty point as well-known, having remembered them from Evan Chen’s list of facts about the symmedian, but managed to forget its name. Jamie tries to find her ‘well-known’ claims online for 15 minutes before realising she has proved them the line below anyway. Everyone but Thomas got the construction and bound for Problem 4. Lingde asks the rest of the team and is relieved that he is not the only one whose immediate thought upon reading Problem 4 is the hit mobile game Among Us - it turns out that Hayden and Thomas have similar thoughts too. During lunch, we discuss with Jamie our solutions and the official solutions, followed by a chaotic frenzy as Sida and Thomas race to be the first to post the problems on AOPS. They are successful.

After a while it becomes clear that Problems 3 and 4 were found difficult by all teams, with very few complete solutions. Then we play Mao with the Greeks and other countries (one of the Greeks, George, now has a strong dislike for this game), as well as Threes (the alternative name shall be left out) and Betting Whist. Thomas is unsure about the rules for all of these, but somehow ends up winning them all. Ben is baffled.

Then is our first walk of many down the mountain. This time it is Thomas and Hayden who miss it. Half-way during the walk, we skirt off the main road and end up climbing some obscure dirt path, eventually leading us to a main road. Lingde proclaims that this is more exercise than he ever normally does, and Sida gets a nosebleed due to the heat. We see goats; more on them later.

³This may or may not be a reference to the 2022 EGMO report...



Figure 6: UK, Greece and Italy teams.

Making it alive and in time for dinner, we then play Palmero, a Greek Mafia variant, and try to guess a story from the Greeks using only yes-or-no questions. The Italians join and we chat with them about problem selection (nobody liked them). Both official solutions to Problem 2 are wrong, and it seems Lifting the Exponent Lemma ends up quite crucial for a 10– solution.

Then Sida goes up to sleep; the rest of them go for a midnight walk.

Day 4.5 - Satfriday 6.5th May

It is Thomas' birthday, and Lingde wishes it to him 7 minutes after midnight. The rest of the team chill in the downstairs lobby, and talk about the excessive use of the word 'like', especially from Eleanor.

Robin and Jamie start marking our scripts and we find them at 1am with Eleanor's sixteen-page solution to Problem 3 spread out across the table. She then spends 2 hours explaining what she had done to Jamie and Robin. Along with Lingde, they then try to fix the issue in it until 4:00 before giving up and going to bed (continued the following morning).

Day 5 - Saturday 7th May

Thinking that we missed the tour of the village, we embark on the 3rd walk down the mountain, this time missing Eleanor. Somehow we take 1 hour going down, but 15 minutes going up. None of us knew where we were going, so that was quite the achievement. Eleanor spends the entire time trying to piece together her solution to Problem 3, and at midday spots a way to fix it, with some input from Mohit over the phone.

We play basketball, tennis and football with some Albanians and Greeks. They are significantly taller than us, so Sida, Eleanor and Lingde played minimal role in the basketball.



Figure 7: Birthday boy with his birthday badge.

Ben, Hayden and Thomas on the other hand demonstrate their athletic prowess.

In the lobby, Jamie comes with our scores, an analysis of which is in Appendix A. Sida ends up with 1 more point than predicted, Hayden is surprised at a full solution to problem 2, Eleanor is happy with 8 on Problem 3, and Jamie is glad he doesn't have to touch her solutions ever again (just kidding, or not?).

After lunch Ben decides to buy Thomas a birthday cake, so sets off with Eleanor and Hayden in secrecy. Unfortunately Thomas notices so attempts to catch up, and Sida tries to force him back with random requests to his room, eventually with success. After visiting three shops, Ben and Hayden secure a cake.

Although not necessarily a traditional competition game, we have a game of Cheat, which is interrupted by the medal boundaries. Sida and Eleanor laugh in celebration that they achieved a silver medal, and excruciatingly Thomas ends up one mark off. The boundaries this year reflect a difficult paper. Then, we time how long it would take between Jamie emailing Joseph the scores and him updating the BMOS site; unfortunately due to Jamie's picture format, it takes 11 minutes.

Thomas, Ben and Hayden head back to their rooms to watch basketball on Thomas' laptop. Sida also heads back to relax. Jamie introduces Lingde and Eleanor to Liar's Poker, and they play with Fidan. Even after a significant blunder, Lingde wins. The Azerbaijanis bring Uno (which Jamie insists is an inferior version of Mao) and there is a group game with the Montenegro team.



Figure 8: At the Roman theatre.

Day 6 - Sunday 8th May

We start the day off with an early excursion to the coast, where we sat with Fidan. There we follow a tour guide around a Roman theatre and Roman baths in some Roman dude's house. Ben is disappointed that we aren't at the beach, and Sida is called upon for his selfie stick.

Next destination is the Marine Harbour, with the sea looking like it is generated using Blender. We chat about other Olympiads, mainly Hayden, Eleanor and Lingde on Chemistry, the possible areas in the 3rd and 4th TST's, and have a debate over levels of 'famous', in particular Breaking Bad vs. Po-Shen Loh, and Sherlock vs. Ankan Bhattacharya.

Upon arriving at the beach, we try to skim stones. Eleanor, being the most experienced, demonstrates a perfect 3-skim, and by the end Thomas and Ben could as well. Hayden resorts to chucking stones as far as possible. Another team, possibly Bosnia, are full on vibing with the beach; they were in swimsuits and in the sea, with no regard for the meet-up time given by the tour guides.

Upon returning, we play tennis with the Azerbaijanis, and some aggressive variant of catch. Thomas is amazing at catching, and Sida is dubbed as a good tennis partner. Ben attempts to climb a signal tower, but once it starts wobbling we deem it too dangerous. We embark on our 4th walk: the first one with everybody. To commemorate this occasion, we go on the same route as the first time, visiting the rose shop for Sida to get souvenirs. There are also a lot of goats. What does that mean? We have goat to take a picture.⁴

At the closing ceremony, everyone is nervous at the fact that we not only have to make sure our flag is the right way around, but also our medal is on top. This is quite the difficult task; gravity can be unrelenting. Fortunately, all but one of us have the flag the right way up. The odd one out shall not be named.

⁴This reference should be clear.



Figure 9: Goats.

By this point, the Greeks are the event celebrities, and receive massive rounds of applause. Naturally, the winner from Bulgaria also receives resounding applause.



Figure 10: Both flags are the correct way up (phew).

After taking photos, we play Mao in the progressively darkening outdoors, until we literally cannot see the cards anymore and everyone is shivering frantically. Now that Thomas has the hang of the game, Robin is our new source of entertainment. For Sida's first Mao, he introduces a new rule that nobody makes sense of; the rest of the team tries to case-bash though.

Sida then gets a signature and selfie with one of his favourite Balkan problem composers, Demetres Christofides, and we proceed to a... 'normal dinner'. Except there is a DJ in the middle of the room. Phanes, who was the little brother of the other George on the Greek team, and who everyone knew at this point, starts dancing on the centre stage, and one by one people join in. The UK team makes a pact to all go up at once, and when we do, it became very wild. At some points, even Jamie joins in.

Just after 22:00, we go back to the lobby to play Threes with Fidan, and Ben once again tries to sabotage Thomas. Sida reveals his Mao rule, and afterwards we play Liar's Poker. Somehow Lingde wins again, having stayed on one card throughout most of the game.

Eleanor heads to bed, whilst Hayden, Ben, Sida and Thomas return to their rooms to pack. Lingde is the only one still up and stays in the lobby downstairs to socialise. He plays Liar's Poker again, this time with the Azerbaijan team and our team leaders. It is here that his winning streak is finally broken. He and Robin then stay in the lobby for even longer, as they chat to a random assortment of people from various countries. Lingde finally heads to bed at 4am in the morning.⁵

Day 7 - Monday 9th May

On the last day we wake up pretty late, and go down to the van having packed up and checked out. After a few dark premonitions about the trip, we begin a karaoke with the Greeks, with Jamie claiming afterwards that our singing was not the best.

Another highlight is the birthday-guessing: Ben claims that each month has its own vibes, giving the example of Eleanor as a January girl because she is decently intelligent (slight understatement). Thomas apparently has May vibes, and portrays himself as a Will or Brandon. Sida gives off October vibes, which is agreed upon by a Greek member, and then we proceed to guess Ina's and George's birthday. Hayden scored a point.



Figure 11: Saying goodbye to the Greeks.

⁵Note from Lingde: I do not recommend staying up until way past midnight for 3 days in a row!

After saying goodbye to the Greek and Azerbaijan teams (having graced them with our beautiful rendition of pi to the tune of Mamma Mia), eating lunch, and making it through passport checks very quickly, we board the plane. Hayden and Sida discuss USAMO 2022/1, and the whole team plays a game of Contact. Robin also talks about Sida's mysterious mentor, and how they are friends but rarely meet perhaps due to his mysterious work.

Upon landing, we look up, and immediately feel gloomy because of the grey skies. Oh how we missed the UK.



Figure 12: Final group selfie in Cyprus!

Acknowledgements

We are extremely grateful to everyone who supported us throughout this journey.

Thank you to Ina, for looking after us, allowing us to go on our random walks, for taking the most efficient photos, and for making us laugh during every meal.

Thank you to Robin and Jamie, for discussing mathematics and strategies, for arguing for us during coordination, for putting up with Eleanor's 16-page Problem 3 solution and Hayden's illegible handwriting, and in general for supporting us through the event.

Thank you to the Balkan Mathematical Olympiad 2022 organisers, including the Cyprus Mathematical Society, for organising such a wonderful event and allowing us to experience Cypriot culture, and make international friends.

Thank you to UKMT for giving us this opportunity, and especially to XTX markets for sponsoring this event.

Thank you to our schools and teachers, for supporting us, in one way or another.

Thank you to Dominic Yeo for all the training throughout the year, and to Geoff Smith, His Royal Highness of UKMT Monarchy.



(a) Our lovely team leaders!



(b) Jamie with a well-deserved medal.

Figure 13: Thank you!

Appendix

A Scores

Code	Name	Q1	Q2	Q3	Q4	Total	Result
UNK1	Benjamin Gillott	10	10	4	1	25	Silver Medal
UNK2	Thomas Kavanagh	10	10	1	0	21	Bronze Medal
UNK3	Hayden Lam	0	10	0	1	11	Honourable Mention
UNK4	Sida Li	10	10	2	1	23	Silver Medal
UNK5	Eleanor MacGillivray	10	4	8	1	23	Silver Medal
UNK6	Lingde Yang	0	4	1	1	6	-
-	Total	40	48	16	5	109	-
UNK7	Robin Bhattacharyya	-	-	-	-	87	Gold Medal
UNK8	Ina Hughes	-	-	-	-	592	Very Gold Medal
UNK9	Jamie Bell	-	-	-	-	2149	Extremely Gold Medal
UNK0	Reserve Evreser	-	-	-	-	-158	-
-	Min	0	4	0	0	4	-
-	Max	10	10	8	1	29	Silver Medal

We came 11th out of 16 teams. Eleanor achieved 1st out of all females.

Our fifth degree polynomial:

$$\frac{91x^5}{120} - \frac{335x^4}{24} + \frac{2279x^3}{24} - \frac{7009x^2}{24} + \frac{23597x}{60} - 158$$