41ˢᵗ Balkan Mathematical Olympiad Unofficial Report

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Satuday 27th April - Thursday 2nd May

Introduction

The 41ˢᵗ Balkan Mathematical Olympiad took place in Varna, Bulgaria between Saturday 27th April and Thursday 2nd May. The following report will detail some of this experience from the perspective of the UK team.

Day -1 - Thursday 25th April

This day was relatively uneventful. We all arrived at different times at the Luton Premier Inn, so some of us went to the Beefeater Restaurant, and some of us went straight to our rooms to work on problems. These problems were from the Balkan Mathematical Olympiad Mock that Jamie had given us. We were all eluded by Question 2: a difficult functional equation. We later went onto trying some interesting inequalities, involving expanding, bashing, AM-GM, and Muirhead's Inequality. Sam arrived late, as he doesn't enjoy inequalities. Unfortunately, Jet forgot his toothbrush, and had to buy another one.

Day 0 - Friday 26th April

We all managed to wake up in time for our 7:40am flight, and Jet went through 3 (not necessarily distinct) outfits. We attempted to grab breakfast at a restaurant before our flight, but only $\frac{5}{9}$ of us were successful in doing so. Then we got up to various activities on the plane, some of which involved maths - we tried Balkan 2022 Problem 4 about Dionysus and his Frogs.

Figure 1: The UK team on the plane

We then took a taxi to our hotel called the "Boutique Splendid", which was both boutique and splendid, rated $\frac{4.8}{6}$ stars, five not being enough to do the splendour of the hotel justice.

In the afternoon, we went on a very long walk next to the beach, enjoying the contrasting weather to Great Britain and the plethora of cats. Samuel mentioned the famous 'Cars' problem (TSTST 2019/3) to Ed and Adavya, warning them not to try and headsolve it (really, don't do this). Ed ignored this warning and later produced a 4 page fakesolve. We had some ice cream, and we then went to a restaurant. While we were waiting for food, Aanya and Ed started feeling cold, and so both ran a lap around the restaurant seating area, before parkouring back to the hotel for warmer clothes.
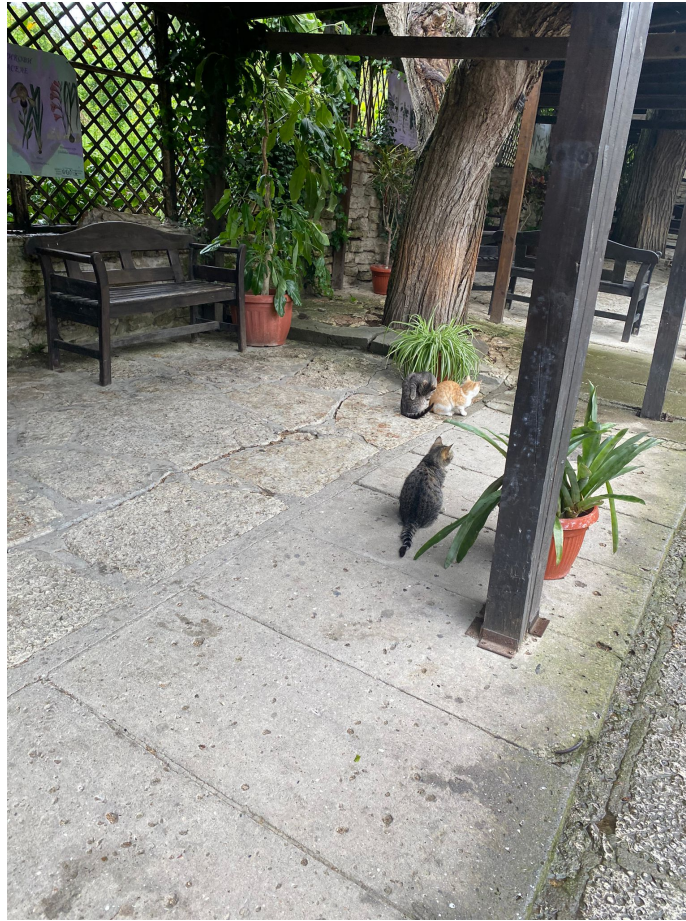
Figure 2: A small fraction of the cats which we saw on the walk

We ended the day off by playing a small amount of Irish Snap (better than Mao!) before not solving the functional equation and then heading to bed.

Day 1 - Saturday 27th April

In the morning, we all did a bit of maths, before embarking on the same walk we did yesterday. We stopped for lunch on the beach, where the team explored the coastline and admired the sea. Ed was deterred from exploring any further when he spotted a creepy looking crab next to one of the tetrapods.

Figure 3: A very creepy crab

Later, we were transported to hotel Koral (rated four stars out of five), where we were acquainted with our guides and given some Balkan MO merchandise. Jet swiftly asked for the location of the gym, but we found much more than just a gym.

Figure 4: Entrance to the Massage Room

This liminal space, which we named the "Massage Room", would be revisited later. Amongst the seemingly alive industral machinery and the disconcerting and discombobulating paintings was a singular chair, perhaps the site of many a massage.

We spent our afternoon and evening doing maths as well as playing table tennis and pool, and doing a Codeforces contest in the hotel hallway, as that was where the Wi-Fi was best.

Just before going to bed, Ed had one final attempt at the functional equation which had stumped the whole team for multiple days. With a stroke of luck, he found the final step of the solution, completing the team's progress on this evil problem.

Day 2 - Sunday 28th April

The UK team had all agreed (or at least were politely encouraged by Aanya) to bring formal wear for the opening ceremony. This consisted of folk music involving bagpipes, which Jet - the Scotsman - found thoroughly amusing. After many speeches detailing the sponsors of the Olympiad, and how 41 was a very

famous prime number, the 41st Balkan Mathematical Olympiad was declared to have begun. The speakers (or perhaps ChatGPT) predicted that the contest would *'slip through our fingers like grains of sand'*. It is also around this time that Jet started the never-ending references to a singular 'best mathematician in the whole world' - a very inspirational man.

Afterwards, we took a photo with the Swiss team, and walked back. It was only when we were close to Hotel Koral when Ed realised that he had left his bag outside. Fortunately, after a mild panic, the bag was uneventfully retrieved by Ed and Jet after lunch.

Figure 5: Swiss-British team photo

In the evening, we all went to do our final preparations for the 41st Balkan Mathematical Olympiad, including Adavya's Functional Equation Marathon. Especially exhilarating was the Functional Equation that appeared twice in the Balkan Mathematical Olympiad - officially considered "the weak point of the BMOs" - appearing as 1997 P4 and 2000 P1. We are sure some participants were very happy with that. We then went to bed early to get a good night's rest for the next day.

Day 3 - Monday 29th April

We left the hotel early for a 9AM contest start, arriving at a fancy-looking hotel, where we were eventually directed to the exam room.

Figure 6: Exam room

A notable thing about the undertaking of the contest was the fact that they intended to have no clocks throughout the exam, and instead decided to aperiodically write the time remaining on whiteboard. This was protested profusely by Aanya who ensured a clock was fetched.

Regarding the actual problems, Problem 2 was not very popular among the UK team, apart from Adavya, who described it as a standard Codeforces problem. Problem 3 was also not enjoyed (apart from by Ed). We tried to predict whether there would be a Polynomial or Functional Equation question on the paper, but we were completely blown away by the invigorating combination that problem 4 greeted us with - the difficulty of the problem meant Adavya's marathon was in vain.

In the afternoon, we played various card games to unwind, and in the evening we had a joint expedition with the Swiss to explore further into the massage room. Throughout this expedition, there were many jumpscares, with Samuel slamming the door of a particularly creepy room with Aanya and two Swiss team

members still trapped inside and then running away to escape their wrath, before suddenly being jumpscared himself by Jet who was hiding in a corner.

After this, we got an ominous message from our leaders - they wanted to discuss the 'state of play' of problem 1. This discussion produced mixed reactions from the team. In an impressive display, they also managed to mark problems 2, 3 and 4 before we went to sleep (although we didn't yet know our scores for certain as the scripts also had to be coordinated).

Day 4 - Tuesday 30th April

The British weather seemed to finally join us this morning (as a certain member of the Bosnian team remarked), as we were met with the familiar patter of rain. This did not prove favourable for the excursion, which also commenced rather early. Our destination for this excursion was the nearby coastal town and palace of Balchik, which we believe would have been a very pleasant place was it not for the weather. On the way back, we had the pleasure of listening to Bosnian pop songs and singing.

Once we got back from the excursion, we all knew our scores, but the medal boundaries had not yet been decided. We went up to Ed, Will and Sam's room, knowing that we had to find something to distract us during the wait.

For the next few hours, we attempted to find something to occupy ourselves, starting with working on problems. After a while, most of the us got bored, so we decided to play a game where two of us would throw bananas off our balcony, whilst two more of us would be on the ground and attempt to catch them. Unfortunately, none of the bananas survived the 3 floor descent, and we didn't catch any of them either. We swiftly disposed of approximately 3.5/4 of the banana remains, losing one or two of the banana chunks to the soil.

In the evening, we heard the medal boundaries which produced mixed reactions from the team. Nonetheless, we went on to play many rounds of table tennis, getting other teams like the Bosnians and Swiss involved in our (lost) game of Around the World.

Day 5 - Wednesday 1st May

With the prospect of another day of free time not being very appealing, we foolishly decided to try a difficult problem. IMO 2017 C8 caught our eye (despite having heard rumours of its 'Brutal' difficulty), and so we set to work. The natural first thing to try was to look at small cases, but unfortunately $n = 1$

and $n = 2$ weren't that useful. After trying some other things and making a small amount of progress, we decided to do the $n = 3$ case. This involved half an hour of counting more squares than in an A level physics paper, but eventually we arrived at the answer. From this, we were able to guess the general answer, but Adavya wrote a computer program to check more cases to make sure (which sort of defeated the purpose of doing $n = 3$ by hand, but by then we didn't really care).

At this point, Samuel, Adavya and Ed had come up with three radically different approaches, but they couldn't get any of these to work. After five hours we eventually stopped working on it (although we vowed to never give up).

In the afternoon, a group of teams consisting of the Romanians, Bulgarians and a few others played football in a nearby pitch, in which Will and Jet joined in. However, this was abruptly stopped when people decided to take a small break. The final score was $1 - 1$.

The closing ceremony was another extremely engaging event. We listened to some more speeches, before receiving our medals. Then, the 41st Balkan Mathematical Olympiad was declared to have officially ended.

Figure 7: The team photo after the closing ceremony

After the closing ceremony, we were invited to the final official dinner. Taking note from previous experience, we were told to be wary of the music, particularly its volume. This warning turned out to be true, as we measured the music to be at around 90 decibels. As the dinner continued, the music appeared to get increasingly loud, with a maximum of 110.6. This caused Sam, Ed and Adavya to return to the hotel, while the others stayed.

Figure 8: The closing dinner group photo

Dancing from the Algerians spurred many other teams to join them, and so Will, Jet and Aanya also took to the dance floor. What then followed was a conga around the hall, people sitting on top of each others shoulders, and lots of jumping and dancing around. Jet demonstrated his physical prowess by doing one arm clap press-ups and a handstand press. A participant from Moldova showed his break-dancing skills, and the Italians were very lively to say the least. Shortly before we left, the Serbian leader took to the stage and gave a very memorable performance.

Meanwhile, that day Ed had been wearing a T shirt which had an IMOSL question written on it (2014 A4). Sam and Adavya decided to attempt it, solving it in under an hour. The impressively short time they spent on the problem lead to an impressively long solution.

Ed then attempted a couple of beautiful geometry problems from a handout we had got at a previous training camp, solving the first one (All-Russian Olympiad 2011 Grade 10 Problem 4) with the assistance of a member of the French team, and a couple of hints. The second problem (All-Russian Olympiad 2011 Grade 11 Problem 8) captivated our attention until half past midnight, by which time we had added so many things to the diagram that it looked like someone's year 7 art homework. Annoyingly, the intended solution was completely unrelated to any ideas we had come up with during the problem solving session.
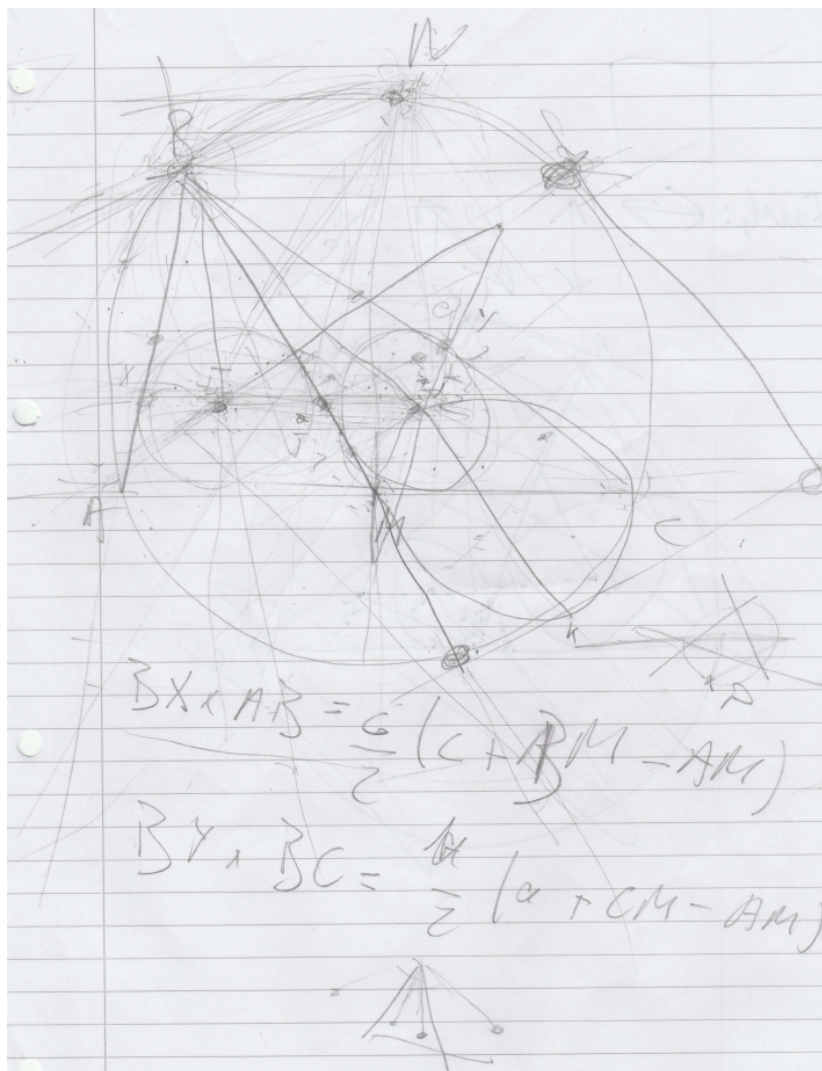

Figure 9: Midnight Geometry

Figure 10: Definitely more fun than parties

Day 6 - Thursday 2nd May

We woke up at about 8, and had an uneventful breakfast (except for Aanya who woke up at 9:50 and ran to get breakfast).

We then took a bus to the airport, arriving shortly after 2, but our flight was only scheduled to leave shortly after 7. This unfortunately meant that we had $(7+\varepsilon)-(2+\varepsilon) \approx 5$ hours to kill, and so we all worked on this report for a while. Also, during this time Aanya decided to take a nap, while Jet went on a wee walk. Aanya and Will befriended a small child whom Will influenced into performing bottle flips, presumably causing the child's mother a future of suffering.

Figure 11: Writing the report

On the plane flight we did some more maths, where Ed attempted IMO 2016/2. Prior to this, there had been some arguments among team members about its difficulty, with some people saying that it was a good difficulty for IMO P2, and others saying it was more like an easy P1. Ed almost fell asleep after trying the problem for a couple of hours, and so he decided to join the "hard difficulty" party. We then looked at some problems from the IMO Longlist 1989, which possessed a particular quality. Some of our favourites were a question which asked to prove the existence of the nine-point circle, and a question which asked to find all pairs of square numbers (S_1, S_2) such that $S_1 - S_2 = 1989$.

After a thankfully uneventful trip through passport control and baggage retrieval, we said our goodbyes, and thus ended the 41$^{\text{st}}$ Balkan Mathematical Olympiad.

Acknowledgements

Statistics

Scores							
Code	Name	Q1	Q2	Q3	Q4	Total (mod 41)	Result
UNK0	Geoff Smith	−30	−160	−117	−3	18	Bronze Medal
UNK1	Aanya Goyal	10	0	0	0	10	Honourable Mention
UNK2	Adavya Goyal	10	10	3	0	23	Silver Medal
UNK3	Jet Guan	8	0	2	0	10	
UNK4	Ed Hart	10	0	7	0	17	Bronze Medal
UNK5	William Harwood	10	0	0	0	10	Honourable Mention
UNK6	Samuel Sturge	10	10	7	3	30	Silver Medal
UNK7	Jamie Bell	40	120	170	18	20	Bronze Medal
UNK8	Ina Hughes	178	560	819	63	21	Bronze Medal
UNK9	Dominic Yeo	570	1760	2544	168	40	Gold Medal; Perfect Score

And the 5th degree Polynomial of our scores:

$$\frac{31}{24}x^5 - \frac{545}{24}x^4 + \frac{3619}{24}x^3 - \frac{11191}{24}x^2 + \frac{7883}{12}x - 310$$