Balkan 2025 Unofficial Report

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"-1 is prime"

1 Introduction

The 42^{nd} Balkan Mathematical Olympiad was held in Sarajevo, Bosnia and Herzegovina from the 25^{th} to the 30^{th} of April. We (the UK team) participated as a guest country. This is the unofficial report written by the team members.

2 Day -3: Arrival day at Stansted

On the 23^{rd} of April we arrived at the Premier Inn at Stansted airport. Nothing interesting happened, except we ended up solving a question about intervals that Elsa gave to us at Trinity camp:

Given n random intervals entirely contained in $[0, 1]$, what is the probability that there is at least one interval that overlaps all others?

Before meeting, Aum had spent about 3 hours trying to evaluate an integral that Sam created at camp, which would theoretically give the correct answer. We decided that there should be a pure combinatorics way of solving the problem, so we tried to find a *trijection* between sets of n intervals that would work. Teri found this after around 30 minutes, and then we finished dinner and went to sleep, since we would have to leave at 4:45 in the morning.

3 Day -2: Travel to Sarajevo

We met in the hotel lobby at 4:45 on the 24^{th} with our stuff, and leave for the airport at around 5:05. At the airport, we get through check-in and security at a very normal pace, except one of Jayden's UKMT handbooks gets pulled by security. We then get breakfast from Pret and eat it in the airport, and start heading for our gate at around 6:45, and get there at around 6:55.

We quickly found out that the gate was closed 5 minutes beforehand and that we managed to miss the last direct flight from London to Sarajevo before the arrival day.

We go back a bit to sit down, and then at around 7:30, Jamie and Natalie book a connecting flight from Heathrow to Sarajevo through Frankfurt, leaving at 16:30 and arriving at 22:30 CET. This meant that we had 9 hours to get to Heathrow, which was plenty of time.

We had to get our luggage back from one of the luggage belts, and then we left for Heathrow. Around 8:00, Jamie sends us some problems from the 2024 BMOSL to do while travelling, and it is also mentioned that the EGMO problems would be worth doing as well. The first one that was sent to us was G4, and the other ones sent were G3, G5, A2, N3, and C4. We started doing the

problems on the train from Stansted to Liverpool St., and Aum solved C4 literally exactly when we got off the train, and also told some of us the solution for N3. On the train ride Jayden also suggested betting our trinity checks on the results of the competition which no one else agrees to. As it turns out, C4 was an entirely number theory problem and we still do not know why it was labelled as combinatorics.

We then took the Elizabeth line to Heathrow terminal 2, and Teri is surprised at how nice the trains are compared to the rest of the tube. We continued trying the problems, and by this point still no one has drawn a usable diagram for G4. We get into Heathrow at around 10:00, and then check in and get through security as usual. We now had 5 hours of free time at Heathrow.

We spend about another hour doing the shortlist problems and then we have lunch at a Chinese restaurant. After eating we end up staying in the restaurant for about 30 minutes, where Teri finishes EGMO P1 and solves P2. At this point, it is still the case that an accurate diagram for G4 has not been drawn (by this point it is already 13:30). We then go down to a lounge area in Heathrow until 15:30, where we solve more problems - we quickly solve G3 with 2 lines of projective geometry, and Teri finally solves G4 with a slightly cursed phantom point solution which ends up proving at least 9 lengths are equal to the radius.

We go to the gate quite early, this time making sure not to miss the flight again, and board at around 16:00. The flight goes as planned and we get into Frankfurt at around 19:00 CET.

Interestingly, the area just before security in Frankfurt airport was completely empty, and we were the only people there. We again drank all of our water and went through security, though there was a machine telling us which security line to go through which changed the line about 3 times a second, so we all tried to go through the same line until they told us we could all go through different ones which would be much faster. Security was much faster than usual for most of us, but Jamie's Jane Street themed playing cards were somehow flagged by security. We then went to the gate and got dinner at the nearest bakery in the airport, where 7 out of 8 of us got pizza, and 6 of us got exactly the same cheese pizza. Again we made sure not to miss this flight, and leave Frankfurt at 20:10.

We get into Sarajevo at around 22:30 as expected, get our luggage and take a taxi to our hotel for the night. We find out that G5 is actually extremely easy, and Jamie says that it was not put in the paper as the French leader last year found a much easier solution than the official solution. We arrive at the hotel and go to sleep.

4 Day -1: "Arrival" Day

We went to breakfast at 9:00, and Teri hears the melody of Thick of it in the random music that was being played, which was in fact, not thick of it. Everybody knows that we don't have anything to do until around 12:00, so we all go to our rooms to do more maths. We then went outside (shocking, we know) to see some of the city, and also to eat lunch. Later on, we took a photo in front of the Sarajevo sign.

Figure 1: #Sarajevo

During lunch, we again discuss betting on the results, but this time Jayden suggests betting on what UNK0 will get (mod 41), which is much more random. Everyone still declines.

Afterwards, we went back to the first hotel to get our luggage and then took another taxi to the olympiad hotel (Hotel Hollywood). When we got there we met with our guide, Tarik, and checked into the hotel. We also got our lanyards and also a backpack with a few things including 2 tshirts and a pack of Jane Street cards.

Figure 2: Photo of us arriving at the hotel and meeting Tarik

Most of the rest of this day was spent in our rooms, with a highlight of a random

person being given the wrong room key, opening Ben's room and saying "you don't look like Myra". Jayden did convince Aum to head downstairs to attempt to socialise with other contestants, but after arriving downstairs they were too scared and quickly went back to their room.

5 Day 0: Opening Ceremony

We met for breakfast at 8:30, wearing the blue shirts that UKMT gave us. We then left for the opening ceremony, but they did not want us to walk through the non-positive quantity of rain so we took a coach to the other hotel (Hills), which took a grand total of $-\sum_{n=0}^{\infty} 2^n$ minutes. We waited for an hour for the opening ceremony to start. During the opening ceremony some people gave speeches, who we deem to be, on a scale of importance from 1 to 1, $\frac{1}{4}(1+\sqrt{5})+i\sqrt{\frac{5-\sqrt{5}}{8}}$. They also showed the state tourism video and we also had a *minute*[1] of silence for the pope. During the presentation of the teams, we did in fact present the flag the correct way around, after much deliberation, to show the organisers that we were in fact not in *distress*[3]. Also, during this time Kelvin went to the toilet for enough time to cause the rest of the team anxiety about whether he would get back in time and returned around $\frac{987654321}{123456789}$ team presentations before we went up.

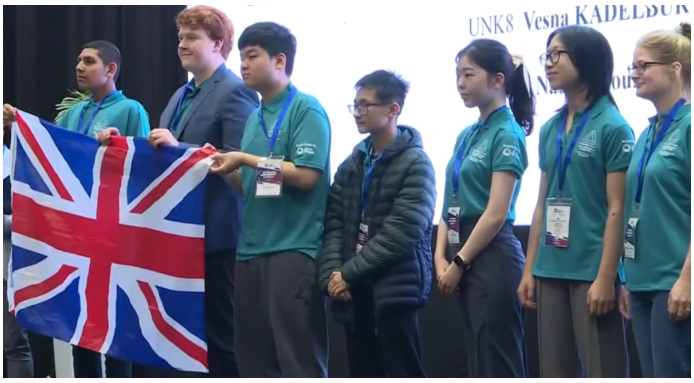

Figure 3: Flag presenting (not in distress)

After lunch we went back to our rooms to do problems, and came back down to the lobby at 17:00 to play card games. Natalie asks if we know any card games to play, and after around 20 minutes we decide on *the game of our glorious leader*, and then play it for 1.5 hours. *No other information about the game of our glorious leader shall be provided.*

After dinner, we went to our rooms to sleep before the contest.

6 Day 1: Competition day

On the day of the competition, we went to breakfast at 7:00, but ended up leaving for the other hotel (where the competition would take place) at 8:20. We got there quite quickly and were told our places to sit, and unlike RMM or EGMO, it was actually the case that everyone was able to see the timer. As planned, the exam started at 9:00, and finished at 13:30.

After the exam we ate lunch and discussed the problems with each other, and also with Jamie and Vesna (who we weren't allowed to talk to after the problem selection). We thought we had 6 solves on Q1, 2 solves on Q2, and 5 solves on Q3 (3 of which ended up missing one of the solutions, which was probably worth 1 mark - we are simply unable to solve $a^2 = 1$), both in classic UK fashion skill issuing on geometry, and also in classic Sam Griffiths fashion getting solutions for questions 1 and 3, but not 2. (though usually this only applies to 3 question papers). However, unfortunately none of us were able to replicate his legendary 10 0 2 8. We mostly decided that we thought Q1 $<$ Q3 $<$ Q2 due to our results (where $<$ denotes difficulty).

After we ate lunch, we went outside again to a park while discussing social resources, racism and trans communists, and also listening to Tarik's friend's playing of the harmonica. After getting back to the hotel, we went to a room to play some games. They did have other games in the room but we still had our packs of cards so we decided to play the *Game of our Glorious Leader* again, but this time Nanako joined us and two guides joined us, so we reset the game. *Again, no further information shall be provided about the Game of our Glorious Leader.* We then got kicked out of the room because other things would be happening.

Then, Teri, Aum, Jayden and Ben decided to do Q4 since it did not look completely impossible, so we first tried to show that there was a cycle through n cities, which Ben ended up proving, and then Teri proved that some inductions worked using some "extremely motivated" constructions, which Aum helped her to find, by stating what the induction would probably be. We did not end up finishing the entire problem, but the only part that was left was just making sure that a few special cases worked, which we could not be bothered to do. While this was happening, the leaders got our scripts at 18:00.

After dinner we again went back to our rooms, and then went back down to the lobby to discuss coordination with Jamie and Vesna at 23:10. We expected to get 10 9 9 8 10 10 for Q1, 10 10 0 8 1 1 for Q2, 7 9 9 7 1 10 for Q3, and that Teri might get 1-3 marks of partials for Q4.

7 Day 2: Coordination day

We met for breakfast at 8:45, and then left the hotel for a tour of Sarajevo at around 9:45. We took a tram to get to where the tour started (which was actually quite close to our original hotel), which weirdly stayed at the stop where we got on for around 5 minutes. Somehow during this, Teri left her phone on the tram, so Tarik managed to get one of the organisers to call the tram company, and get the driver to stop the tram and get the phone. Tarik then went to get the phone, and we got it back after about 2 hours, after the tour. During the tour we saw a symbol of each of the main religions present in Sarajevo, some of the old town, some of the centre and also the place where Franz Ferdinand was shot.

Figure 4: A photo taken at the aptly named "Pigeon Square"

Meanwhile when we were at the excursion, Jamie and Vesna were discussing with the coordinators to finalise the score. They sent the scores on Whatsapp after each problem are coordinated so we can keep track of our score.

For Q1, there were subtle issues addressed by Jamie the previous day that may have caused marks to be docked, including Kelvin forgetting to justify the construction on $2p$ (where p is an odd prime) actually being a permutation, but the coordinator luckily agreed that it was trivial enough for any deductions. Some others also risked a 1-mark deduction due to their flaw in the difference of two squares argument that may not handle $a_1 = 2$ well, but this was removed from the mark scheme so they didn't get penalised. At the end, all UK team members got 10/10 for Q1.

For Q2, we had 2 guaranteed solutions from Aum and Ben. Ben's solution was the closest to the official solution using Pascal's Theorem, which easily got 10/10. Aum used a trig bash approach after proving some cyclic quadrilaterals via an angle chase. Vesna wrote a lengthy neatened version of Aum's solution with detailed diagrams and the coordinators were looking through his solution

very seriously. But his solution was flawless so he got 10/10 as well.

Kelvin also had an attempt at solving the question using moving points, which simplifies the problem into checking 3 cases to show 2 projective transformations are equal. However, the proof was missing one line that could be quite easily proven through angle chasing. Initally, Vesna said a mark of 8 was expected to Kelvin, saying it was a minor flaw, but the coordinators thought that it was trivial enough so Kelvin somehow obtained 10 on Q2 despite having an incomplete, unintended solution. Others, such as Nanako and Teri, got 1 or 2 marks of partials based on proving the cyclic quadrilaterals needed for the proof.

For Q3, the main idea was to show that 0 was in the image of f, which most of us did in very different ways. Aum's solution turned out to not be completely correct however, as at some point he used f^n to mean two different things. However, since this was quite fixable the solution got 5/10, which was entirely unexpected.

For Q4, most of us didn't submit any meaningful scripts for it. Teri got a partial for Q4 though, because the leaders found some useful well-written observations within her scratch paper.

After that we added up our marks from the messages from our leaders so we knew our marks - 25, 30, 19, 28, 13 and 23.

On the way back from the tour, we started trying C8 from ISL 2017, since the team from Balkan 2024 tried it for 5 hours (apparently, around 10 minutes of which was in the basement). We did find a basement in our hotel, but it wasn't as cursed as it was last year so we just continued in one of our rooms rather than staying in the basement. During the tram ride back (which was around 100 *minutes*), we made approximately no progress on the question, other than drawing the diagrams for the first 3 cases.

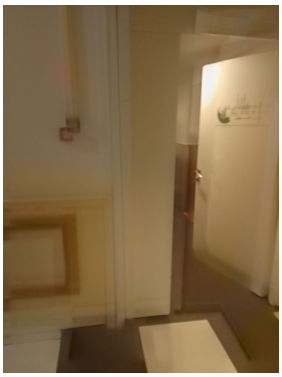

Figure 5: An extremely low quality photo of the basement

After going to the room, we made some conjectures about what was happening (including a multi-hour debate about whether the shape of the curve was a circle or two parabolas), and after around 4 hours of trying the problem, Teri found the first extremely important observation (which is not mentioned here as to not spoil the problem for the Balkan 2026 team if they read this report, also if you are reading please continue the tradition). Aum and Jayden wrote code for testing larger cases, which was useful for confirming this rather unmotivated observation. During this, we also got our Q3 results back - 5 10 9 8 2 10.

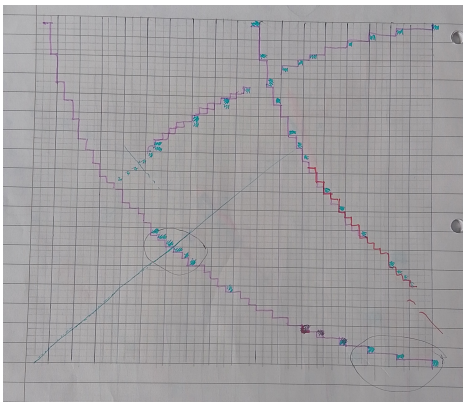

Figure 6: 3 failed attempts to draw the $n = 5$ case

We went to a 4^{th} hotel for dinner that day, where we met with Jamie and Vesna again, after they had a long day of coordination. Around half an hour before we left, Loek (a.k.a. *bulko*) from the Italian team told us on discord that his guide told him that the medal boundaries would be 13/23/33 (so we would get 4 silvers and 2 bronzes). However, when we were leaving he told us that the boundaries might not have been accurate, which was slightly concerning for us considering two of us were just on the boundary, but when we got to the other hotel, they told us that the boundaries were 11/23/33 instead (very close to the previously thought 13/23/33), and confirmed our medal results - Ben, Kelvin, Aum and Teri with silver and Jayden and Nanako with bronze.

8 Day 3 - Excursion and Closing Ceremony

We met for breakfast quite early, to get a coach that would bring us to Mostar, which took 300 *minutes* (that's $\sum_{n=1}^{\infty} \frac{12}{(\pi n)^2}$ hours if you can't be bothered to work out what a *minute* is), and we got in at around 10:20. During the coach ride, Aum also found a way to sum any rational function over the integers, and also a way to sum a few more types of functions, which turned out to be quite useful:

For any coprime polynomials P, Q such that $\deg Q \geq \deg P + 2$ and Q has no integer roots and all of its roots are distinct, if $\alpha_1, ...\alpha_r$ are the distinct roots of Q, $\sum_{n=-\infty}^{\infty} \frac{P(n)}{Q(n)} = \sum_{i=1}^{r} \frac{-\pi}{\tan(\pi\alpha_i)} \lim_{z\to\alpha_i}(\frac{P(z)}{Q(z)}(z-\alpha_i))$

First we were shown the tallest bell tower in Bosnia and Herzegovina, then part of the old town, a mosque, and a bridge that some of the people in the town are paid to jump from.

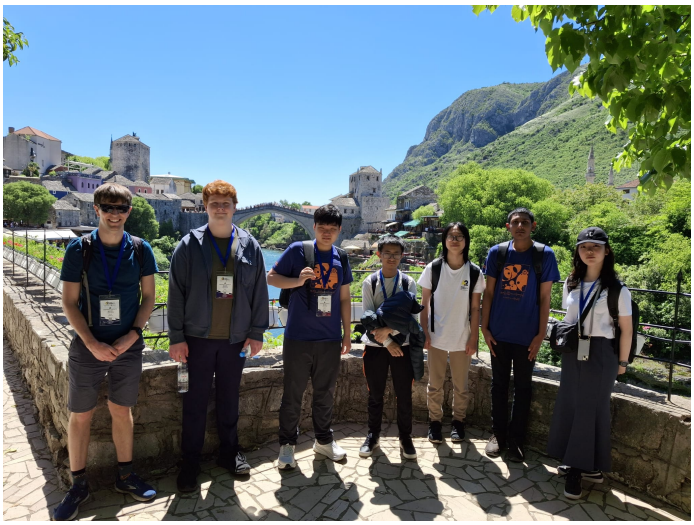

Figure 7: Group photo taken from an area near the mosque

During our free time after the tour (which ended at around 11:40), we decided to climb up the minaret of the tallest mosque in the town. We found the stairs quite thin, and for some of us it was slightly tiring by the end - but at the top we were able to see pretty much the entire town. After that, we went down, walked around a bit and ate lunch at a restaurant.

Since the organisers already ordered lunch for us, we were extremely confused why there was no menu and they just gave us the food. After lunch, we walked around some more and went down to the beach to watch the jump, which would happen at exactly 15:00. A bit before the jump, everyone at the olympiad went to the same area on the beach where we all displayed our flags and waved to the jumper.

The jump was about 2 seconds long, and the sound of the splash was quite loud, but after about 10 to 20 seconds, we saw him get out of the water onto the beach we were at. After this, we got ice cream and went back to the coach to travel back to Sarajevo. We arrived back at our hotel at 17:50, only around $\frac{3}{4}$ of an hour before we had to leave for the closing ceremony.

We left for the closing ceremony at 18:40, and got to hotel Hills only a little later, for the closing ceremony which would start at around 19:00. Ben and Nanako did bring more formal clothing for this, and the rest of us just wore the shirts again. During the closing ceremony, more people spoke, who on a scale of 1 to 1, we deem to be of $\frac{1}{4}(\sqrt{5}-1)+i\sqrt{\frac{5+\sqrt{5}}{8}}$ importance (slightly more than in the opening).

There was a flag at each table at the closing ceremony to tell us where to sit, but the flag that was printed was an incorrect version, as 3 of the 4 diagonal stripes were far too wide (which we saw due to our extensive deliberation of how to make sure we did not hold the flag in a way that would cause *distress*), and at this point we remembered that the flag shown on screen in the opening ceremony was the exact same incorrect version. Though thankfully our own flag was the correct version.

Figure 8: A horrifyingly incorrect rendition of the UK flag

We each went up to collect our medals, all at different times. None of us ended up being in *distress*, but Aum did end up holding the flag rotated 180 about the flag's intermediate axis. We were excited to see if there would be the 4 Uzbek golds that Loek jokingly "predicted", especially after the first one we saw, but unfortunately, there ended up only being 1 Uzbek gold. Romania won the competition, managing to score 60/60 on the first 3 questions, though not a single person in the entire competition managed to get a full solution for Q4 during the exam.

Figure 9: Aum holding the flag backwards but not being in distress

During the closing ceremony we also saw two performances of folk music and dance, and during the second one Teri and Aum were extremely confused as to what the time signature was - but at the end we figured out it was 9/8 but split into 2/2/2/3 rather than 3/3/3. We then had dinner in the same room, and took photos of us individually holding the flag with our medals in front of the competition banner, and also a photo of us all next to the competition banner. Initially, Ben was in *distress* even though the flag was the correct orientation as viewed in the photo, since the pole side of the flag was on the right rather than the left, so we ended up taking a photo the correct way as well.

Figure 10: Picture of us at the closing ceremony

9 Day 4: Returning to the UK

We brokefast[4] at 8:00, and we then took our luggage to the lobby at around 9:00, when we then took a taxi to the airport, early enough to be there before check-in opened. While waiting for check-in, Teri continues writing the report, very confused about how to create a table in LaTeX, and realising that she initially made an arithmetic mistake in the score analysis (claiming $30 - 19 = 21$). When check-in opened, we all go through (though with quite a lot of administrative issues) fast enough to have about half an hour after security. We then board the flight to Copenhagen at around 11:30 and depart at 12:06, slightly earlier than planned.

Originally, we had a 40 minute connection in Copenhagen to get back to Heathrow, which was quite tight, but now we had an extra 6 minutes to make the connection so we would most likely be able to make it. We got through extremely quickly (in around 10 minutes, so we got to the gate around 30 minutes before departure. The gate opened just 10 minutes after, and we went on the flight.

At the UK border control, Kelvin had to go through a seperate queue with Natalie as only his passport was not eligible for automatic entry; annoyingly for the rest of us, that queue was actually shorter. Teri did get rejected by the machine and had to go through an immigration officer, but overall there weren't major issues and we headed back home.

10 "Score analysis"

As is tradition for all maths olympiads, we created the 5^{th} degree polynomial that went through all of our scores for each question, so that we could extrapolate the scores that UNK0, UNK7, UNK8 and UNK9 would get.

Contestant code	Name	Q1	Q2	Q3	Q4	Total
UNK0	Dominic Yeo	10	−236	−58	−1	−285
UNK1	Aum Bagla	10	10	5	0	25
UNK2	Ben Fellows	10	10	10	0	30
UNK3	Jayden Joo	10	0	9	0	19
UNK4	Kelvin Lam	10	10	8	0	28
UNK5	Nanako Ueda	10	1	2	0	13
UNK6	Teri Xu	10	2	10	1	23
UNK7	Jamie Bell	10	247	110	6	373
UNK7.82475	Geoff Smith	10	[5]	[5]	[5]	1434
UNK8	Vesna Kadelburg	10	1312	474	21	1817
UNK9	Natalie Redman	10	4252	1403	56	5721

Taking the scores mod 41, Dominic got 2, Jamie got 4, Vesna got 13, and Natalie got 22. To calculate the scores, we could not be bothered to find the actual polynomials and instead just used metadifferences to extrapolate the values for each question (this gives the same result as finding the polynomial, but can only be used on integer values quite inefficiently). The 5th metadifference for our scores in Q2 was 137, which was coincidentally also our total score for the first 3 questions.

Since this year's Balkan Mathematical Olympiad was the 42^{nd}, it was clear that this number - 137 - was the answer to life, the universe and everything, which certainly fits the fact that this is the closest integer to the reciprocal of the fine structure constant. 137 also appears when we take Vesna's final score (mod the answer to life, universe and everything), as $137 \equiv 1817 \pmod{42}$. Many other things are also congruent to 137 mod 41, such as the the number of whole *fur/c* in $\frac{\pi}{3}$ *minutes*[6] being congruent to 137 mod 41, and $1 \times 3^7 \equiv 137$ (41).

11 Acknowledgements

We thank the organisers of the event, Ministry of Education of Sarajevo, and the Mathematical society of Sarajevo.

We thank the sponsors of the event, the Global Talent Fund and Jane Street, the latter also for sponsoring our team.

We thank our team leaders for their work with marking, coordination and accompanying us throughout the olympiad.

We thank Tarik for being a wonderful guide and showing us around Sarajevo.

We thank the coordinator who decided that the missing line in Kelvin's solution was trivial.

Jayden thanks Aum for teaching him -1 squareness the morning of the contest, without which he would likely have pursued other incorrect solutions for P1.

12 Quotes

"If you live longer you are wasting social resources"

"I don't want to waste social resources"

"Child labour decreases life expectancy, which wastes less social resources"

"-1 squareness is theory heavy"

"-1 is prime"

"how do you prove there are infinitely many primes"

"It looks easy so it must be misplaced"

"The Collatz Conjecture can't be that hard"

"wlog racism does not exist"

"I could probably beat Alex Chui in at least 1 of 5 SMCs"

13 Bilboigrapyh

[1] $\sum_{n=2}^{\infty} \frac{2}{n(n+1)}$ *minutes* is defined as exactly 4470777.5292293 *fur/c*. [2]

[2] where *fur/c* is defined as a British Furlong per speed of light.

[3] Distress causes and is caused by holding the flag backwards from the perspective of the pole, but not necessarily by holding the flag backwards from the perspective of the people.

[4] The past tense 1^{st} person plural conjugation of *to breakfast.*

[5] Extrapolating the polynomials for each question and plugging in 7.82475 to each would yield decimals too long to include in the table as there is not enough horizontal space on the page. Therefore finding the values here is left as an exercise for the reader.

[6] Without loss of generality, we can assume $\pi = 3$.